Reasoning about Discontinuous Change

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ABSTRACT

Intuitively, discontinuous changes can be seen as very rapid continuous changes. A couple of alternative methods based on this ontology are presented and compared. One, called the approximation method, approximates discontinuous change by continuous function and then calculates a limit. The other, called the direct method, directly creates a chain of hypothetical intermediate states (mythical instants) which a given circuit is supposed to go through during a discontinuous change. Although the direct method may fail to predict certain properties of discontinuity and its applicability is limited, it is more efficient than the approximation method. The direct method has been fully implemented and incorporated into an existing qualitative reasoning program.

I. Introduction

Continuous change is a notion in which quantities are assumed to take a certain amount of time to change value. Discontinuous changes are those to which this assumption does not apply; quantities can change value in a moment.

Notion of discontinuous change plays a crucial role in characterizing the behavior of dynamic systems, such as nonlinear oscillators or flip-flops, without worrying about unmotivated details. At the commonsense level, the notion of discontinuous change seems to be natural; things appear to suddenly stop moving, collide, disappear and so on.

Unfortunately, analysis of discontinuous changes is not easy. This is mainly because ordinary models for physical systems (*e.g.*, circuit equations) do not always specify the system's behavior under discontinuous change in full detail. In textbooks, this problem is often solved by using an ontology in which discontinuous change is very rapid continuous change.

A couple of alternative methods are possible to implement this view. One, called *the approximation method*, approximates discontinuous change by a continuous function and then calculates a limit. The other, called *the direct method*, uses a notion of mythical instants to describe hypothetical intermediate states which a given circuit is supposed to go through during a discontinuous change.

In this paper, we present and compare these two algorithms. We base our theory on qualitative reasoning, a formal theory for causal understanding, and we choose electronic circuits as a subject domain. In the next section, we study properties of discontinuous changes. In section III, we will briefly overview previous work in qualitative reasoning and see how discontinuity has been handled. In sections IV and V, we will describe the two algorithms separately, and in section VI, we will compare the two and summarize the discussion.

II. Properties of Discontinuous Change

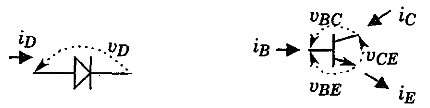
A. Origin of Discontinuity

The varieties of discontinuous changes depend on the physical model employed. In this paper, we study discontinuous changes arising in piecewise linear equation models for electronic circuits, since the use of piecewise linear equations is one of the most popular techniques in the electronic circuit domain. In this modeling, nonlinear circuit elements, such as diodes or transistors, are described with multiple operating regions. Circuit devices modeled with multiple operating regions will be called *multiple-mode devices*. Figure 1 shows the models for diodes and transistors we employ for explanation in this paper. Although they might appear too simple, they suffice for the discussion below, since the same kind of phenomena arise even when more complex models are used, as will be seen below.

Possible causes of a single occurrence of discontinuous change arising in piecewise linear circuit models can be classified into three categories:

(A1) discontinuous input
(A2) mode transition of a multiple-mode device
(A3) positive feedback without time delay.

(a) A model for diodes. (b) A model for transistors.



[OFF] $v_D < v_o$, $i_D = 0$. *[ON]* $v_D = v_o$, $i_D \geqq 0$

[OFF] $v_{BE} < v_o$, $i_B = i_C = i_E = 0$. *[ON]* $v_{BE} = v_o$, $v_{CE} = 0$, $i_B \geqq 0$, $i_C \geqq 0$.

$i_B + i_C = i_E$, $v_{BE} = v_{BC} + v_{CE}$

(v_o is a positive constant.)

Figure 1. Device models in piecewise linear equations.

Discontinuous change caused by cause category A*i* will be referred to as type A*i* discontinuity.

Properties of type A1 discontinuity have been studied in depth in the field of transient analysis. Mathematical techniques such as Laplace or Fourier transform methods are studied to see how discontinuous input affects a given circuit. However, humans seem to use much simpler method when they characterize the behavior of many pulse circuits at a commonsense level.

