A CORNER FINDING ALGORITHM FOR IMAGE ANALYSIS AND REGISTRATION

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ABSTRACT

In our algorithm the gradient of θ (the gradient direction) in a grey level image is employed for detecting candidates for corner points. Two theorems show that this quantity attains a local maximum at a corner point. Two approaches, median filtering and Hough transform are used effectively to distinguish corner points from noise points. The corner points formed by our algorithm are used in interframe matching.

Keywords: Corner Detector, Corner Finder, Median Filter, Hough Transform, Correspondence Problem.

I INTRODUCTION

Several papers have mentioned the importance of corner detection in the analysis of time-varying imagery and stereo imagery. Moravec [77], Beaudet [78], Kitchen and Rosenfeld [80], and Yam and Davis [81] proposed different operators for detecting interest points or corner points directly from the grey values of images and those methods are employed by Barnard and Thompson [80], Dresehler and Nagel [81], and Yam and Davis [81] in disparity analysis of images and image registration.

Our corner detector is similar to that of Kitchen and Rosenfeld, but we put it on a solid mathematical basis and found effective ways to cope with the truncation errors in discretization and the noise in real-life digitized images.

II CORNER DETECTOR

We employ the gradient of θ, the gradient direction of the grey value function $f(x,y)$ as a corner detector.

$$\theta(x,y) \triangleq \arctan (f_y/f_x) \tag{2.1}$$

$$\begin{bmatrix} \theta_x \\ \theta_y \end{bmatrix} = \frac{1}{\rho^2} \begin{bmatrix} f_{xx} & f_{xy} \\ f_{xy} & f_{yy} \end{bmatrix} \begin{bmatrix} -f_y \\ f_x \end{bmatrix} \tag{2.2}$$

where ρ is the gradient magnitude

$$\rho^2 = f_x^2 + f_y^2 \tag{2.3}$$

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We use the local maxima of the product of the gradient magnitude of θ to be denoted by c and ρ to characterize corners. For a digitized image, we use the Sobel operator to approximate the derivatives.

Theorem 1

For an image containing a step edge along a curve defined by the equation

$$z(x,y) = 0 \tag{2.4}$$

if the greyvalues of this image are given by the function

$$f(x,y) = \int_{-\infty}^{z(x,y)} g(t)\, dt \tag{2.5}$$

where function $g(t)$ is nonnegative, first-order differentiable, absolutely integratable, unimodal and having a maximum at zero, and if the function $z(x,y)$ has second order derivatives, then the gradients of θ, the gradient direction of greyvalue function $f(x,y)$, can be represented in terms of the derivatives of the function $z(x,y)$ as follows:

$$\begin{bmatrix} \theta_x \\ \theta_y \end{bmatrix} = \frac{1}{z_x^2 + z_y^2} \begin{bmatrix} z_{xx} & z_{xy} \\ z_{xy} & z_{yy} \end{bmatrix} \begin{bmatrix} -z_y \\ z_x \end{bmatrix} \tag{2.6}$$

Note: A simple example of the function $g(t)$ is

$$g(t) = \frac{1}{\sqrt{2\pi}\sigma} e^{-\frac{t^2}{2\sigma^2}} \tag{2.7}$$

Corollary

For an image having a step edge along a straight line defined as in Theorem 1, the gradients of θ in the neighboring area of the edge are always equal to zero.

Theorem 2

We approximate the tip of an ideal corner bounded by two straight edges by a continuous curve. For simplicity, we assume the corner tip is at (0,0) and the two edges of the corner are symmetrical with respect to the y-axis (see Figure 1). Then, we can express the curve by

$$z(x,y) = by + p(x) \qquad -h \le x \le h$$

$$z(x,y) = ax + by \qquad x < -h$$

$$z(x,y) = -ax + by \qquad x > h$$

where p(x) is even and has a second order derivative which reaches a minimum at 0, and makes z(x,y) approximate the original corner within a distance in the range $-h \le x \le h$. If the approximate corner curve $z(x,y) = 0$ satisfies the above condition, then the maximum of the value of ρc of the grey-value corner

$$f(x,y) = \int_{-\infty}^{z(x,y)} g(t)\, dt$$

in the vicinity of the corner point is larger than

$$\frac{1}{h} \cos\left(\frac{\alpha}{2} \right) g(0)$$

where $0 < \alpha < \pi$ is the angle of the ideal corner and g(x) is the same as given in Theorem 1.

