A SYSTEMATIC APPROACH TO CONTINUOUS GRAPH LABELING WITH APPLICATION TO COMPUTER VISION*

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ABSTRACT

The discrete and continuous graph labeling problem are discussed. A basis for the continuous graph labeling problem is presented, in which an explicit connection between the discrete and continuous problems is made. The need for this basis is argued by noting conditions which must be satisfied before solutions can be pursued in a formal manner. Several cooperative solution algorithms based on the proposed formulation and results of the application of these algorithms to the problem of extracting line drawings are presented.

I THE CONTINUOUS GRAPH LABELING PROBLEM

A graph labeling problem is one in which a unique label, λ from a set Λ of possible labels must be assigned to each vertex of a graph $G = (V,E)$. The assignment must be performed given information about the relationship between labels on adjacent vertices and incomplete local information about the correct label at each vertex. In a *discrete* graph labeling problem [1,2,3], the local information consists of a subset, $\Lambda_i \subseteq \Lambda$, of the label set associated with vertex $v_i \in V$, from which the correct label for each vertex must be chosen. The contextual information consists of binary relations $R_{ij} \subseteq \Lambda\times\Lambda$, referred to as *constraint relations*, assigned to each edge $v_i v_j \in E$. The function of the constraint relations is to make explicit which labels can co-occur on adjacent vertices. The graph, label set, and constraint relations together form a *constraint network* [2,5]. An (unambiguous) *labeling* is a mapping which assigns a unique label $\lambda \in \Lambda$ to each vertex of the graph. A labeling is *consistent* if none of the constraint relations is violated, that is, if label λ is assigned to vertex v_i and label λ' is assigned to vertex v_j then the pair (λ,λ') is in the constraint relation R_{ij} for the edge $v_i v_j \in E$.

Given initial labeling information, several search techniques have been developed which can be used to derive consistent labelings. The original backtracking search described by Waltz [1] was later implemented in parallel by Rosenfeld et al. [6], resulting in the discrete relaxation operator. At the same time a continuous analogue, the continuous graph labeling problem was proposed, as well as a continuous relaxation algorithm for its solution, and since then several other relaxation algorithms have been proposed [7,8].

In a continuous graph labeling problem, the initial information consists of strength measures or figures of merit, $p_i(\lambda_j)$, given for each label $\lambda_j \in \Lambda$ on each vertex $v_i \in V$. The strength measures are assumed generated by feature detectors which are making observations in the presence of noise. They usually take on values in the range [0,1], a 0 indicating no response, and a 1 indicating a strong response. The contextual information, which is represented in terms of constraint relations for the discrete graph labeling problem, are replace by measures of compatibility, usually taking values in the range [-1,1] or [0,1], which serve to indicate how likely the pairs of labels are to co-occur on adjacent vertices.

Several problems have resulted in the extension of the graph labeling problem from the discrete to continuous case. In the discrete case the presence or absence of a pair in a constraint relation can be determined with certainty depending on what labelings are to be considered consistent. In the continuous case, however, there is apparently no formal means to assign specific numeric values to the compatibility coefficients, particularly for shades of compatibility between "impossible" and "very likely", although several heuristic techniques have been proposed [7,9,10]. Furthermore, with respect to a constraint network, the concept of consistency is well defined. The objective the continuous relaxation labeling processes has often been stated to be that of improving consistency, however, the definition for consistency has not been given explicitly. This latter issue is circumvented in several of the optimization approaches which have been proposed [11,12,13], where the an objective function, defined in terms of the compatibility coefficients and the initial strength measures is given. However, because of the dependence of the objective functions on the compatibility coefficients, and because no real understanding of the role which these coefficients play yet exists, it is often difficult to describe the significance of these approaches in terms of what is being achieved in solving the problem.

II A REFORMULATION OF THE CONTINUOUS GRAPH LABELING PROBLEM

In an alternate approach to the continuous graph labeling problem [14] an attempt has been made to maintain the characteristics of the original problem while allowing for more systematic approaches toward a

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solution. It is felt that solutions to the reformulated problem will be more useful because it will be easier to relate the results of the solution algorithm to what is being achieved in the problem domain. In order to develop this approach, we review the characteristics of the solutions to the graph labeling problem which have appeared so far (refer to Fig. 1).

