A Qualitative Approach to Mechanical Constraint*

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Abstract

This paper provides a qualitative analysis of instantaneous, constrained motions in rigid bodies. We develop a symbolic spatial representation to describe the effects of configuration on the dynamic behavior of rigid objects. We also explore the way symbolic shape information may be used to reason about force transmission. This information may be used to provide a static analysis for a given configuration and is an important component of the calculation of behavioral transitions when envisioning device behavior. All results are based on an implementation.

1 Introduction

The goal of Qualitative Mechanics (QM) is to produce a commonsense theory of mechanical analysis sufficient to describe the behavior of rigid body devices. We want theories which describe both the behavior of common mechanisms such as gear trains, pistons, and ratchets, as well as mechanisms which contain unusual or variant devices such as clock escapements. These descriptions may be used to predict the behavior of an unknown mechanism, determine the suitability of a given device for a task, diagnose mechanical failures, and critically analyze new mechanisms.

There is a great deal of interest in developing AI tools to assist in mechanics, both by mechanical engineers [Dixon, 1986] and AI researchers [Davis, 1986; deKleer, 1975; deKleer and Brown, 1984; Forbus, 1981; Gelsey, 1987; Kuipers, 1986; Laughton, 1985; Shoham, 1985; Stanfill, 1985]. In addition, for a robot to interact with the physical world and manipulate its environment it must be able to accomplish tasks such as turning knobs, opening doors, lifting boxes, and stacking objects. Except in a highly artificial environment these tasks all require deep knowledge of the basic underlying principles of statics discussed in this paper.

We assume as input a specification of the set of objects involved, a set of possible configurations of these objects, and the external forces acting on the device, if any. The results of this analysis are the instantaneous directions in which an object can and must move. For example if a scape wheel is moving clockwise and the fore pallet is in contact with a tooth on the scape wheel, how might the scape wheel move?

This result covers a wide range of statics problems and is an initial step in understanding the qualitative mechanics of motion. All of these ideas have been implemented in a program called *ALEX*, and the examples are taken from that program.

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1.1 Overview

Section 2 describes the methods used to represent spatial descriptors and rigid body objects. Section 3 presents the theory of qualitatively constrained motion. This section is divided into three parts: motion prevented by contact with immovable objects, motions required by contact with moving objects, and motion allowed by forces external to the device. Section 4 briefly describes how these ideas are used in mechanism analysis. Section 5 provides a summary and discusses other research on QM.

2 Representations

2.1 Spatial Representations

2.1.1 Translational Direction

A concept of direction is essential to spatial reasoning. When people describe direction in space, without resorting to diagrams or mathematics, they typically use words such as "right, left"; "up, down"; and "front, back" relative to some frame of reference. We use this order and assign "+" to the first of each of these pairs and "–" to the second with "0" meaning center. This description corresponds to the signs of the numerical values in a Cartesian coordinate system or to the sign of the cosine and the sine in a polar coordinate system.

Directions in space are described by combining these values over several dimensions. A vector which corresponds to an axis may be represented exactly, for example, (+ 0 0) indicates the vector lies along the positive X axis. Vectors which do not correspond to axes are represented by continuous spatial regions. For example any location to the lower left of some reference will be written (– –) or (– – 0) where 0 indicates there is no magnitude along the Z axis.

Definition 1 (Translational Direction) `trans-dir(x)` *is defined only over vector quantities. It is the ordered set of the signs of the direction vector of* x.

Definition 2 (Possible Directions) *The set of all possible translational directions τ is the Cartesian product of the sets of all possible directions for each dimension. The set of all possible translational directions for a single dimension is* {+, 0, –}.

