Extending the Mathematics in Qualitative Process Theory

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Abstract

We present a semi-quantitative extension to the qualitative value and relationship representations in Qualitative Process (QP) theory. Examination of a detailed example reveals a number of limitations in the current ability of QP theory to analyze physical situations. The source of those limitations is traced in part to the qualitative mathematics used in QP theory. An extension to this mathematics is then presented and shown capable of eliminating many of these limitations, at the price of requiring additional system specific information about the system being modelled.

I. Introduction

Qualitative Process (QP) theory [Forbus, 1984] describes the form and structure of naive theories [Hayes, 1979] about the dynamics of physical systems. A key component of QP theory is the qualitative mathematics used to represent values of continuous parameters and relationships between them. A research strategy for developing this mathematics has been to search for a qualitative mathematics capable of yielding significant results from a minimum of information about the situation being modelled. In the work described here, we ask a slightly different question: what kinds of information can we add to the base theory, and what new questions can we answer with this additional information?

A. Mathematics in QP theory

The representation for a continuous parameter in QP theory is a *quantity*. A quantity has four parts:

1. The magnitude of the amount of the quantity.
2. The sign of the amount {-, 0, +}.
3. The magnitude of the derivative.
4. The sign of the derivative.

The use of the sign as a significant qualitative abstraction is adopted from DeKleer [deKleer, 1979] [deKleer and Brown, 1984]. Magnitudes are represented in a *quantity space*. The quantity space for a number consists of all those amounts to which it is potentially related in the situation being modelled. The special value ZERO is always included in every quantity space, and relates the quantity space representation with sign information.

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Quantities are related to one another through *Relations*, which can be either ordering relations, functional relations, or influences. Functional relations are a qualitative analog of normal mathematical functions whose domain and range are real numbers. The following states that the level of water in a container is qualitatively proportional to the amount in the container:

level(p) Q+ amount-of(p)

These are called Qualitative Proportionalities (Qprops). A *Process* is the mechanism of change in QP theory. A process acts to change a situation by *influencing* some parameter(s) of objects in the situation. An *Influence* is similar in information content to a qualitative proportionality, but affects the derivative of the range variable, rather than its amount. For example, the primary effect of a fluid-flow process is on the derivatives of the source and destination fluid quantities. Qprops are often referred to as *indirect* influences, since they provide pathways through which *direct* influences propagate.

Forbus' implementation of QP theory combines this basic domain information with an initial system description to perform measurement interpretation and envisioning. The basic inferences required are: *Elaboration*, *View and process structure determination*, *Influence resolution*, and *Limit analysis* We will primarily be concerned with influence resolution in this paper. For a discussion of the other inferences, see [Forbus, 1984].

II. Example

We now analyze a hypothetical model of a typical continuous flow industrial process, in order to demonstrate these steps and identify the capabilities and limitations of QP theory. Fig. 1 shows a simplified sketch of the process. Reactants in granular form enter through the port at the top left (a material flow process), and are heated to reaction temperature within the vessel (a heat-flow process). When the reactants reach reaction temperature, they undergo a state change (a reaction), in which they disappear and a fluid product and an off-gas are created. The off-gas exits through the port at the upper right (another material flow process). As the hot off-gas flows out of the reaction vessel, heat is transferred to the cool incoming reactants (counter-current heat flow). We will ignore the processes by which the product is extracted from the vessel and simply allow it to accumulate at the bottom.

The four basic processes crucial to understanding of the system described above, basic heat flow, the reaction,

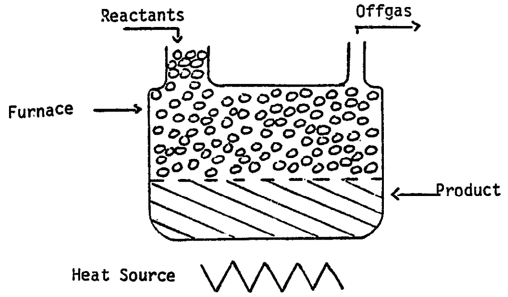

Figure 1: Reaction Vessel

material flow, and counter-current heat flow, are described in detail in [D'Ambrosio, 1986]. Given a suitable initial state description, the first two QP inferences identify three possible states for the situation described, (1) that nothing is happening, (2) that the only thing occurring is that the reaction vessel is being heated, or (3) that all processes are active. The state of interest is the one in which all processes are active. Using the third basic QP deduction, we can determine various facts about this state, such as:

- If the heat input is increasing, the off-gas generation rate will be increasing also.
- If the incoming reactant temperature is decreasing, the off-gas temp will be decreasing.

