Primitives and Units for Time Specification *

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Abstract

We work in a calculus of intervals, formulated by James Allen for convex intervals, and by ourselves for unions of convex intervals [*All2,Lad2*]. We investigate the primitive relations and operations needed for implementing such calculi in a system which includes some set theory, and which allows the assertional definition of operators in Horn clause fashion. We indicate how standard temporal logic may be rephrased in the interval calculus, and present a formalisation of a system of time units in the interval framework. We are implementing the primitives in the $REFINE^{TM}$ system[1].

Introduction

James Allen introduced an interval calculus for reasoning about time, and we have proposed an extension of this calculus to enable the representation of time by non-convex intervals. Recent work on the convex interval calculus is by Allen, Pat Hayes, and Henry Kautz [*All2, All3, AllHay, AllKau*]. Recent work on the non-convex calculus is by ourselves and Roger Maddux[*Lad2, Lad3, LadMad*].

Convex intervals are those intervals considered by Allen and Humberstone [*All2, Hum*], which span a period of time, without gaps of any sort. In a formalism based on points, these are 1-dimensional convex sets of points. We are concerned with intervals that are arbitrary unions of these, which we need for expressing temporal properties of intermittent events [*Lad1, Lad2*]. We consider the interval formulation to be an abstraction of time periods, and in this view, sets of time points would be just one way of modelling intervals.

Mathematically, Allen's calculus of convex relations is a particular ***relation algebra*** in the sense of Tarski [*JoTa1, JoTa2*]. Allen's algebra has thirteen atoms, and thus generates a relation algebra of size 2^{13}. By contrast, Ladkin's algebra is infinite, as well as having only infinite representations. We argue in [*Lad2*] that the relations, which are a strict subset of all relations between unions of convex intervals, are not only convenient but necessary for expressive power. Mathematical results concerning Allen's and our own calculi are contained in [*LadMad*].

We present a specification of primitives which can be used to implement time intervals represented as unions of convex intervals. It is important to us to allow only relation structure in the interval calculus, so that we are able to maintain the structure of a relation algebra, and to restrict the proliferation of intervals denoted by basic terms in our model [*LadMad, Lad2*]. We obtain the effect of operators on intervals by using a correspondence between an interval and a certain set of convex subintervals of that interval. Sets of convex intervals are needed in any case for our model of time units.

We show how to express standard temporal logic primitives in the interval calculus, and finally we develop a general model of time units in the interval framework.

We assume throughout that time is linearly ordered, with respect to the relation ***precedes***, although this work is equally applicable to branching time models. Some modification would be needed to the measure functions, and other modifications would be of a minor nature only.

Other references to time representation by intervals are [*vBen, Dow*]. Another representation of time for AI purposes using a points-based model rather than an interval model is described in [*McDer1, McDer2*].

Notation

We assume the reader is familiar with standard logical notation, in particular the connectives and quantifiers

- $\wedge \ \vee \ \neg \ \Rightarrow \ \forall \ \exists$

We use certain terminology from [*All2, Lad2*], in particular

- We refer to the relations in [*All2*] as *convex* relations. All convex relations are irreflexive and antisymmetric, except for equality. *Non-convex* intervals or relations are those for unions of convex intervals in [*Lad2*]. A picture of such a beast will look like a sequence of lines with gaps, when drawn in one dimension. The lines are the *maximal convex subintervals*, or *maxconsubints*.

- $\|$ is the convex relation *meets*.
  $(i \| j)$ iff i is before j with no interval occurring between them

- $\ll$ is the convex relation ***contained-in***
  $(i \ll j)$ iff i is a strict subinterval of j, i.e.
  i starts j $\vee$ ***i during j*** $\vee$ ***i ends j*** in the terminology of [*All2*]

- $\prec$ is the convex relation ***precedes***
  $(i \prec j)$ iff i is before j, with some other interval occurring between

- $\oslash$ is the convex relation ***overlaps***
  $(i \oslash j)$ iff, intuitively, i starts before j, and finishes before j.

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[1] $REFINE^{TM}$ is a trademark of Reasoning Systems Inc.