2.1.2 Rotational Direction

People typically describe direction of rotation either by the direction of a tangent or using *counter-clockwise* and *clockwise*. We represent a counter-clockwise rotation as "+" and a clockwise rotation as "–" when looking along a positive axis toward the origin. Thus the way we normally perceive the movement of the hands of a clock is (–) in 2 dimensions or (0 0 –) in three dimensions (clockwise about the Z axis). Note that this representation of rotational directions corresponds to the signs of

		sign(x)		
		−	0	+
sign(y)	−	−	−	N1
	0	−	0	+
	+	N1	+	+

N1 :	if X > Y then sign(X) if X < Y then sign(Y) if X = Y then 0

Table 1: [X] + [Y]

		sign(x)		
		−	0	+
sign(y)	−	+	0	−
	0	0	0	0
	+	−	0	+

Table 2: [X] * [Y]

numerical values in a right handed Cartesian coordinate system.

Definition 3 (Rotational Direction) `rot-dir(x)` *is the ordered set of the signs of* x*'s rotation about each axis.*

2.2 Vector Arithmetic

The calculations we perform on vector quantities include computing open half planes and ninety degree rotations of vectors which are represented in the manner specified above. These calculations make use of the vector dot product $(a_1a_2 + b_1b_2 + c_1c_2)$ and the vector rotation formulas. In the vector rotation formulas a rotation of ninety degrees causes the cosines to become zeros and the sines to become ones, so that the only math we need consider is addition and multiplication of signs. The qualitative arithmetic(cf., table 1 2) [deKleer and Brown, 1984] provides these results.

Definition 4 (Half Plane) *The predicate* `Half-Plane(x, y)` *is true if* x *and* y *are both vector quantities, and the sign of the vector dot product of* x *and* y *is "+".*

Definition 5 (Rotate-90) `Rotate-90(x, y, r)` *is true iff* y *is the vector which is perpendicular to* x *by a rotation in the rotational direction* r*.*

2.3 Objects

Rigid objects are represented by the set of their surfaces, and surfaces, in turn, are represented by both the qualitative direction of the surface normal *and* the direction from the surface to the center of rotation. For example, the bottom of a two dimensional block consists of three qualitatively distinct surfaces all of which have a surface normal in the *down* direction but whose directions to the center of mass are *up-right*, *up*, and *up-left*. Places where the surface normal is not defined (corners) are represented by the set of adjacent surfaces. For tractability we assume the centers of rotation to be fixed.

Definition 6 (Surface) *The predicate* `Surface(x, p)` *is true if* p *is a point (or set of qualitatively equivalent points) on the perimeter of object* x*.*

Definition 7 (Surface Normal) `Surface-Normal(p, d)` *is true if* d *is the direction of the surface normal at the surface* p*.*

Definition 8 (Origin Direction) `Origin-Dir(p, d)` *is true iff* d *is the translational direction from a point or surface,* p*, on an object to the center of rotation of that object.*

2.4 Contact

In order for an object to affect another object there must be contact between the two objects in some sense. (If we think of effects such as gravity and magnetism as a field, we then can then reason about the contact between this field and an object.) The `Contact` relation shows which parts of objects are in contact.

Definition 9 (Contact) *The predicate* `Contact(x, y)` *indicates that the distance from* x *to* y *is less than* ϵ*. Where* x *and* y *are both surfaces of objects.*

3 Mechanical Motion

3.1 Blocking

This subsection answers two questions. Given contact between an object and an obstacle :

1. How will the motion constraints of the obstacle block the object?
2. What motions of the obstacle must be constrained to block the object?

A *constraint* is a reaction force which absolutely prevents a body from moving a certain way. Constrained motion is essential to understanding mechanics because a *machine* is defined as "any device consisting of two or more resistant, relatively constrained parts which may serve to transmit and modify force and motion so as to do work [Cowie, 1961]." The opposite of a constraint is a *freedom*. In three dimensions there are six *degrees of freedom* (an object can rotate about any of the three axes or translate along any of the axes), and in two dimensions there are three degrees of freedom (two translational and one rotational). In our analysis an object is assumed free to move in each direction unless it is specifically constrained.