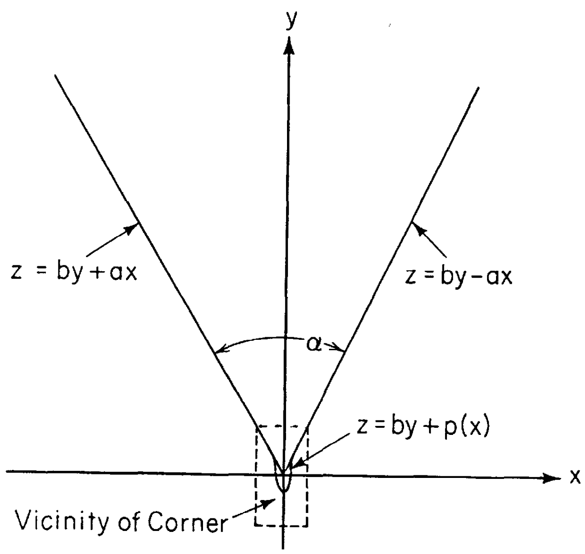

Figure 1

Corner and its approximation.

III DISTINGUISHING CORNER POINTS FROM NOISE POINTS

Because of noise and discretization errors, the candidates obtained by our algorithm contain noise points as well as corner points. We tried two approaches to pick out the corner points from the candidates.

A. Median Filtering

The first approach is applying median filter in a window centered at a candidate, then calculating the values of c again to check if there is still a corner point near the original point. We use separable median filters for preserving edges and corners better and saving time.

The merit of the median filter is that it can eliminate isolated, irregular noise points such as spots or highlights on edges. These noise points always yield an abnormal value for the corner detector. But the median filter approach has difficulty in eliminating false corners which are near edges. This kind of false corner point occurs due to the truncation errors in discretization. See the upper right part of Figure 2.

B. Hough Transform

Using Hough transform in a window centered at a candidate not only differentiates between the corner points and the noise points, but also acquires more information of the corner, such as the number of edges which form the corner, the slope of each edge and the angle between edges. We made some modifications in the O'Gorman and Clowes [76]' method. The first modification is using thinned gradient magnitude in place of the original one. We employed Cheng's [80] method for thinning edges. The second modification is considering more values of the angle θ in the Hough space than only one value as in the O'Gorman's method. We tried four Hough Transforms:

J: the O'Gorman + Clowes' method with Both modifications

I: the O'Groman + Clowes' method with modification 1

H: the O'Gorman + Clowes' method with modification 2

K: the original O'Gorman and Clowes' method

More precisely, if we denote the gradient magnitude and direction at the point (x_i,y_i) by W_{ij} and θ_{ij} respectively, and use the variables r and ψ as the distance and the angle in the equation of a straight line

$$x \cos \psi + y \sin \psi = r$$

then we have the following (FORTRAN-like) expressions for calculating the values of Hough transforms J, I, H and K:

$$J: \quad h(r,\psi) = h(r,\psi) + e^{-|\psi-\theta_{ij}|/10},$$

if (x_i,y_i) is a skeletal pixel after thinning in a map of gradient magnitude.

$$I: \quad h(r,\psi) = h(r,\psi) + 1,$$

if (x_i,y_i) is a skeletal pixel and $\psi = \theta_{ij}$.

$$H: \quad h(r,\psi) = h(r,\psi) + W_{ij}\, e^{-|\psi-\theta_{ij}|/10}$$

if W_{ij} is above the threshold.

$$K: \quad h(r,\psi) = h(r,\psi) + W_{ij}$$

if W_{ij} is above the threshold and $\psi = \theta_{ij}$.