- $(R_1 \vee R_2 \vee \ldots)$ is the non-convex relation of *disjunction*, where the R_i are convex relations.
  $(i\,(R_1 \vee R_2 \vee \ldots)\,j)$ iff corresponding maxconsubints of i and j are one of the R_i to each other (different subintervals may be related by different R_i).
  We assume a 1-1 correspondence between the maximal convex subintervals is available.
- (i *always-R* j), where R is a convex relation, iff maxconsubints of i are in each case R to the corresponding maxconsubints of j
- (i *sometimes-R* j), where R is a convex relation, iff some maxconsubint of i is R to the corresponding maxconsubint of j

Operators for Intervals

We attach intervals to actions, tasks, events and assertions, representing the periods of time over which an action takes place, a task is performed, an event happens, or an assertion is true. We note here that certain supposed problems with the definition of truth-on-an-interval have been adequately answered in [*Hum*].
We also provide the correspondence between an interval and a set of convex intervals, a means of measuring the *duration* of an interval, and the length of time over which it happens, here called the *diameter*. *Duration* and *diameter* are, of course, the same for convex intervals.
In mathematics, duration is usually called *measure*, and *diameter* is terminology from topology.

The operators

We use the word *type* to indicate a domain of objects of the same sort. There is no implied type theory in the use of this terminology.

interval-of(P): *P is of type task / action / event / assertion* returns i of *type interval* such that *occupies(P,i)* (see below)
Since we reason assertionally about time intervals, this gives us a way of passing between the task domain and the time domain.

dissect(i): the *set* of maximal convex subintervals of i *Dissect* is somewhat like a selector for the data domain of non-convex intervals even though it returns a *set*, not an interval.

combine(S): S is a *set* of intervals
makes an interval out of S, rather like a *union* operator. In general, this interval will be non-convex. All the intervals in S are subintervals of *combine(S)*. *Combine* is the constructor for the data domain of non-convex intervals.

duration(i): *type real*, the measure of i

convexify(startint,endint): *type interval*, the smallest convex interval containing *startint* and *endint*. Note that it follows that *convexify* is commutative and associative.

alltime: *type interval*, the global time interval that includes all time.

Note that we have omitted from the list of primitives the function

diameter(i): *type real*, the largest distance between two subintervals of i (including the duration of the subintervals)

Diameter may be defined in terms of *duration* by the equation

- *diameter(i) = duration(convexify(i,i))*

We do not include *diameter* as a primitive, since it is definable, but it is a basic part of the constraint-expression language.

Similarly, *duration* need only be defined for *convex* intervals, since its extension to non-convex intervals follows from the additive property below.

Axioms

In this section, we give the axioms that specify the operators described above. We note that the only unbounded quantifiers appearing in the axioms are universal, and that the bounded existential quantifiers that appear are restricted to range over sets or intervals that are parameters in the formula. We envisage that these objects will be finitely bounded in most applications, in such a way that the quantifiers may be realised by an enumeration. This is indeed the case for $REFINE^{TM}$.

Note *(i convex)* and *(convex i)* are both shorthand for *i bars i* [*Lad2*].

- $(\forall i)(i \ll alltime \vee i = alltime)$ characterises *alltime* as the global time interval. All intervals are *contained-in* or *equal-to alltime*

The next three axioms characterise *dissect(i)*.

- $(\forall\ convex\ j \ll i)(\exists k \in dissect(i))(j \ll k \vee j = k)$
- $(\forall j \in dissect(i))(j\ convex \wedge (j \ll i \vee j = i))$
- $(\forall j)(\forall k \in dissect(i))(j \ll k \Rightarrow \neg\, j \in dissect(i))$

The first axiom states that all convex subintervals of i are contained in some interval k in *dissect(i)*, or are equal to such an interval. The second ensures that *dissect(i)* contains only convex subintervals of i, which in the presence of the first ensures that *dissect(i)* contains at least the maxconsubints of i. The third axiom is not in positive Horn clause form, since the consequent is negated. It may easily be turned into the right form by observing that the negation of the antecedent is equivalent to a positive disjunction of the other twelve interval relations enumerated by Allen, who gave the exhaustive list of possible relations between convex intervals. This observation allows us to take the contrapositive statement for our axiom. This has the correct form, even though its intent is more obscure. Our relations include those that are the disjunction of convex interval relations, so this disjunction reduces to a single predication in the consequent.

