

MODELING HUMAN KNOWLEDGE OF ROUTES:  
PARTIAL KNOWLEDGE AND INDIVIDUAL VARIATION

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**Abstract**

Commonsense knowledge of large-scale space (the *cognitive map*) includes several different types of knowledge: of sensorimotor, topological, and metrical spatial relationships. Sensorimotor knowledge is defined as that knowledge which is necessary to reconstruct a route from memory after travel along that route in a large-scale environment. A representation for route knowledge is proposed with sufficiently robust performance properties to be useful as commonsense knowledge. Its states of partial knowledge are shown to correspond to those observed in humans. We also define and explore the space of all possible variants of this representation, to derive empirical predictions about the nature of individual variation.

**1. Introduction**

This paper presents part of a computational theory of spatial knowledge. We focus our attention on knowledge of *large-scale* space: space whose structure cannot be perceived from a single vantage point, and which is learned by integrating local observations gathered over time. There are three major categories of spatial knowledge:

**Sensorimotor Procedures:** Knowledge of a set of actions, and their sequence, required to travel from one place to another.

**Topological Relations:** Knowledge of non-metrical properties of the external environment, such as containment, connectivity, and order.

**Metrical Relations:** Knowledge of, and the ability to manipulate, magnitudes such as distance, direction, and relative position.

This paper concentrates on spatial knowledge of the first type. The other types of knowledge in the cognitive map also exhibit interesting behavior and structure, but they are discussed elsewhere [Kuipers 1978, 1982].

The structure of the representation is motivated by empirical observations of the states of partial knowledge exhibited by human spatial knowledge. Knowledge of large-scale space is an attractive, accessible domain for the study of knowledge representations because changes caused by acquiring and assimilating new observations must take place slowly, constrained by the speed of physical travel. The intermediate states of

knowledge are particularly long-lived and visible, compared to more rapid cognitive processes such as vision and language understanding.

**2. The Problem: Sensorimotor Knowledge of Space**

The most fundamental information processing problem solved by the cognitive map is to store a description of a route travelled in the environment so that it can be reconstructed later. Even with this apparently simple kind of spatial knowledge, there are interesting states of partial knowledge that reveal the structure of the representation. One of the most interesting is indicated by the familiar response, "I could take you there, but I can't tell you how!"

Physical presence in the environment makes an important difference to the way information can be retrieved. Lynch (1960) observed styles of navigation that depend on the environment to evoke the next action. Similarly, in a study of experienced taxi drivers, Chase (1982) found that routes selected while travelling in the environment were better than routes selected during a laboratory interview.

Another type of partial knowledge is the asymmetrical storage of apparently symmetrical spatial relationships. Piaget, Inhelder, and Szeminska (1960) observed that young children are frequently able to follow a route correctly from beginning to end, but are unable to travel the same route in reverse, or to start it in the middle. Hazen, Lockman, and Pick (1978) studied this effect in detail, and found that their young subjects were able to travel a well-learned route in reverse, but while they could anticipate up-coming landmarks in the original direction, they could not do so in the reverse direction.

We can formalize the "I could take you there, but I can't tell you how!" effect by observing that the human knowledge representation is able to express a state of knowledge capable of solving problem 1 but not problem 2, for the same route.

**Problem 1:** Assimilate knowledge of a route by travel in the environment, then reconstruct the route from memory *while travelling in the environment*.

**Problem 2:** Assimilate knowledge of a route by travel in the environment, then reconstruct the route from memory *in the absence of the environment*.

In order to specify these problems precisely, we must define the

inputs received during assimilation and the outputs provided during recall. We will describe the sensorimotor world of the traveller in terms of two types of objects, *views* and *actions*.

A *view* is defined as the sensory image received by the observer at a particular point, and may include non-visual components. The internal structure of a view, while undoubtedly complex, is not considered at this level of detail. The only operation allowed on views is comparison for equality.

An *action* is defined as a motor operation that changes the current view, presumably by changing the location or orientation of the traveller. For the present purposes, the only operation allowed is comparison for equality.

Views and actions are egocentric descriptions of sensorimotor experience, rather than descriptions of fixed features of an external environment. The actual environment, and the sensory system for observing it, are assumed to be rich enough so that each different position-orientation pair corresponds to a distinguishable view. When this assumption is false, as for the blind traveller or a stranger lost in the desert, we can model the consequences by positing some frequency of false positive matches between views.

The observations during travel, then, can be defined as a temporal sequence of sensorimotor experiences consisting of alternating views and actions.

$$V_0 \quad A_1 \quad V_1 \quad . \quad . \quad . \quad A_n \quad V_n$$

Reproduction of the sequence can be accomplished either by performing the actions to travel the correct route in the environment, or by recalling the views and actions from memory and expressing them verbally.