The inputs to the process are the initial strength measures $\{p_i^0(\lambda_j): i=1,\ldots,n,\ j=1,\ldots,m\}$ which can be represented by an $n \times m$ dimensional vector:

$$\bar{p} = (p_1^0(\lambda_1), p_1^0(\lambda_2), \ldots, p_n^0(\lambda_m)) \in R^{nm}.$$

Since the selection of a particular label at a given vertex is related to the label selections made at other (not necessarily adjacent) vertices, information about the label selection at that vertex is contained in the initial labeling values distributed over the extent of the network. The function of the global enhancement process, g, is to accumulate this evidence into the labeling values at the given vertex. The output vector:

$$\bar{q} = (q_1(\lambda_1), q_1(\lambda_2), \ldots, q_n(\lambda_m))$$

is used by a process of local maxima selection [15], s, to choose a labeling:

$$\bar{\lambda} = (\lambda_1, \lambda_2, \ldots, \lambda_n),$$

where λ_i is the label assigned to vertex v_i. Thus g is a function, $g: R^{nm} \to R^{nm}$ and s is a function is $s: R^{nm} \to \Sigma_n(\Lambda)$, where $\Sigma_n(\Lambda)$ is the set of possible labelings. The hope is that labeling resulting from the process $s(e(\bar{p}))$ is an improvement over the labeling resulting from direct local maxima selection $s(\bar{p})$.

If a numerical solution is to be sought for this problem, then a formal definition must be given to the concept of an improved labeling. In previous work, particularly with respect to computer vision, improvements were rated subjectively, or in the case of an experiment where the solution was known in advance, by the number of misclassified vertices. In our formulation this issue is resolved by assuming that the problem domain specifies an underlying constraint network, or can be modeled to do so. The objective is then to use the initial information to choose a labeling that is (a) consistent with respect to this constraint network, and (b) which optimizes a prespecified objective function. In this extension from the discrete to the continuous graph labeling problem, the constraint relations remain intact.

We are currently investigating optimal solutions to this formulation of the graph labeling problem based on a ***maximum-sum*** decision rule, that is, the rule is to choose a consistent labeling such that sum of the initial labeling values is maximal. A solution to this problem could be extended in a straightforward manner to certain well established decision rules such as is found, for example, in nearest neighbor classification.*

Though the decision rule serves to make explicit what is meant by an improved labeling, it is defined globally. The problem remains to implement it in terms of a cooperative process. The concept of a cooperative process, although not well defined, can be characterized in terms certain general properties [Ul179]. Our research is into algorithms which exhibit certain of these properties, such as locality, and simplicity. In an ***optimal*** solution, the labeling algorithm must, furthermore, perform the label selection in accordance with the given decision rule. Other important issues, such as speed of convergence are also being addressed. Two approaches which have some of these properties are demonstrated in the following section. The first is an heuristic approach based on dynamic programming [14] which converges very rapidly and with good results, but does not guarantee a consistent labeling. The second approach is based on linear programming. Details on the latter algorithm will be presented at a later date.

III EXPERIMENTAL RESULTS

In this section, we demonstrate the application of the two approaches discussed above to the problem of extracting polygon approximations of the outlines of objects in a scene. The experiments described here are based on the reconstruction of simple closed curves (Fig. 2) when noise has been added to the initial labeling values.

The graph used in this experiment is a 16 by 16 raster. Each vertex is represented by a pixel, and is adjacent to its eight immediate neighbors. The associated label set is shown in Fig. 3. A pair of labels on adjacent pixels are consistent if an outgoing line segment is not broken across a common border or corner, and inconsistent otherwise. Examples of consistent pairs of labels are given in Fig. 4, and examples of inconsistent pairs of labels are given in Fig. 5.

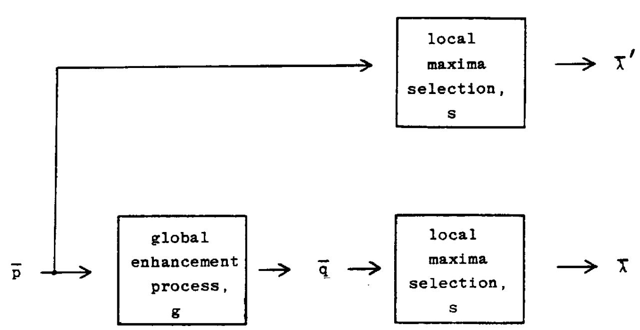

Fig. 1: Function of the global enhancement process: $\bar{\lambda}$ represents an improved labeling with respect to $\bar{\lambda}'$.

*In terms of decision theory, every consistent labeling constitutes a class and the input vector $\bar{p}$ is a point in an $n \times m$ dimensional feature space.