- $(\forall j)(\forall k \in dissect(i))((j \in dissect(i)) \Rightarrow (j(\| \vee \oslash \vee \ldots\ldots)k))$

In the presence of the first two axioms for *dissect*, the third *dissect* axiom ensures that *dissect(i)* contains *only* maxconsubints of i.

The next three axioms characterise *combine*. The first says that if you *combine* a *dissected* interval, that you get back the original interval under all circumstances. The second asserts that if the set S consists of disjoint, non-*meeting* convex intervals, then *combine* is indeed the full inverse of dissect. Note that *dissect(i)* consists only of disjoint, non-*meeting* intervals, i.e. this is derivable from the *dissect* axioms. The third *combine* axiom ensures that *combine*-ing a set of intervals which *overlap* or *meet* does not give more than you want in the resulting interval. If there were more, then the extra would form an interval that was a subinterval of *combine(S)*, but which was disjoint from anything in S. The axiom rules this out. An alternative way of axiomatising this property is

- $(\forall i \in dissect(combine(S)))$
  $(((\exists j \in S)(j\ starts\ i) \wedge (\exists k \in S)(k\ ends\ i)) \vee$
  $(\exists j \in S)(j = i))$

This property ensures that there are no "little bits hanging off the end" of an interval in *combine(S)* that aren't there in some interval in S. Notice that every convex interval in *dissect(combine(S))* is at least as big as any convex interval in S, thus reducing the number of cases we have to worry about stating in the axiom.

The cleanest way to implement the correct *combine* function is probably to iteratively *convexify* intervals in S which *overlap* or *meet*. Call the resulting set S_1. All the intervals in S_1 are disjoint, and non-meeting, and hence *dissect(combine(S_1))* $= S_1$. Furthermore, *combine(S_1) = combine(S)* by the axioms. However, in this paper, we are only concerned with a correct assertional specification of the functions in a limited logical language, and we have given this for *combine*. The assertions may be compiled any appropriate way.

- $combine(dissect(i)) = i$
- $(\forall i \in S)(\forall j \in S)((i \prec j \vee j \prec i \vee i = j) \wedge$
  $(i\ convex \wedge j\ convex))$
  $\Longrightarrow dissect(combine(S)) = S$
- $(\forall j)(((j \oslash combine(S)) \Rightarrow (\exists i \in S)(j \oslash i)) \wedge$
  $((combine(S) \oslash j) \Rightarrow (\exists i \in S)(i \oslash j)))$

Next, we have the axioms for *duration*, which specify only that duration is a fully additive function. Giving values of *duration* on the convex intervals will then specify *duration* completely. The purpose of the special axiom for the case of *meeting* is to be able to derive different time units. For example, one can specify that there are seven *days* in a *week*, by adding a condition that seven *day*-type intervals which meet consecutively form a *week*-type interval. One can then count in *week* units or *day* units merely by adding axioms of the form

- $(\ i \in DAYS \Longrightarrow duration(i) = 1\ )$
- $(\ i \in WEEKS \Longrightarrow duration(i) = 1\ )$

as appropriate. Given the *days* definition, the specification of *week*, and the axiom, will then guarantee that the *duration* of a *week* is seven *day* units.

- $duration(i) = \sum duration(dissect(i))$

  The sum is taken over all members of *dissect(i)*.
- $(i \parallel j) \Rightarrow duration(combine(dissect(i) \cup dissect(j))) =$
  $duration(i) + duration(j)$

Finally, the axioms for *convexify* specify that *convexify(i,j)* is the smallest convex interval containing i and j. *Convexify* is a total, and thus a commutative operation. It is also associative, and we prefer to include both these properties explicitly, even though they follow from the minimal property.