Note: in Hough transforms I, K, each point in an image space only maps to one point in the Hough space (r,ψ), where $\psi = \theta_{ij}$, while in Hough transforms J and H, we take the points near θ_{ij} into account by including the factor

$$e^{-|\psi-\theta_{ij}|/10}.$$

IV EXPERIMENTS AND CONCLUSIONS

Images for tests Three pairs of images: polyhedra, model planes and robot arms were used. The procedure consisted of three steps: (i) Detection of candidates for corner points, (ii) Distinguishing corner points from noise points, (iii) Matching between the corner points in two frames. The first two pairs are stereo images, whereas the last one is of moving objects.

Results of the experiments The candidates found by the corner detection algorithm include most real corner points. However, half of the candidates are noise points. Both of the two methods for distinguishing corner points from noise points worked well. They eliminated 90-95% of the noise points. Matching between the corner points in the two frames was successful. (We used Ranade and Rosenfeld's [80] method for matching.) See Figures 2 and 3.

Note: In Figure 2, □ and x represent, respectively, the corner points and the noise points recognized by the step 2, distinguishing corner points from noise points.

Comparison between Median Filter and Hough Transform By carefully checking the results of our experiments, we found that the Hough transform approach worked better in most cases than the median filter approach. The median filter approach has difficulty with near-edge error points (see the upper right of Fig. 2). The Hough transform approach is sensitive to line-shaped noise as occurred at the wrist of the robot (see Figure 2). This kind of noise is caused by the saturation of the digital camera. One shortcoming of the Hough transform approach (compared with the median filtering approach) is that it is more time-consuming.

Parallel processing All formulas employed in our algorithm can be evaluated parallelly and the basic calculation in these formulas is simple, so that it is fitted for multi-processors.

Limitation of the algorithm This algorithm has been tested only with indoor scenes containing man-made objects. Because of the sensitivity of the algorithm to noise and other errors, one may have trouble in applying it to pictures of natural scenes or complicated objects.