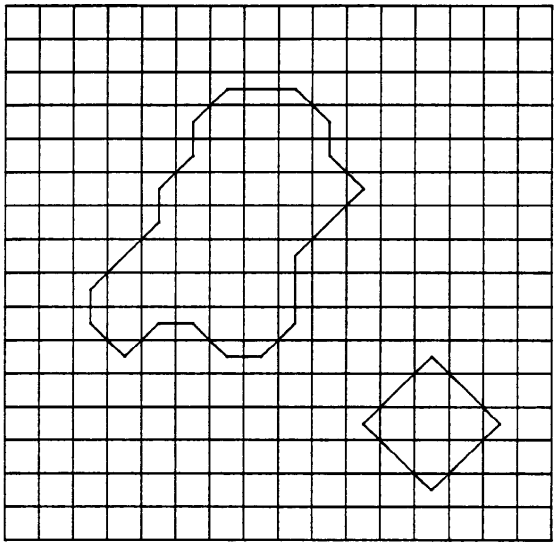

Fig. 2: Initial labeling.

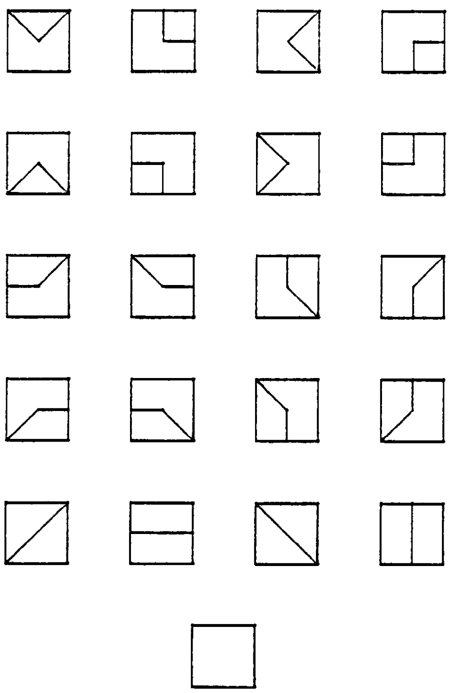

Fig. 3: Label set for line drawing description.

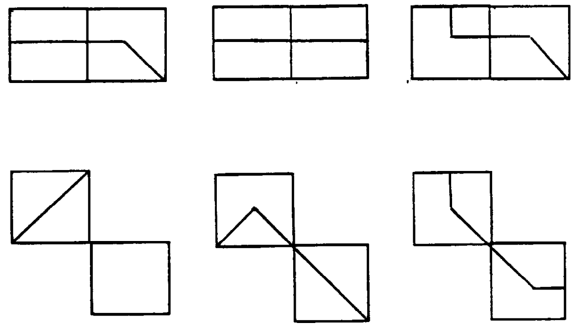

Fig. 4: Examples of locally consistent label pairs.

Uniformly independently distributed noise was added to the labeling values at each pixel resulting in the labeling, by local maxima selection, shown in Fig. 6. The two cooperative algorithms were applied to the initial labeling in attempt to reconstruct the original curves. The first is the dynamic programming approach with data moving along the eight major directions of the raster (two horizontal, two vertical, and four diagonal). The second is the algorithm based on a linear programming approach. The performance of these algorithms are presented in Fig. 7 and Fig. 8, which show the resulting labeling (by choosing the label with greatest strength at each pixel) after 2 and 4 iterations. The dynamic programming approach reaches a fixed point after 2 iterations, however, the result is not a consistent labeling. The linear programming algorithm reconstructs the original labeling after six iterations.

IV DISCUSSION

Our interest here has been to restate the continuous graph labeling problem in a manner which allows for a systematic approachs to a solution. The formulation which we have presented amounts to the classification of

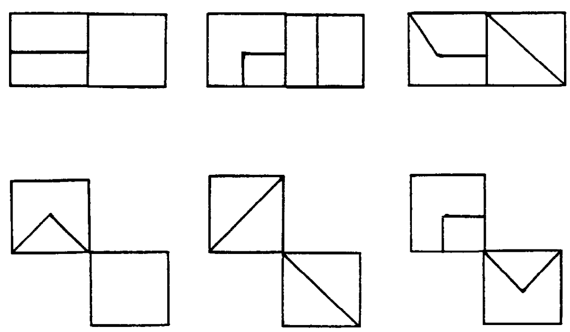

Fig. 5: Examples of inconsistent label pairs.

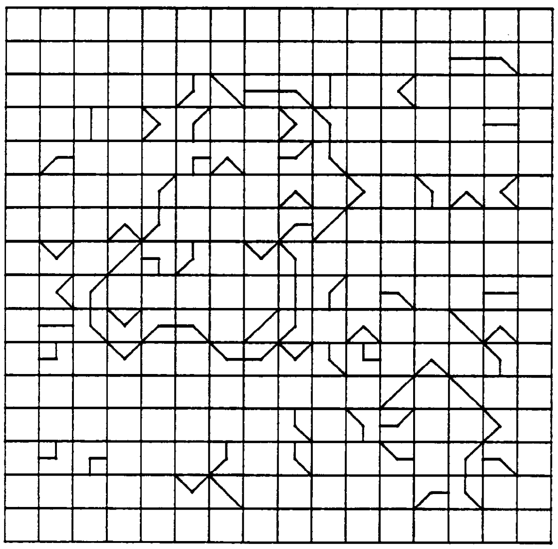

Fig. 6: Initial labeling plus noise.