- $i\ (begins\text{-}at \vee ends\text{-}at)\ convexify(i,j) \wedge$
  $j\ (begins\text{-}at \vee ends\text{-}at)\ convexify(i,j)$
- *convexify(i,j) convex*
- $(\forall k\ convex)(\forall i)(\forall j)((i \ll k \wedge j \ll k) \Rightarrow$
  $(convexify(i,j) \ll k \vee convexify(i,j) = k))$
- $convexify(i,j) = convexify(j,i)$
- $convexify(i, convexify(j,k)) = convexify(convexify(i,j),k)$

  Note that the associative and commutative properties of *convexify* actually follow from its minimal property. However, any reasonable theorem prover would probably prefer to know this explicitly (as ours does).

Additional Noteworthy Properties

The following properties are all consequences of the axioms:

- $i\ convex \Rightarrow duration(i) = diameter(i)$

  this follows from the definition of *diameter* and the property:
- $i\ convex \Rightarrow convexify(i,i) = i$
- $i \ll j \Rightarrow duration(i) < duration(j)$

  which follows from the additive property of *duration*, given enough subintervals in the universe.

Predicates

The predicates we wish to use on intervals are specified in [*Lad2, Lad3*]. They form a relation algebra in the sense of Tarski [*JoTa1, JoTa2, Mad1*]. We mention them here only for completeness, since it is not the purpose of this paper to explain the interval calculus.

interval relations: We include all the relations between intervals defined in [*Lad2*].

Axioms

- We include the relation product table, and the other axioms needed for specifying the algebra of relations in [*Lad3*]. See [*LadMad, Mad1, JoTa2*].

Additional Defined Entities

We need the basic (but not primitive) function *diameter* to adequately express properties of, and constraints on, non-convex intervals. We repeat here the definition and properties of *diameter*

diameter(i): *type real*, the largest distance between two subintervals of i (including the duration of the subintervals)

- $diameter(i) = duration(convexify(i,i))$

We also need, for purposes of specification, the predicates

occupies(P,i): *P is type task / action / event / assertion*
i is type interval
i is the exact interval over which *P takes place / holds / occurs / is true*

occurs-in(P,i): *true* of all *i* such that $i \ll$ *interval-of(P)*

which have the properties

- *occupies(P,interval-of(P))*
- *occupies(P,i)* $\Longrightarrow$ *occurs-in(P,i)*
- *occurs-in(P,i)* $\Longrightarrow (\exists j \ll i)(occupies(P,j))$

These predicates are provided by, and their properties follow from, the definitions

- *occupies(P,i)* $\Leftrightarrow i =$ *interval-of(P)*
- *occurs-in(P,i)* $\Leftrightarrow$ *interval-of(P)* $\ll i$

Temporal Logic

Interval Constants

We introduce three interval constants, which correspond to McTaggart's A-series notion of time [*McT, Lad1*], and the standard syntax of temporal logic. The A-series notion conceives of time as consisting of the moving, changing present, the past and the future. This corresponds to the interpretation of the temporal operators in classical tense logic, except that 'present' is implicit. The semantics of tense logic, however, is similar to McTaggart's B-series, which consists of immutable points of time, like timestamps, at which there is no change. Change is represented in the B-series by moving from one point to another. The evaluation of a tense-logical formula relative to a point, which is the standard semantical definition, is similar to connecting the A-series and the B-series notions. We show how to capture the A-series notion in interval calculus. The standard time-of-day clock functions as an A-series to B-series converter.

We refer the reader to [*All2*] for the terminology and calculus of interval relations.

The constants are:

now: intuitively, an interval of smallest granularity. In any practical domain of application, intervals will not be infinitely divisible. If they are, there is still no logical contradiction in the axioms presented, as can be shown by a compactness argument from model theory. In this case, *now* would function like an interval of measure 0.

future: intuitively, for those who like their intervals to contain points, the interval (now, ∞); all future time

past: intuitively, the interval $(-\infty, now)$; all past time

Axioms

- *now convex*
- *future convex*
- *past convex*
- $past \parallel now \parallel future$
- $(\forall\ convex\ i)(\neg\ now \ll i \Longrightarrow ((i \ll past) \vee (i \ll future)))$
- $(\forall i)(i \prec now \Longrightarrow i \ll past)$
- $(\forall i)(now \prec i \Longrightarrow i \ll future)$
- $(\forall\ convex\ i)((now \ll i) \vee (now \prec i) \vee (i \prec now) \vee (i \parallel now) \vee (now \parallel i) \vee (i = now))$

These axioms characterise the constants. They state that the three convex intervals meet in the right ways, that they include all time, and that time is linearly ordered with respect to the *now* interval.

