CIRCUMSCRIPTION IMPLIES PREDICATE COMPLETION (SOMETIMES)

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ABSTRACT

Predicate completion is an approach to closed world reasoning which assumes that the given sufficient conditions on a predicate are also necessary. Circumscription is a formal device characterizing minimal reasoning i.e. reasoning in minimal models, and is realized by an axiom schema. The basic result of this paper is that for first order theories which are Horn in a predicate P, the circumscription of P logically implies P's completion axiom.

Predicate completion [Clark 1978, Kowalski 1978] is a device for "closing off" a first order representation. This concept stems from the observation that frequently a world description provides sufficient, but not necessary, conditions on one or more of its predicates and hence is an incomplete description of that world. In reasoning about such worlds, one often appeals to a convention of common sense reasoning which sanctions the assumption - the so-called closed world assumption [Reiter 1978] - that the information given about a certain predicate is all and only the relevant information about that predicate. Clark interprets this assumption formally as the assumption that the sufficient conditions on the predicate, which are explicitly given by the world description, are also necessary. The idea is best illustrated by an example, so consider the following simple blocks world description:

A and B are distinct blocks.
A is on the table. (1)
B is on A.

These statements translate naturally into the following first order theory with equality, assuming the availability of general knowledge to the effect that blocks cannot be tables:

$$\begin{array}{lll} BLOCK(A) & BLOCK(B) & \\ ON(A,TABLE) & ON(B,A) & \\ A \neq B & A \neq TABLE & B \neq TABLE \end{array} \qquad (2)$$

Notice that we cannot, from (2), prove that nothing is on B, i.e., (2) $\not\vdash (x) \sim ON(x,B)$, yet there is a common sense convention about the description (1) which should admit this conclusion. This convention holds that, roughly speaking, (1) is a description of all and only the relevant information about this world. To see how Clark understands this convention, consider the formulae

$$(x).x = A \vee x = B \supset BLOCK(x) \qquad (3)$$
$$(xy).x = A \;\&\; y = TABLE \vee x = B \;\&\; y = A \supset ON(x,y)$$

which are equivalent, respectively, to the facts about the predicate BLOCK, and the predicate ON in (2). These can be read as "if halves", or sufficient conditions, of the predicates BLOCK and ON. Clark identifies the closed world assumption with the assumption that these sufficient conditions are also necessary. This assumption can be made explicit by augmenting the representation (2) by the "only if halves" or necessary conditions, of BLOCK and ON:

$$(x).\ BLOCK(x) \supset x=A \vee x=B$$
$$(xy).\ ON(x,y) \supset x=A \;\&\; y=TABLE \vee x=B \;\&\; y=A$$

Clark refers to these "only if" formulae as the completions of the predicates BLOCK and ON respectively. It now follows that the first order representation (1) under the closed world assumption is

$$(x).\ BLOCK(x) \equiv x=A \vee x=B$$
$$(xy).\ ON(x,y) \equiv x=A \;\&\; y=TABLE \vee x=B \;\&\; y=A$$
$$A \neq B \quad A \neq TABLE \quad B \neq TABLE$$

From this theory we can prove that nothing is on B - $(x) \sim ON(x,B)$ - a fact which was not derivable from the original theory (2).

Circumscription [McCarthy 1980] is a different approach to the problem of "closing off" a first order representation. McCarthy's intuitions about the closed world assumption are essentially semantic. For him, those statements derivable from a first order theory T under the closed world assumption about a predicate P are just the statements true in all models of T which are minimal with respect to P. Roughly speaking, these are models in which P's extension is minimal. McCarthy forces the consideration of only such models by augmenting T with the following axiom schema, called the circumscription of P in T:

$$T(\phi) \;\&\; [(x).\phi(x) \supset P(x)] \supset (x).\ P(x) \supset \phi(x)$$

Here, if P is an n-ary predicate, then ϕ is an n-ary predicate parameter. $T(\phi)$ is the conjunction of the formulae of T with each occurrence of P replaced by ϕ. Reasoning about the theory T under the closed world assumption about P is formally identified with first order deductions from the theory T together with this axiom schema. This enlarged theory, denoted by $CLOSURE_P(T)$, is called the closure of T with respect to P. Typically, the way this schema is used is to "guess" a suitable instance of ϕ, one which permits the derivation of something useful.

To see how this all works in practice, consider the blocks world theory (2), which we shall denote by T. To close T with repect to ON, augment T with the circumscription schema

$$[T(\phi) \ \& \ (xy).\phi(x,y) \supset ON(x,y)] \supset (xy).\ ON(x,y) \supset \phi(x,y) \quad (4)$$

Here ϕ is a 2-place predicate parameter. Intuitively, this schema says that if ϕ is a predicate satisfying the same axioms in T as does ON, and if ϕ's extension is a subset of ON's, then ON's extension is a subset of ϕ's, i.e., ON has the minimal extension of all predicates satisfying the same axioms as ON.