Type A2 discontinuity results from the use of piecewise linear equations as a circuit model. For example, the diode model shown in figure 1(a) causes derivatives of current or voltage to change discontinuously when the operating region of the diode transitions. If detailed, nonlinear description of a circuit device is used, discontinuity of this type might disappear. However, abstract, less precise models are more useful in characterizing the behavior of nonlinear devices in such circuits as regulators, TTL, or Schmitt triggers. Note also that as long as we use a piecewise linear model, we cannot avoid discontinuous change (at least of first order derivatives) on mode transition.

Positive feedback without time delay may accelerate any small disturbance *ad infinitum*, resulting in type A3 discontinuity. Positive feedback is not rare in electronic circuits; positive feedback is observed in circuits containing devices such as tunnel diodes which exhibit negative incremental resistance in some operating mode, or one can design a circuit with positive feedback to create a memory (or a stable state), to generate pulses, and so on. In fact, the latter is an important technique in digital circuit design. Note that a positive feedback will not result in value jump, if the feedback factor is less than *1*. Note also that ordinary qualitative reasoners may produce an undesired result unless positive feedback is correctly recognized and handled.

B. Properties of Discontinuous Change

Discontinuous change arising from piecewise linear models has at least two properties.

(Property 1) *Causal structure of the system may change during discontinuous change.* Theories of qualitative reasoning view causality as a value dependency among variables. More computationally, it can be seen as information flow from the cause to the effect [de Kleer and Brown, 1984]. When a circuit consists only of passive elements, such as resistors or capacitors, its causal structure will not change in general. In contrast, when multiple-mode devices are involved, the causal structure of a circuit may change drastically, due to the transition of operating mode. An analysis program should be able to recognize and keep track of the change of causal structure so as to generate a causal explanation.

(Property 2) *A number of discontinuous changes of different types may occur one after another.* Discontinuous change applied to the input of a given circuit will be propagated into other parts, possibly creating a complex chain of events. The problem here is that ordinary circuit equations do not contain sufficient information to explain this process in stepwise, causal terms. Consider the hypothetical circuit shown in figure 2. In textbooks of electronic

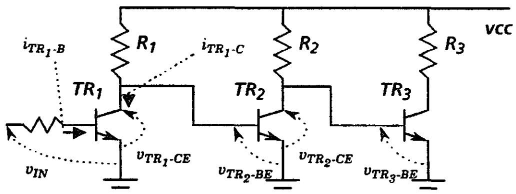


Figure 2. A hypothetical transistor circuit.

engineering, the behavior of this circuit with v_{IN} being raised from *zero* is usually explained as follows:

> when v_{IN} rises and reaches some level, TR_1 will turn *ON*, causing TR_2 to turn *OFF*, causing TR_3 to turn *ON*.

This explanation implicitly assumes the following states:

- *state-1*: TR_1: *OFF*, TR_2: *ON*, TR_3: *OFF*
- *state-2*: TR_1: *ON*, TR_2: *ON*, TR_3: *OFF*
- *state-3*: TR_1: *ON*, TR_2: *OFF*, TR_3: *OFF*
- *state-4*: TR_1: *ON*, TR_2: *OFF*, TR_3: *ON*.

Among these, *state-2* and *state-3* are *mythical* in the sense that they do not satisfy circuit equations employed here. By definition, mythical instants terminate instantaneously. Although one might want to use only legal states (namely, *state-1* and *state-4*), a resulting explanation would be acausal and magical. This kind of situation arises as long as one uses an abstract model to capture the physical world. The direct method attempts to use mythical states to create a causal explanation.

III. Previous Work

So far, the mainstream of qualitative reasoning has been analysis under continuity assumption [de Kleer, 1984] [Kuipers, 1984] [Williams, 1984]. Analysis of discontinuous change has received an inadequate treatment. In the domain of electronic circuits, J. de Kleer and B. Williams each have given algorithms for analyzing operating mode transition. De Kleer's EQUAL does not allow discontinuous change even when a mode transition occurs [de Kleer, 1984, p. 272]. Williams allows discontinuous change only when discontinuity is explicitly indicated in a device model. But neither of the two deals with type A1 or A3 discontinuity or property (2).

Qualitative Process (QP) theory [Forbus, 1984] has more flexibility with discontinuous changes. Discontinuous change is allowed when processes are switched or when a special process, one for collision for instance, comes into play. Unfortunately, the process centered ontology of QP theory does not seem to match the device centered view taken by circuit engineers. Aggregation theory [Weld, 1985] also handles discontinuity, but from a different perspective.