However, we cannot determine:

1. Is the product temperature increasing, decreasing, or constant?
2. If the heat input is increasing, is the off-gas exit temperature increasing or decreasing?
3. If we increase the heat input a little, how much will the generation rate increase?
4. If the available observations do not uniquely identify a single state, which of the possible states is more likely?

These limitations are the result of ambiguity in the conclusions derived using QP theory.

III. Ambiguity in QP theory

We identify two types of ambiguity in QP theory, *Internal* and *External* ambiguity. Internal ambiguity occurs when use of QP theory produces multiple descriptions of a single physical situation. External ambiguity is the dual of this, namely when a single QP theory description corresponds to several possible physical situations which must be distinguished. Internal ambiguity is of two types. First, given a situation description, there may be ambiguity about which of several possible states a system is in (e.g., given a leaky bucket with water pouring in, is the water level rising or falling?). Second, given a specific state, there may be ambiguity about what state will follow it (e.g. - given a closed container containing water, and a heat source heating the container, will it explode?).

External ambiguity is the inability to determine, on a scale meaningful to an external observer, the duration of a situation, as well as the magnitude and intra-situation evolution of the parameters of the situation (e.g., how fast is the water rising? How long before the container explodes?)

These ambiguities are the result of four fundamental limitations in QP theory representations and inference mechanisms:

1. Inability to resolve conflicting functional dependencies. This is caused by the weak representation for functional form of dependencies, which captures only the sign but no strength information.
2. Inability to order predicted state changes. This is caused by lack of ordering information on change rates, as well as lack of quantitative information on the magnitude of change needed for state change.
3. Inability to quantify, even approximately, parameters significant to external observers during times between major state transitions. This is caused by a weak model of *intra*-state situation evolution. Time, quantity values, and functional dependencies are all represented qualitatively in QP theory.
4. Inability to represent non-boolean predicate and state possibilities.

Solving these problems requires extending QP representations to capture more information about the system being modelled. We have studied three classes of extensions: extensions to the quantity representations, the relationship representations, and the certainty representations. Specifically, we have developed an extension to QP theory which utilizes:

- Belief functions certainty representations - these will permit capture of partial or uncertain observational data, and estimates of state likelihood.
- Linguistic descriptions of influence sensitivities - to reduce undecidability during influence resolution.
- Linguistic characterizations of parameter values and ordering relationships - to permit capture of partial or uncertain observational data, and enable estimates of the effects of adjustments to continuous control parameters.

These extensions reason at the appropriate level of detail for the kinds of control actions typically needed, draw the needed distinctions, are computationally tractable, and can reason with the imprecise or uncertain data typically available. In this paper we concentrate on the second of these extensions, linguistic influence sensitivities, and present a way of annotationing the relationship representation in QP theory to reduce ambiguity. Discussion of the integration of Dempster-Shafer belief functions with QP theory and the underlying ATMS can be found in [D'Ambrosio, 1987]. Discussion of parameter value extensions can be found in [D'Ambrosio, 1986]. See [Simmons, 1987] for an alternate extended quantity representation.

IV. Linguistic Influence Sensitivities

The influence resolution rule used by Forbus states that if opposite influences impinge on a single parameter, then the net influence on the parameter is unknown. In order to reduce the number of situations in which conflicting functional dependencies cannot be resolved, we extend QP

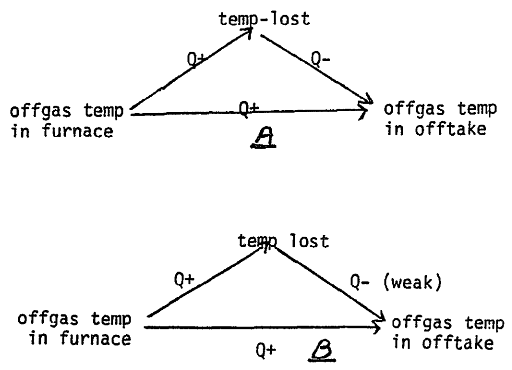

Figure 2: Conflict Triangle

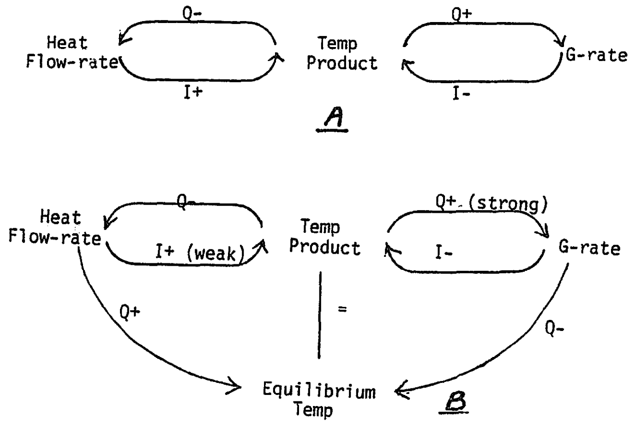

Figure 3: Feedback Loops

theory functional descriptions with a linguistic influence sensitivity. Intuitively, this corresponds to distinguishing between first order, second order, etc., dependencies. With this extension we can now address the second question unanswerable earlier: if we increase the heat input, will the offgas temp increase or decrease?