To see how one might reason with the theory $CLOSURE_{ON}(T)$, consider the following choice of the parameter ϕ in the schema (4):

$$\phi(x,y) \equiv x=A \ \& \ y=TABLE \lor x=B \ \& \ y=A \quad (5)$$

Then $T(\phi)$ is

$$BLOCK(A) \ \& \ BLOCK(B) \ \& \ [A=A \ \& \ TABLE=TABLE \lor A=B \ \& \ TABLE=A] \ \& \ [B=A \ \& \ A=TABLE \lor B=B \ \& \ A=A] \ \& \ A \neq B \ \& \ A \neq TABLE \ \& \ B \neq TABLE$$

so that, for this choice of ϕ,

$$CLOSURE_{ON}(T) \vdash T(\phi)$$

It is also easy to see that, for this choice of ϕ

$$CLOSURE_{ON}(T) \vdash (xy).\phi(x,y) \supset ON(x,y).$$

Thus, the antecedent of (4) is provable, whence

$$CLOSURE_{ON}(T) \vdash (xy).ON(x,y) \supset \phi(x,y).$$

i.e.

$$CLOSURE_{ON}(T) \vdash (xy).ON(x,y) \supset x=A \ \& \ y=TABLE \lor x=B \ \& \ y=A \quad (6)$$

i.e. the only instances of ON are (A,TABLE) and (B,A). It is now a simple matter to show that nothing is on B, i.e.

$$CLOSURE_{ON}(T) \vdash (x).\sim ON(x,B)$$

a fact which is not derivable from the original theory T.

Notice that in order to make this work, a judicious choice of the predicate parameter ϕ, namely (5), was required. Notice also that this choice of ϕ is precisely the antecedent of the "if half" (3) of ON and that, by (6), the "only if half" - the completion of ON - is derivable from the closure of T with respect to ON. For this example, circumscription is at least as powerful as predicate completion.

In fact, this example is an instance of a large class of first order theories for which circumscription implies predicate completion. Let T be a first order theory in clausal form (so that existential quantifiers have been eliminated in favour of Skolem functions, all variables are universally quantified, and each formula of T is a disjunct of literals). If P is a predicate symbol occurring in some clause of T, then T is said to be Horn in P iff every clause of T contains at most one positive literal in the predicate P. Notice that the definition allows any number of positive literals in the clauses of T so long as their predicates are distinct from P. Any such theory T may be partitioned into two disjoint sets

T_P: those clauses of T containing exactly one positive literal in P, and

$T-T_P$: those clauses of T containing no positive (but possibly negative) literals in P.

Clark (1978) provides a simple effective procedure for transforming a set of clauses of the form T_P into a single, logically equivalent formula of the form $(x).\ A(x) \supset P(x)$. The converse of this formula, namely $(x).\ P(x) \supset A(x)$, Clark calls the completion axiom for the predicate P, and he argues that augmenting T with P's completion axiom is the appropriate formalization of the notion of "closing off" a theory with respect to P. Our basic result relates this notion of closure with McCarthy's, as follows:

Theorem

Let T be a first order theory in clausal form, Horn in the predicate P.Let $(x).P(x) \supset A(x)$ be P's completion axiom. Then

$$CLOSURE_P(T) \vdash (x).\ P(x) \supset A(x)$$

i.e. P's completion axiom is derivable by circumscription.

Discussion

Circumscription and predicate completion are two seemingly different approaches to the formalization of certain forms of common sense reasoning, a problem which has recently become of major concern in Artificial Intelligence (see e.g. [AI 1980]). That circumscription subsumes predicate completion for a wide class of first order theories is thus of some theoretical interest.

Moreover, circumscription is a new formalism, one whose properties are little understood. Predicate completion on the other hand, has a solid intuitive foundation, namely, assume that the given sufficient conditions on a predicate are also necessary. The fact that predicate completion is at least sometimes implied by circumscription lends support to the hypothesis that circumscription is an appropriate formalization of the notion of closing off a first order representation.

Finally, the theorem has computational import. Notice that in order to reason with McCarthy's circumscription schema it is first necessary to determine a suitable instance of the predicate parameter ϕ. <u>This is the central computational problem with circumscription</u>. Without a mechanism for determining "good ϕ's", one cannot feasibly use the circumscription schema in reasoning about the closure of a representation. This problem of determining useful ϕ's is very like that of determining suitable predicates on which to perform induction in, say, number theory. Number theory provides an induction axiom schema, but no rules for instantiating this schema in order to derive interesting theorems. In this respect, the circumscription schema acts like an induction schema.

Now the above theorem provides a useful heuristic for computing with the closure of first order Horn theories. For we know a priori, without having to guess a ϕ at all, at least one non trivial consequence of the circumscription schema, namely the completion axiom. Clearly, one should first try reasoning with this axiom before invoking the full power of circumscription by "guessing ϕ's".

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