IV. The Approximation Method

A. Outline of the Method

If discontinuous changes are very rapid continuous changes, it would be natural to approximate them by continuous change and then to calculate a limit. This idea can be implemented using a simple version of infinitesimal calculus. The analysis is carried out in two stages:

1. Replacing discontinuous input by qualitatively continuous change in infinitesimal calculus.

2. Carrying out envisionment using an infinitesimal calculus. A number of techniques [Robinson, 1966] [Nishida *et al.*, 1985] [Raiman, 1986] are available for this. For the purpose of this paper, our simpler method will do. We use a set of symbols {0, ε(infinitesimal), M(medium), ∞(infinitely large)}, to represent order of magnitude. Among these symbols, we can define some obvious rules, such as $\varepsilon+\varepsilon=\varepsilon$, $\varepsilon+M=M$, $\varepsilon\times M=\varepsilon$, $\varepsilon\times\infty=?$, *etc.* We use a value interval to represent the range of the changing value. For example, $(-\varepsilon, M)$ means that the value is changing between some negative infinitesimal and some positive mid-range value. We abbreviate (a, a) as a.

In order to see the possible behavior of a given system over time, we can make use of the following qualitative integration rule (on time):

given a time interval I: $[t_0, t_1]$ and a function $f(t)$, the value of $f(t_1)$ is constrained by the following formula:

$$[f(t_1)]\subset[f(t_0)]+[length(I)]\times[range_I(\partial f)]$$

where, $length(I)$: the length of the interval I,

$range_I(\partial f)$: the value range of ∂f during the interval I.

Although this rule seems to be too underconstrained, it is useful when the length of interval I is infinitesimal.

B. Application of the Method

This method directly applies to the type A1 discontinuity. Suppose a step input is applied to the circuit shown in figure 3(a). This discontinuous input is approximated to the second order derivatives, as shown in figure 3(c). Table 1 shows the result of

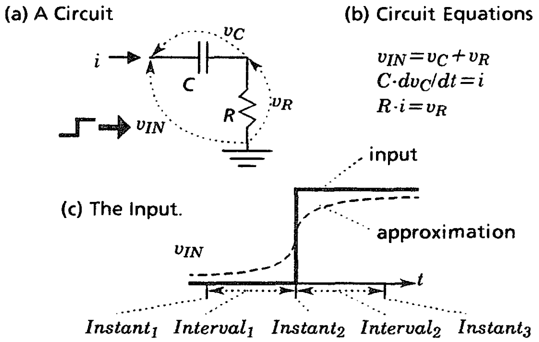


Figure 3. A sample circuit and approximation of discontinuous input.

envisionment. The three rows headed by $\partial^i v_{IN}$ represent the approximated input. The six rows headed by $\partial^i v_C$ or $\partial^i v_R$ are the result of envisionment. They are derived by integrating various constraints.

Table 1. Analysis of type A1 discontinuity by the approximation method.

	$Instant_1$	$Interval_1$ (length ε)	$Instant_2$	$Interval_2$ (length ε)	$Instant_3$
$\partial^0 v_{IN}$	$+\varepsilon$	$(+\varepsilon, M)$	$+M$	$+M$	$+M$
$\partial^1 v_{IN}$	$+\varepsilon$	$(+\varepsilon, +\infty)$	$+\infty$	$(+\varepsilon, +\infty)$	$+\varepsilon$
$\partial^2 v_{IN}$	$+\varepsilon$	$(+\varepsilon, +\infty)$	0	$(-\infty, -\varepsilon)$	$-\varepsilon$
$\partial^0 v_C$	0^*	$(-\varepsilon, +\varepsilon)$	$(-\varepsilon, +\varepsilon)$	$(-\varepsilon, +\varepsilon)$	$(-\varepsilon, +\varepsilon)$
$\partial^1 v_C$	$+\varepsilon$	$(-\varepsilon, +M)$	$+M$	$+M$	$+M$
$\partial^2 v_C$	$(-\varepsilon, +\varepsilon)$	$(-\varepsilon, +\infty)$	$+\infty$	$(-M, +\infty)$	$-M$
$\partial^0 v_R$	$+\varepsilon$	$(-\varepsilon, +M)$	$+M$	$+M$	$+M$
$\partial^1 v_R$	$(-\varepsilon, +\varepsilon)$	$(-\varepsilon, +\infty)$	$+\infty$	$(-M, +\infty)$	$-M$
$\partial^2 v_R$	$(-\varepsilon, +\varepsilon)$	$(-\infty, +\infty)$	$-\infty$	$(-\infty, +M)$	$+M$

*) $\partial^0 v_C(instant_1)=0$ is assumed.