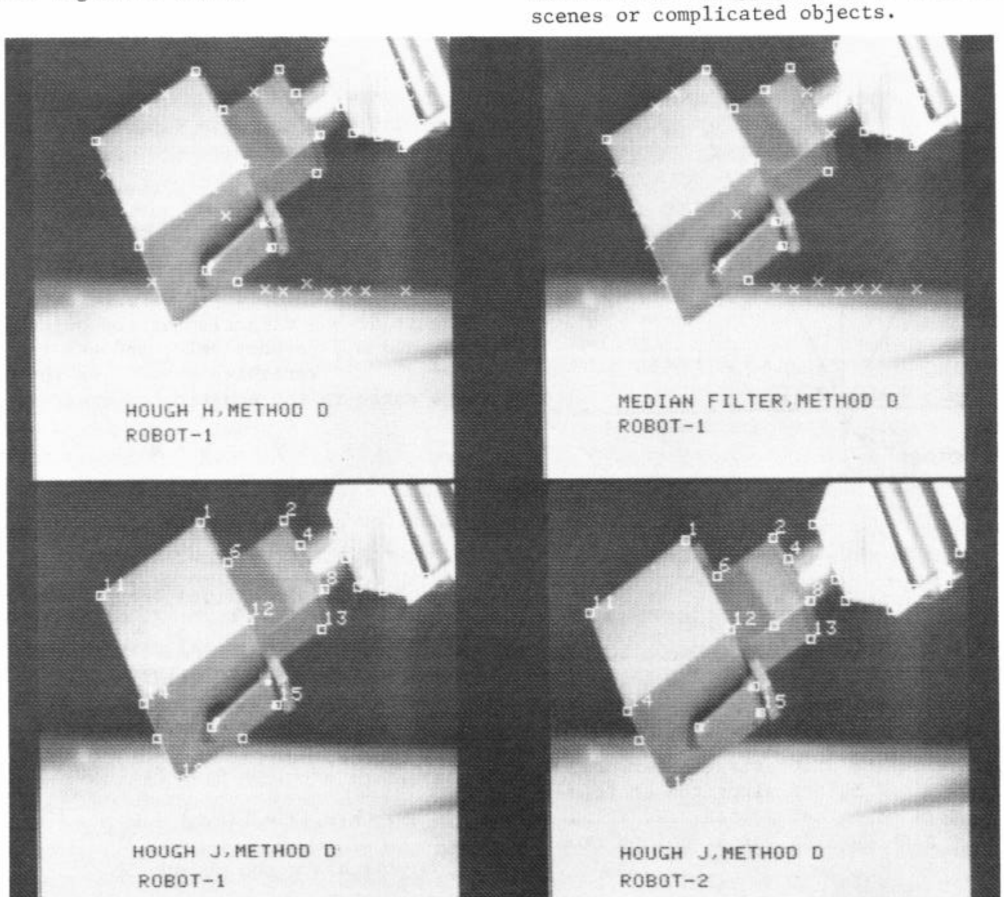

Figure 2. Upper: results of the different approaches to corner finding. □ denotes a corner point, x represents a noise point. Lower: results of the point matching between two frames of a robot arm (the robot arm moved), corresponding points are represented by the same number.

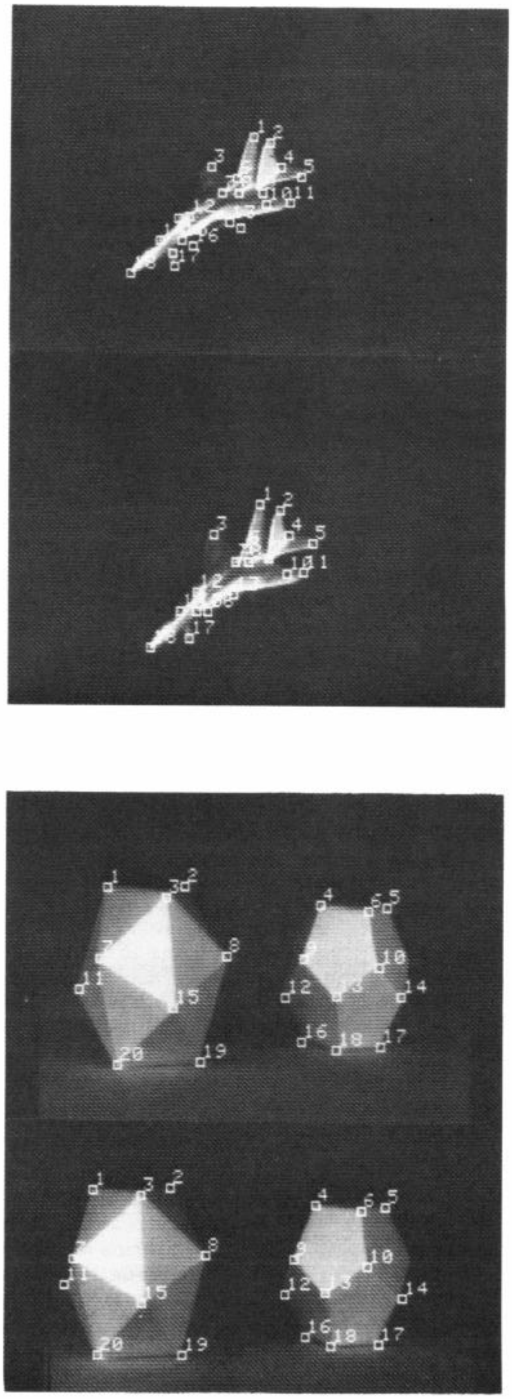

Figure 3. Results of point matching between the stereo pair. Upper part: correspondence between the left and right image of a model plane, corner points found by the median filter; Lower part: correspondence between the left and right image of polyhedra, corner points found by Hough transform J.

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