Forbus claims that if actual data about relative magnitudes of the influences is available, it can be used to resolve conflicts. We might attempt to achieve this by extending direct and indirect influences with a strength parameter. This is inadequate, however, for two reasons. First, the overriding influence may not be local. Information may have to be propagated through several influences before reaching the parameter at which it is combined. Second, various sources of strength information have varying scopes of validity. In the following sections we first identify two basic influence subgraphs responsible for the ambiguity in our example, and argue that the ambiguity can be eliminated by annotating the subgraphs with influence sensitivity and adding additional situation parameters. We then present extensions to the influence resolution algorithm for utilising the sensitivity annotations, and finally describe a control structure for managing acquisition and use of annotation information.

A. Identifying internal causes of conflict in influence graphs

We have identified two basic patterns of influences which account for the ambiguity previously encountered. These are the conflict triangle (Fig. 2) and the feedback loop (Fig. 3). The reason, for example, that the change in offgas temp in the offtake cannot be resolved is that there are two conflicting paths through which a single parameter (offgas temp in the reaction vessel) affects the target parameter. But the effect on temperature-lost is in this case smaller than the direct effect on the offtake temp, and can be ignored. We can indicate this by adding to the influence arc an annotation indicating temp-lost in counter-current heat flow is relatively insensitive to offgas temp in the furnace (Fig 2b).

Another ambiguity in the QP theory analysis of the furnace is in the generation rate and associated variables. One of the causes of this ambiguity is the set of influences on Product temperature shown in Fig. 3. Since both the generation rate and heat-flow rate are positive, the qualitative derivative of the product temperature is undecidable. This network is similar to one Kuipers [Kuipers, 1986] identifies as introducing a new *landmark value*, not in the original quantity space for the product temperature. This new value represents an equilibrium value towards which the temperature will tend. Recognition of the existence of an equilibrium value permits resolution of the effects of the conflicting influences on product temperature, depending on the assumed ordering between the actual product temperature and the equilibrium value. analysis can be taken one step further. Kuipers adds the equilibrium value to the set of fixed points in the quantity space for the original variable. We, however, add it as a *new parameter of the model*, subject to influences similar to those of the original quantity. Thus, we can represent and reason about change in both the actual value and the equilibrium value in response to active processes. For example, if the actual temperature is only slightly sensitive to the heat-flow rate, but the equilibrium temperature is very sensitive, then we might conclude that the system will be slow in returning to equilibrium once perturbed. The extended influence diagram for the feedback loop is shown in Fig 3b.

B. Sensitivity Annotations

An influence is a partial function from the controlling variable to the controlled variable. In QP theory, computing a value for a controlled variable takes place in two phases:

1. All of the individual influences on the controlled variable must be identified and the effect of each of these must be computed.
2. The various effects must be combined to determine the composite effect on the controlled variable.

This procedure relies on local propagation to perform influence resolution. If local propagation is to carry the burden of our extended influence resolution, then the propagated value must somehow be extended to represent the sensitivity information. The value being propagated in influence resolution is a quantity, and the representation used in sign abstraction. Given our model of extended influences as describing the normalised sensitivity of one variable to changes in another, we can simply extend the quantity representation for the influence quantity and use a discrete scale of influence magnitudes. We then represent the actual value as a fuzzy set over this value space, to model the imprecision in the available sensitivity information. While this procedure is conceptually simple, the question arises of how an appropriate discretization for this normalised change value, henceforth referred to as influence, can be determined.

If we start with an n-level influence discretization and an m-level sensitivity discretization, then after k influence propagation steps we seemingly might need an nm^k discretization to avoid information loss. This worst case complexity can be substantially reduced, however, by the following four observations:

1. We are only interested in the result at a resolution equivalent to the original n-level discretization.
2. Additional detail is only relevant when two annotated influences are being combined, to aid in influence resolution if they conflict.
3. Rather than annotating all influences in a graph, we will only annotate those necessary to disambiguate parameters of interest in a specific query.
4. The basic fuzzy relational influence algorithm can be designed so that failure to maintain a fully detailed discretization only increases the ambiguity of the result, rather than produce incorrect results.