For example, the value of $\partial^0 v_C$ at $instant_2$ is constrained using the qualitative integration rule, as follows:

$$\partial^0 v_C(Instant_2)\subset\partial^0 v_C(Instant_1)+\int_{Interval_1}\partial^1 v_C dt.$$

The right handside will be simplified as follows:

$$0+\varepsilon\times(-\varepsilon, +M)=(-\varepsilon, +\varepsilon).$$

For the value of $\partial^0 v_R$ at $instant_2$, the value range $+M$ obtained by applying the equation:

$$\partial^0 v_R(Instant_2)=\partial^0 v_{IN}(Instant_2)-\partial^0 v_C(Instant_2)$$

is more precise than that from applying the integration rule (namely, $(-\varepsilon, +\infty)$). Hence we have employed the former.

For type A2 and A3 discontinuity and property (1), however, the above idea does not suggest a solution. In order to handle these problems, we use a technique called dynamic causal stream analysis [Nishida *et al.*, 1987], which has some similarity to the transition analysis [Williams, 1984]. Details will be mentioned in section V-B, since the same algorithm is also used in the direct method.

Difficulty arises when a discontinuous change evolves from the inside of a given system. Suppose for instance the situation in figure 4, in which discontinuous change evolves at variables v_2, v_3, and v_4, due to positive feedback. In order to apply the idea

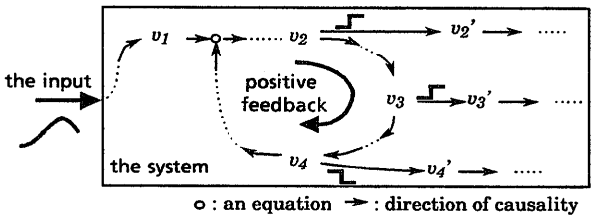


Figure 4. A situation in which discontinuous change evolves internally.

mentioned above likewise to this kind of situation, we must replace each internal occurrence of discontinuous change by continuous change before propagating them into other parts (involving v_2', v_3', and v_4' in this case) of the system. For this purpose, the notion of local evolution of time [Williams, 1986] seems to provide an adequate ontology, though it is not explored in this paper.

It is not obvious, however, whether or not the above algorithm will eventually terminate with a correct state description for the next stable state. The problem is complicated since mode transition may change the structure of the causal network during the above process. Although we have not proved, this algorithm seems to work correctly for normal circuits.

V. The Direct Method

A. Outline of the Method

The direct method produces stepwise causal explanations for discontinuous changes by admitting intermediate mythical states which are not consistent with circuit equations. Mythical instants result from assuming as a default that the operating mode of multiple-mode devices and the value of variables constrained by integral will not change unless otherwise specified. These assumptions are called *persistence of operating mode* and *persistence of integrated quantity*, respectively. Analysis with these default assumptions seems to coincide with our intuitions, at least in the electronic circuit domain.

Unlike the approximation method, the direct method does not place any hypothetical time intervals of infinitesimal length between adjacent instantaneous states during discontinuous change; instead, it directly predicts the next instant by analyzing the current instant. This process is repeated until the algorithm encounters a normal instant without any inconsistency. As a result, the algorithm will produce a chain of successive mythical instants followed by a normal instant for each successive occurrence of discontinuous change.

Our discontinuity-as-a-very-rapid-continuous-change ontology is used in various forms in this process. For example, we use the following rules:

[Continuity in discontinuous change] When the value of a variable x changes (either continuously or discontinuously) from one value a to another b ($b \neq a$), change to any value c between a and b always occurs before that to b.

[Adjacency in operating mode transition] A muliple-mode device cannot transit from one operating mode to another in one transition, unless the next mode is adjacent to the original.

Like canonicality heuristics [de Kleer and Brown, 1984], these rules provide a basis of canonical explanation.