The Temporal Operators

We don't intend to present a full development of temporal logic in the interval framework in this paper. That work is still in progress. We provide here definitions of the two temporal operators $\Box$ and $\Diamond$.

The formula $\Box P$ is true if and only if P is true everywhere in the future. Similarly, $\Diamond P$ is true if and only if P is true at some evaluation object in the future. We deliberately avoid the word "point" since in our formulations, the evaluation objects are intervals, which may or may not have point-like properties depending on the application. [*Hum*] shows how truth may be defined for tense-logical formulae evaluated on convex intervals.

The definitions of $\Box P$ and $\Diamond P$ can be

- $\Box P \equiv$ (*int(P)* $=$ *future* $\vee$ *future* $\ll$ *int(P)*)
- $\Diamond P \equiv$ (*int(P) sometimes-(overlaps* $\vee$ *contained-in) future*)

By observing that $\Diamond P \equiv \neg\Box\neg P$, we can obtain alternative definitions of either within the interval framework.

Conversion Functions

Standard temporal logic has a syntax that corresponds to McTaggart's A-series time, and a semantics that corresponds to his B-series notion of time (roughly, timestamps).

We indicate that conversion by noting that it is provided already in the standard facilities available on most AI workstations, as a real-time clock, which converts *now* into a timestamp. We also need to construct *past* and *future* from the timestamp. We assume that the clock runs to a certain granularity, say microseconds, and point out that the clock does, in fact, specify a time interval, whose duration is one microsecond in this case. Essentially, the clock tells you which interval the query interrupt is contained in. In this context, there are *next* and *previous* operators, which return the next and the previous timestamps. They may be implemented by *increment* and *decrement* respectively. The representation is therefore just

timenow(): the call to the clock implements the conversion of *now* to its B-series timestamp.

If there are *infinity* intervals at both ends of the time structure, (∞ and $-\infty$ for want of better names) then *past* and *future* may both be represented by

past: *convexify(*$-\infty$*, previous(now))*

future: *convexify(next(now),* ∞*)*

It is consistent to add such *infinity* intervals. If you don't want them, the properties of the past and future B-series intervals may then be inferred from the axioms alone.

Defining Time Units

We need to reason about years, months, days, minutes and microseconds. We introduce a standard form for an interval which represents an instance of one of these units. All the units will be convex intervals, and we then show how to develop the types of units from these standard forms.

Standard Time Units

We use sequences of integers to represent our standard units. We use a linear hierarchy of standard units, **year, month, day, hour, minute, second**, arranged as a sequence. We illustrate its use down to seconds, hence our sequences will have lengths of up to six elements. It should be clear that the hierarchy is easily extendable to smaller units such as microseconds. We illustrate the meanings of sequences of integers, rather than giving an obvious definition.

- [**1986**] represents the year 1986
- [**1986,3**] represents the month of March, 1986
- [**1986,3,21**] represents the day of 21st March, 1986
- [**1986,3,21,7**] represents the hour starting at 7am on 21st March, 1986
- [**1986,3,21,7,30**] represents the minute starting at 7:30am on 21st March, 1986
- [**1986,3,21,7,30,32**] represents the 33rd second of 7:30am on 21st March, 1986 (the first second starts at 0)

We conceive of these intervals as being closed at the left end and open at the right, since this corresponds with normal usage. Notice, as we mentioned, that the standard clock times returned by a time-of-day clock in fact returns the interval in which the interrupt occurred. (It's not really possible to determine from this which end of an interval should be open and which closed, since the interrupts are serialised).

Axioms for Units

Certain relations hold between these intervals. All of these intervals are convex, and hence the vocabulary of relations is Allen's [*All2*].

We give examples only of the axioms, since the nature of the rest may easily be inferred from the examples.