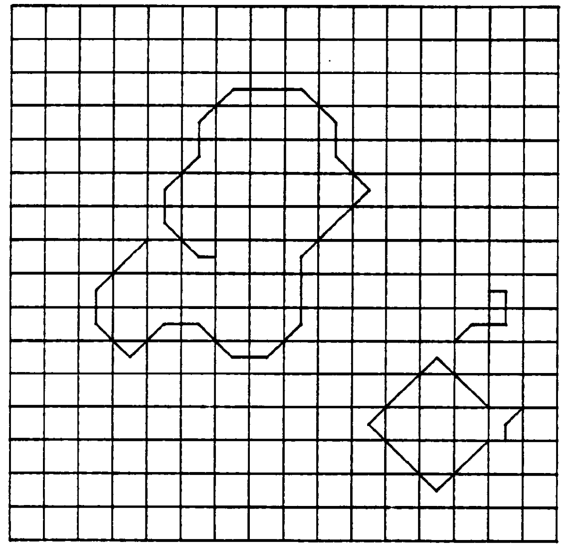

Fig. 8a: Output of the linear programming algorithm after two iterations.

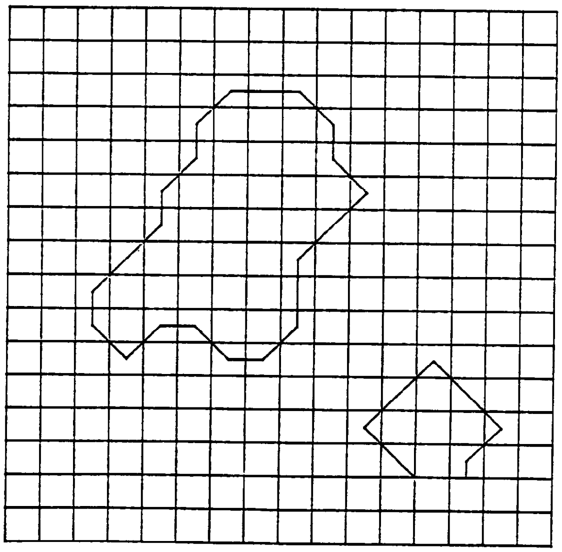

Fig. 7: Output of the dynamic programming algorithm after two iterations. Note: the algorithm has reached a fixed point.

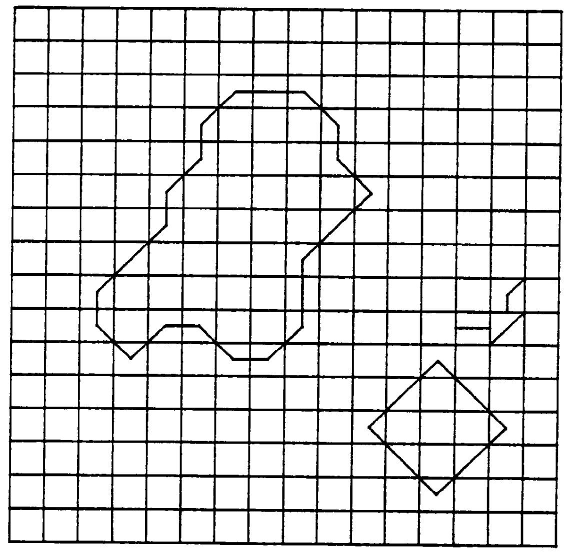

Fig. 8b: Output of the linear programming algorithm after four iterations.

consistent labelings according to a prespecified decision rule. As with previous approaches, consistency is defined on a local basis to make sense with respect to a particular problem. For example, if the objective is to extract continuous curves as in the experiment described above, consistency is maintained between pairs of labels when the scene events they represent do not allow for broken lines. The global nature of the decision rule leads to a more intuitive description of what the techniques accomplishes with respect to the original problem. However, as a consequence, the problem of implementing this rule on a local basis arises.

Two approaches to the reformulated problem have been demonstrated above. Our present feeling is that a linear programming approach should yield an optimal solution to the continuous graph labeling problem based on a maximum-sum decision rule. However, the restriction that the algorithm must be implemented in a local manner has led to some theoretical problems, such as resolving cycling under degeneracy which remain to be solved. Our investigation into these problems is continuing. Obviously, the value of this approach and any techniques which may be derived from it will depend on whether or not real world applications can be modeled in such a manner so that the absolute consistency between pairs of labels is meaningful. We hope to demonstrate this in at least one problem, deriving line drawings from real world scenes, in forthcoming results.

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