B. Analyzing Discontinuity Using the Direct Method

The key idea in analyzing type A1 discontinuity is to identify variables which will not be instantaneously affected by the discontinuous input. Causal analysis [Williams, 1984] [Nishida *et al.*, 1987] helps us do this. If it is possible to assign to equations for a given circuit causal directions in such a way that no differential causality (*i.e.*, data flow from a variable to its derivative) is involved, we can safely say that the output of each integral causality will remain unchanged during discontinuous change. For example, we can predict that the voltage v_C across the capacitor C in the circuit in figure 3 will not be affected by a discontinuous input, since we can consistently think of the value of v_C as being determined by integrating i/C. Notice that during the above process the predicted state may turn out to be inconsistent with the circuit equations.

Inconsistencies encountered during the analysis are analyzed so as to predict the next state. Type A2 and A3 discontinuity is recognized during the analysis of inconsistency. It goes in three steps:

1. Singling out an incorrect assumption. First, a set of assumed equations or inequalities that are relevant to the inconsistency (*the suspect set*) is built by tracing back the causal structure from a constraint in contradiction; and then, if the suspect set contains

more one element, it is filtered by using canonicality heuristics for discontinuous change (some of which are mentioned in the last section) and the preference rules as follows: an inequality supported by a persistence-of-operating-mode assumption is most preferred as a *culprit*, then comes an equation supported by a persistence-of-operating-mode assumption, and finally an equation supported by a persistence-of-integrated-quantity assumption.

It is possible that the suspect set may still contain more than one element. This will produce an ambiguous result.

2. Predicting the next state. If it turns out that a persistence-of-integrated-quantity assumption is supporting the culprit, the assumption is simply retracted. This will loosen the current constraints by one degree of freedom, resolving inconsistency.

If a persistence-of-operating-mode assumption is blamed for supporting the culprit, the next operating mode is sought by examining how circuit equations or inequalities get violated. In order to make this process run efficiently, we associate a suggestion about the next operating mode with each assumption-based constraint. For example, associated with an inequality $v_D < v_\sigma$ (a condition for a diode to be *OFF*) is a note which suggests that if this condition is violated due to the rise of v_D, then the next operating mode of the diode will be *ON*. Notice that the search for the next operating mode becomes crucial when multiple-mode devices are modeled with many operating modes.

3. Constructing a state description for the next state. The state description for the next state is obtained from the current state description, rather than recomputed from the beginning. First, the current set of circuit equations and inequalities are modified by retracting those depending on the culprit and adding those associated with a new assumption. Then, the causal structure for the next state is reconstructed, which is used to compute the state description. If the state description is obtained successfully, the next state is judged as a normal instant, followed by an interval. Otherwise, the causal structure for a new state is checked for a positive feedback, to see a possibility of type A3 discontinuity. If this is the case, a special procedure is applied to determine the direction of jump and to foresee a possible conclusion of the jump. Otherwise analysis of inconsistency is repeated.

A rule for predicting the direction of value jump caused by a positive feedback is as follows:

if a variable in a positive feedback loop depends positively (negatively) on the primary cause, the value will jump to the reverse (same) direction.

This rule is derived from an ontological ground. Consider for example a system which is modeled by equations: $x=y+z$, and $z=K \cdot y$ (x: input, K: a constant), and let the constant K be set to K_0 (>0). A positive feedback comes into play if the constant K is changed to K_1 (<-1) as a result of a mode transition. Although in piecewise linear models, K changes instantaneously, it would be beneficial to think about a hypothetical situation in which K changes gradually from K_0 to K_1. The closer K comes to -1, the bigger becomes y/x and $-z/x$, since $y=(1/(1+K)) \cdot x$ and $z=(K/(1+K)) \cdot x$. Notice that the above rule for jumping values is exemplified, since y depends negatively on x and z positively on x when K reaches K_1 and a positive feedback comes into play.

C. An Example

In general, the direct method provides a simple but a powerful means for dealing with chains of discontinuous change. Let us see how it works for an unstable multi-vibrator shown in figure 5.

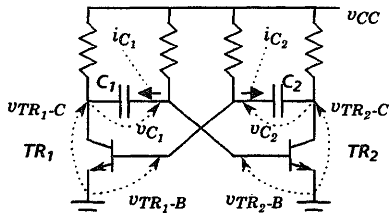


Figure 5. An unstable multi-vibrator.