It is obvious that not all integer sequences of appropriate lengths are going to name units in our formalism. We shall not bother with checking bounds on elements of a sequence, since this is a detail of no theoretical interest. We shall assume bounds are checked somehow.

We shall use **x**, **y**, **z**,..... for integer variables, and α, β,.... for sequence variables. Concatenation of sequences is denoted by $\frown$. All the axioms are quantifier-free statements.

- ([**x,1**] *starts* [**x**]) $\wedge$ ([**x,12**] *ends* [**x**])
  January is the first month and December is the last month of the year
- [**x,y,1**] *starts* [**x,y**]
  All months begin with day 1
- [**x,1,31**] *ends* [**x,1**]
  January ends on the 31st
- **x** divisible by 4 $\Rightarrow$ ([**x,2,29**] *ends* [**x,2**]) *else* ([**x,2,28**] *ends* [**x,2**])
  February has 28, or sometimes 29 days
- appropriate cases for the other months
- ([**x,y,z,0**] *begins* [**x,y,z**]) $\wedge$ ([**x,y,z,23**] *ends* [**x,y,z**])
  constraints for hours in a day
- appropriate cases for minutes and seconds
- for length(α) = 0 to 5 and **x** less than the ending numbers for the appropriate length,
  $(\alpha \frown [\mathbf{x}]) \parallel (\alpha \frown [\mathbf{x+1}])$
  the **x**th year/month/.. *meets* the **x+1**st year/month/..
- for length(α) $\leq$ 5,
  $(\alpha \frown [\mathbf{x}]) \ll \alpha$

We also need to be able to coalesce representations, to gather months into years, and seconds into minutes. The axiom for this is

- $(\alpha \frown \delta \ \textit{starts}\ \alpha) \wedge (\alpha \frown \gamma \ \textit{ends}\ \alpha) \Rightarrow$
  $\textit{convexify}(\alpha \frown \delta, \alpha \frown \gamma) = \alpha$

Interval Types Definable From Unit Types

We can now define classes of intervals, based upon the units.

- YEARS = { α | length(α) = 1 }
- MONTHS = { α | length(α) = 2 }
- DAYS = { α | length(α) = 3 }
- Similarly for HOURS, MINUTES, SECONDS,.....

We may also define units which are not in the list of basic units. Firstly, let us assume that all variables and sequences range over the set DAYS. This will simplify notation for our examples. We define

- $\phi_0(\alpha,\beta) \equiv (\alpha \parallel \beta)$
- $\phi_{i+1}(\alpha,\beta) \equiv (\exists\gamma)(\phi_i(\alpha,\gamma) \ \wedge \ (\gamma \parallel \beta))$ for $0 \leq i$
- ϕ^* is the symmetric, transitive closure of ϕ, for any binary relation ϕ

The ϕ_i are the iterated *meet* relations for DAYS. Note that, as we have defined them, a given α in DAYS *meets* exactly one β in DAYS, and *is-met-by* exactly one γ in DAYS.

- WEEKS =
  { $\textit{convexify}(\alpha,\beta)$ | $\phi_5(\alpha,\beta)$ }
  defines all 7 day intervals as weeks
- MONDAYS =
  { α | $(\phi_6)^*$([**1986,3,31**], α) }
  Needless to say, [**1986,3,31**] is, in fact, a Monday

We may define other days of the week in a similar way to MONDAYS, or we may choose to use an implicit definition, such as

- (SUNDAYS $\subset$ DAYS) $\wedge$ (*combine*(SUNDAYS) *always-meets* *combine*(MONDAYS))

Our definitions of the interval types show that we need to maintain the distinction between a non-convex interval I and the set of its maximal convex subintervals *dissect(I)*. All of the unit classes YEARS, MONTHS, ..., as well as some of the defined classes such as WEEKS, satisfy the condition

- *combine(S) = alltime*

and cannot thus be distinguished purely as interval objects. One of the major reasons for introducing sets into the time structure must be to distinguish between the different classes of time units. Since the set theory is needed, we see no reason not to make cautious use of it, and we may then avoid the need for a proliferation of interval operations, since we may use *dissect* on an interval, perform set theoretic operations, and then use *combine* to create the new interval.

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