Given this, we model sensitivity annotations as parameters of a standard *fuzzy relational influence algorithm* [Zadeh, 1973]. We choose a fuzzy representation to allow simple modelling of the imprecision of these annotations[2]. We next detail the algorithms used to compute the consequences of this fuzzy algorithm.

1. Computing individual influences

An influence of the form:

(Influenced-variable Q+/- Influencing-variable, Sensitivity)

is taken to specify a fuzzy relation between three amounts: C, the amount of the influencing variable; S, the amount of the influence sensitivity; and Iv, the amount of the influence on the influenced variable. The value of Iv can be computed as follows:

$$Iv = \sum_{C,S}(\min(\mu_C, \mu_S, \mu_Q)/Q_{I,C,S}(C,S))$$

$$Q_{I,C,S}(C_j, S_k) = sign(C_j * S_k) * (abs(C_j * S_k)^{1/2})$$

2. Combining influences

Sensitivity annotations provide us with a means of estimating influence magnitudes, which are directly comparable. Below we show an algorithm for computing the combined effect of two influences. A rough translation is that an element is definitely a member of the set of possible values for the combined influence if that element is a member of the value sets for both input values, or if it is a member of the value set for one input, and a weaker element of the same sign is a member of the value set for the other input. Also, an element of the discretization may be an element of the result set under two conditions. First, if it is a member of the value set of one input, and a element of the same magnitude but opposite sign is a member of the value set for the other input. Second, if an element of the same sign but greater magnitude is a member of one value set, and an element of the opposite sign and greater magnitude is a member of the other value set:

$$\begin{aligned}\mu_{Iv}(i) \; = \; & (\mu_{Iv1}(i) \wedge \mu_{Iv2}(i)) \\ & \vee(\vee_{j,|j|<|i|}(\mu_{Iv1}(i) \wedge \mu_{Iv2}(j))) \\ & \vee(\mu_{Iv1}(i) \wedge \mu_{Iv2}(-i) \wedge unknown) \\ & \vee(\vee_{j,j>i} \vee_{k,k<-i} (\mu_{Iv1}(j) \wedge \mu_{Iv2}(k) \wedge unknown))\end{aligned}$$

Subsrcipts i, j, and k are assumed to be 0 for no influence, increasing positive for positive influence elements, and increasing negative for increasing negative influence elements (e.g., -3, -2, -1, 0, 1, 2, 3 for a seven element discrete scale, with -3 the strongest negative influence). The above is only half of the formula actually used. The actual relation is symmetrical in the two influences Iv1 and Iv2.

C. Annotation Management

In examining the sources of ambiguity in the reaction vessel example, we note that many of the annotations which could resolve the ambiguities are not universally valid. In fact, we identify four levels of validity for an annotation. These validity levels are determined primarily by opportunities in the implementation:

1. An annotation is *universally* valid when it can be incorporated directly into a view or process description, and correctly describes the functioning of a particular influence in all situations in which an instance of the view or process participates. These are rare.
2. An annotation is *scenario* valid when it correctly describes the operation of a particular influence in a particular view or process instance, for all qualitative states in which the instance is active. Product temperature annotations in the example are an instance of this annotation type.
3. An annotation is *state* valid when it correctly describes the operation of a particular influence in a view or process instance, only for a defined subset of the qualitative states of a system.
4. Annotation is *query* valid when it correctly describes the operation of a particular influence in a view or process instance, only for a particular query. The conflict triangle annotation for determining off-gas temperature in the offtake is an example of this type of annotation.

The first type of annotation can simply be part of the basic view or process definition. The other three are added to the QP description of a scenario as needed during problem solving. A four step algorithm extends the basic QP theory influence resolution algorithm:

[2] The underlying model is of a set of independent, linear influences. Fuzzy set models of sensitivities permit us to allow for the inaccuracies of this model.

1. Execute the basic influence resolution.
2. Check results for ambiguities in parameter values of interest. If all interesting parameter values are determined uniquely, then problem solving is complete.
3. Otherwise, search the influence graph for instances of ambiguity causing subgraphs. If one is found, and the parameter for which it might create an ambiguity is ambiguous, then annotate the subgraph with influence sensitivity information if available.
4. Re-execute the basic influence resolution algorithm on the now annotated graph.

This algorithm assumes the extended QP reasoner is embedded in a larger system which has or can obtain the necessary problem specific information to resolve ambiguities. It provides a problem directed way of selecting aspects of the larger system's problem specific knowledge relevant to the query being processed.

V. Summary

We have described an extension to QP theory which increases the precision of results available, and still retains the inherent advantages of qualitative modelling. This extension derives its power from influence sensitivity annotations, and a fuzzy mathematical model of influences which permits propagation of the effects of these sensitivities throughout the influence chart.

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