1. Initial condition. We assume TR_1 and TR_2 are initially *ON* and *OFF*, respectively, and both v_{C_1} and v_{C_2} are involved in an interval $(v_\sigma - v_{CC}, v_\sigma)$, where v_σ is a threshold (see figure 1(b)), and $v_{CC} > v_\sigma > 0$.

2. Analyzing the initial state. It follows that the capacitor C_1 is being charged, raising the base voltage $v_{TR_2\text{-}B}$ of the transistor TR_2. Thus, it is foreseen that the condition $v_{TR_2\text{-}B} < v_\sigma$ for TR_2 to be *OFF* will eventually be violated, turning TR_2 *ON*. Notice that C_2 is being discharged, keeping v_{C_2} less than v_σ. This fact will be used in the next step. Notice also that although the base current into TR_1 is positive and is decreasing, it will not reach *zero* since before that happens the capacitor C_2 would be saturated.

3. Constructing a state description for the next instant, say $instant_1$. It is assumed as a default that TR_1 remains *ON* (persistence of operating mode), and the values of v_{C_1} and v_{C_2} will not be affected by the transition of operating mode (persistence of integrated quantities). Unfortunately, these assumptions turn out to be inconsistent, because $v_{TR_1\text{-}B}$ must be below v_σ, on the one hand, since $v_{TR_1\text{-}B} = v_{C_2} + v_{TR_2\text{-}C}$, $v_{C_2} < v_\sigma$ and $v_{TR_2\text{-}C} = 0$, and $v_{TR_1\text{-}B}$ must be equal to v_σ, on the other hand, since TR_1 remains *ON*. Relevant to this

contradiction are the assumptions that v_{C_1} remains unchanged and that TR_1 remains ON. The latter is preferred as a culprit (see the last section) and is retracted. Notice that it also follows that TR_1 will turn OFF since it is now assumed that $v_{TR_1\text{-}B}$ drops below v_0. Thus, it has turned out that the current state is mythical and immediately followed by another instant, say $instant_2$.

4. Constructing a state description for $instant_2$. This time no inconsistency is encountered and $instant_2$ is declared to be normal. Hence, it is followed by an interval, in which TR_1 is OFF, TR_2 ON, C_1 being discharged, and C_2 being charged, just symmetric with the initial situation.

Notice that the above analysis predicts that a number of variables change their value discontinuously. For example, $v_{TR_2\text{-}C}$ is expected to rise discontinuously from $zero$, $v_{TR_1\text{-}B}$ drop discontinuously from v_0, and so on.

Ⅵ. Comparison and Concluding Remarks

These two methods differ in terms of preciseness and efficiency.

Preciseness. In general, the approximation method seems to implement the discontinuity-as-a-very-rapid-continuous change ontology with more fidelity. The direct method may fail to characterize certain properties of the response to discontinuous input. If the direct method is applied to the example shown in figure 3, it will predict that $\partial^0 v_R$, $\partial^1 v_R$ and $\partial^2 v_R$, will jump to $+$, $-$ and $+$, respectively, as a result of discontinuous input. Unlike the result shown in table-1, prediction by the direct method does not make explicit the fact that $\partial^i v_R (i \geqq 1)$ has several keen peaks of infinitely large magnitude. Fortunately, those peaks do not cause serious problems in the electronic circuit domain. In ordinary models of electronic circuits, the operating mode of each circuit element is determined only by variables on ∂^0 level (those that stand either for voltage or for current). Therefore, a peak at ∂^1 level plays a critical role only when differential causality is involved and the value of a ∂^0 level variable is determined by that of a ∂^1 level variable. First of all, circuits with differential causality are relatively rare. Second, existence of differential causality can be detected by causal analysis. It serves as a warning.

Efficiency. A naive implementation of the approximation method will result in an inefficient algorithm because the approximation-limit process will be carried out uniformly for a discontinuous input irrespective of necessity. In contrast, the direct method is more efficient because the computation process is invoked only when inconsistency is detected. Compare also how type A1 discontinuity is handled by each method (see table-1 and description in section Ⅴ-B).

We have incorporated an algorithm based on the direct method into an existing qualitative reasoning program QR-1 [Nishida *et al.*, 1987]. It can analyze all the examples shown in this paper. Our future direction is twofold: extension for differential causality and ill-formed circuits. The robustness against ill-formed circuit is crucially important in ICAI environments where students use the program for reviewing their circuits.

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