### 3. A Representation for Sensorimotor Routines

Knowledge of sensorimotor routines is represented in terms of two types of associative links [Kuipers 1979a, 1979b].

The link  $V \rightarrow A$  has the meaning that when the current view is  $V$ , the current action should be  $A$  to follow the route.

The link  $(V A) \rightarrow V'$  has the meaning that if the action  $A$  is taken in the context of view  $V$ , the result will be view  $V'$ .

A sequence of observations during travel corresponds to a set of associative links of the two different types, as shown in figure 1. If the route description consists of a complete set of both types of links, then the route can be reproduced in the absence of the environment (figure 1). The states of partial knowledge of the representation consists of exactly the subsets of the complete set of links. The full description of a route with  $n$  actions consists of  $2n$  links, so there are  $2^{2n}$  possible partial descriptions. Of course, some are behaviorally more distinctive than others.

The sequence of observations:

$$V_0 \quad A_1 \quad V_1 \quad . \quad . \quad . \quad V_{n-1} \quad A_n \quad V_n$$

The set of associative links:

$$V_0 \rightarrow A_1$$

$$(V_0 A_1) \rightarrow V_1$$

$$V_{n-1} \rightarrow A_n$$

$$(V_{n-1} A_n) \rightarrow V_n$$

**Figure 1.** The sensorimotor routine representation allows a sequence of views and actions to be reconstructed from links of type  $V \rightarrow A$  and links of type  $(V A) \rightarrow V'$ . The first  $V \rightarrow A$  link allows the first action  $A_0$  to be retrieved, given the starting point  $V_0$ . The  $(V_0 A_1) \rightarrow V_1$  link allows the predicted result of that action to be retrieved from memory. Another link of the first type,  $V_1 \rightarrow A_2$ , can then be retrieved to specify the next action, and so on, to the end of the route.

If the route description consists entirely of links of the  $V \rightarrow A$  type, the route can still be followed, but only while travelling physically in the environment. The environment itself contains information equivalent to the link  $(V A) \rightarrow V'$ , since it will always reveal the result of performing an action in a particular context. Thus, this representation for sensorimotor routines is capable of expressing a state of knowledge that solves Problem 1 but not Problem 2, as required. It also exhibits the directional asymmetry of route descriptions observed in young children.

There is a simple learning theory that explains why routes consisting of  $V \rightarrow A$  links but not  $(V A) \rightarrow V'$  links are likely to arise. Consider the two rules:

(R1) If working memory holds the current view  $V$  and the current action  $A$ , then store the link  $V \rightarrow A$  in long-term memory.

(R2) If working memory holds a previous view  $V$ , the action  $A$  taken there, and the resulting view  $V'$ , then store the link  $(V A) \rightarrow V'$  in long-term memory.

The working memory load required for R2 is clearly greater than that for R1, and it is required during the time needed to carry out action  $A$ . In travel through a large-scale space, actions can take many seconds, greatly increasing the probability of an internal or external interruption that would destroy the contents of working memory, preventing rule R2 from succeeding. R1, of course, is much less vulnerable to interruptions and resource limitations.

Since the representation supports assimilation of individual links into a partial route description, leading incrementally to a complete description, we say it supports *easy learning*; since it supports successful travel even when some links are unavailable, we say it supports *graceful degradation of performance under resource limitations*. Both of these are aspects of the robust behavior we expect of commonsense knowledge.

The range of individual variation can be expressed naturally within this representation. We expect that individuals, with different collections of cognitive processes competing for the use of working memory, will vary considerably in the frequency with which rule R2 can run to completion. A second dimension of variation must be the individual choice of "imageable landmarks" (Lynch, 1960) which will affect the selection of views involved in links and their density along a particular route. A third dimension of variation is the selection of types of associative links used to represent the route. This dimension is considered in the next section.

#### 4. The Set of Possible Variants

The representation proposed above for knowledge of routines is expressed in terms of a particular pair of associative links among views and actions. It meets the performance criteria we defined, and exhibits states of partial knowledge corresponding to our observations of human behavior. Assimilation requires a relatively small "window" onto the sequence of observations during travel: at most two views, V and V', and the action A that leads from one to the other. The question remains, Are there *other* solutions to the same problem that meet these constraints? In order to answer this question, we must explore the space of *all* possible solutions to the route representation problem.

We only consider combinations of associative links involving the three adjacent observations V, A, and V', since the assimilation of more complex links imposes prohibitive working memory loads. Each distinct type of link involving these three observations can be represented by a triple of three integers, indicating that the corresponding element in <V A V'> is:

- 0 = not involved in the link;
- 1 = retrieved by the link;
- 2 = acts as a retrieval key for the link.

Thus, for example, the link V->A is encoded as <2 1 0>, and (V A)->V' is encoded as <2 2 1>.

There are twenty-seven possible cases. After removing useless or trivial ones such as <0 0 0> or <2 2 2>, nine potentially useful links remain. For each link, we want to know:

- (1) what combinations of link types can be used to reconstruct a route from memory;
- (2) whether assimilation of the link requires a working memory load to be preserved while the action A is being performed;

Table 1 presents the possible links and their properties with respect to these questions.

	<V A V'>	Route Reconstruction	Working Memory Load During A
1.	<2 1 0>	1 + 2; 1 + 4;	no
2.	<2 0 1>	1 + 2; 2 + 6; 2 + 7; 2 + 8;	yes
3.	<2 1 1>	3;	yes
4.	<2 2 1>	1 + 4;	yes
5.	<1 0 2>	no	yes
6.	<0 1 2>	2 + 6;	yes
7.	<1 1 2>	2 + 7;	yes
8.	<2 1 2>	2 + 8;	yes
9.	<1 2 2>	no	yes

**Table 1.** The possible links for encoding information about a route. There are six viable combinations of one or two link types that will support reconstruction of a route from memory. Link 1 is the only link type that can be stored without a long load on working memory. The route representation previously discussed is coded here as 1 + 4.

The combinations of link types listed are not the only combinations that might be found in a person capable of following routes from memory, but they are the minimal ones. For example, link type 3 can support route reconstruction by itself, but the combination 1+3 is more robust. Similarly, combination 1+4 has the robustness of the V->A link along with the fact that the (V A)->V' links can be interpreted as a context-independent assertion about the result of an action in the environment as well as an instruction in the current route procedure.

As we observed above, it is clear that some people occasionally exhibit a state of knowledge that corresponds to link type 1 (V->A), and to none of the other links. Since link 1 is the only link whose assimilation does not impose working memory overhead during travel, there are also reasons of computational robustness for using that type of link.

This tabulation of possible link types and their combinations allows us to make empirical predictions about how each apparently viable combination of links would manifest itself in behavior (Table 2).

Links	Predicted Behavior
1+2 V->A V->V'	Occasional recall of A without V'. Occasional recall of all Vs with no As.
1+3 V->A V->(A V')	Occasional recall of A without V'. No recall of V' without A.
1+4 V->A (V A)->V'	Occasional recall of A without V'. No recall of V' without A.
3 V->(A V')	No recall of A without V'. Working memory vulnerable during A.
2+6 V->V' V'->A	Occasional recall of all Vs with no As. "Backward" recall of A given V'. Working memory vulnerable during A. Reference to action A without context V.
2+7 V->V' V'->(V A)	Occasional recall of all Vs with no As. Working memory vulnerable during A.
2+8 V->V' (V V')->A	Occasional recall of all Vs with no As. Working memory vulnerable during A.

**Table 2.** The different theoretically viable combinations of link types that support route reconstruction, and some of their behavioral consequences. All of the minimal combinations are presented here, but others, such as 1+3 shown here, can be produced by adding links to a minimal set.

Rather than providing us with a definitive answer as to why one combination of links is used and the others rejected, this tabulation defines the space of possible individual variants. Table 2 shows the predicted behavior corresponding to some of the combinations, including all of the minimal ones. For example, any of the combinations including link 1 will occasionally show the "I could take you there, but I can't tell you how!" phenomenon. Similarly, any combination including link 2 should occasionally produce the phenomenon of being able to enumerate the landmarks on a route, but not the actions needed to get from one to the next. This is unlikely to occur in an individual with combination 1+2, since link 2 is always more vulnerable to interruptions than link 1. However, if an individual variant exists with combination 2+8, link 8 is very vulnerable to interruptions, so we would expect an occasional route to be described purely in terms of V->V' links.

Some of the combinations, such as 2+6 (i.e. V->V' and V'->A), have such peculiar behavioral consequences that we would not be surprised to find it missing, or at least very rare in the population. On the other hand, we would expect that all of the genuinely viable ways of representing knowledge of routes will exist in the population. A missing variant would suggest that we have overlooked a computational constraint. This approach to the "ecology" of individual variation bears considerable further study.

## 5. Conclusion

The representation for knowledge of routes consists of *sequences of sensorimotor observations*, expressed as egocentric experiences distributed over time. Much of human spatial knowledge, however, is concerned with *fixed features of the environment* such as places and paths distributed over space. This dichotomy is inescapable, because sensory input necessarily consists of egocentric observations, while a description of the environment in terms of external places and paths is much more effective for problem-solving and for communication via maps and verbal descriptions.

This paper has discussed a representation for storing procedural descriptions of routes in the long-term memory of the cognitive map. The remainder of a theory of the cognitive map must show how the route descriptions we have defined can be transformed into descriptions of the environment in terms of places, paths, regions, and their topological and metrical relationships.

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