Definition 10 (Motion) `TransMotion(o, t)` *indicates* o *has instantaneous linear motion in direction* t*.*
`RotMotion(o, r)` *indicates* o *has instantaneous rotational motion in direction* r*.*

Definition 11 (Constraint) `TransConstraint(o, t)` *is true when object* o *is absolutely prevented from moving translationally in direction* t*.* `RotConstraint(o, r)` *is true when object* o *is absolutely prevented from moving rotationally in direction* r*.*

Definition 12 (Freedom) `TransFreedom(o, t)` *is true when object* o *is not prevented from moving translationally in direction* t*.* `RotFreedom(o, r)` *is true when object* o *is not prevented from moving rotationally in direction* r*.*

The constraints which may be imposed when two objects are in contact are given in figure 1. This says that if an obstacle is "sufficiently" constrained it will prevent the following motions of an object in contact:

- translational motion into the open half plane centered on the object's surface normal at the point of contact,

$$
\begin{array}{l}
(\forall\ obj,\ obst,\ p,\ q,\ sn) \\
\quad \{RigidBody(obst) \wedge RigidBody(obj) \wedge \\
\quad Surface(obst,\ p) \wedge Surface(obj,\ q) \wedge \\
\quad Contact(p,\ q) \wedge Surface\text{-}Normal(p,\ sn) \wedge \\
\quad Origin\text{-}Dir(p,\ o_1) \wedge Origin\text{-}Dir(q,\ o_2) \wedge \\
\quad (\forall d_1)\ [Half\text{-}Plane(\text{-}sn,\ d_1) \\
\qquad \Rightarrow TransConstraint(obst,\ d_1)] \wedge \\
\quad (\forall r_1 \exists x_1)\ [Rotate\text{-}90(\text{-}sn,\ x_1, r_1) \\
\qquad \wedge\ Half\text{-}Plane(x_1, o_1) \\
\qquad \Rightarrow RotConstraint(obst,\ r_1)]\ \} \\
\Longrightarrow \\
\{(\forall d_2)\ [Half\text{-}Plane(\text{-}sn,\ d_2) \\
\qquad \Rightarrow TransConstraint(obst,\ d_2)] \wedge \\
\quad (\forall r_2 \exists x_2)\ [Rotate\text{-}90(\text{-}sn,\ x_2, r_2) \\
\qquad \wedge\ Half\text{-}Plane(x_2, o_2) \\
\qquad \Rightarrow RotConstraint(obst,\ r_2)]\ \}
\end{array}
$$

Figure 1: The law of contact constraint

- rotational motion clockwise about any axis which lies in the open half plane centered ninety degrees clockwise from the object's surface normal at the point of contact, or
- rotational motion counter-clockwise about any axis which lies in the open half plane centered ninety degrees counter-clockwise from the object's surface normal at the point of contact.

An obstacle is "sufficiently" constrained if it is unable to move in the any of the following ways:

- translational motion into the open half plane centered on the object's surface normal at the point of contact,
- rotational motion clockwise about any axis which lies in the open half plane centered ninety degrees clockwise from the object's surface normal at the point of contact, and
- rotational motion counter-clockwise about any axis which lies in the open half plane centered ninety degrees counter-clockwise from the object's surface normal at the point of contact.

These constrained motions of the obstacle are the minimum required to describe motion of a link relative to some fixed frame of reference, not just the adjacent link. This allows an obstacle to be only partially constrained yet still prevent other object from moving in some directions. Classical kinematics [Reuleaux, 1876] and related AI approaches [Davis, 1986; deKleer and Brown, 1984; Forbus, 1981; Kuipers, 1986; Laughton, 1985; Shoham, 1985] assume that only one object can move, i.e. all objects are fixed except the object of interest. Those approachs simplify analysis, but oversimplify the problem.

Figure 2 illustrates this law graphically for the two dimensional case. The surface normal of object **B** (inverse surface normal of the obstacle) at the point of contact is to the right. If object **W** cannot move ***up-right***, ***right***, or ***down-right***; cannot rotate ***counter-clockwise*** about an axis above the surface normal; and cannot rotate ***clockwise*** about an axis below the surface normal then the object **O** cannot move ***up-right***, ***right***, or ***down-right*** (Fig. 2 B) cannot rotate ***counter-clockwise*** about an axis above the surface normal (Fig. 2 C); and cannot rotate ***clockwise*** about an axis below the surface normal (Fig. 2 D).

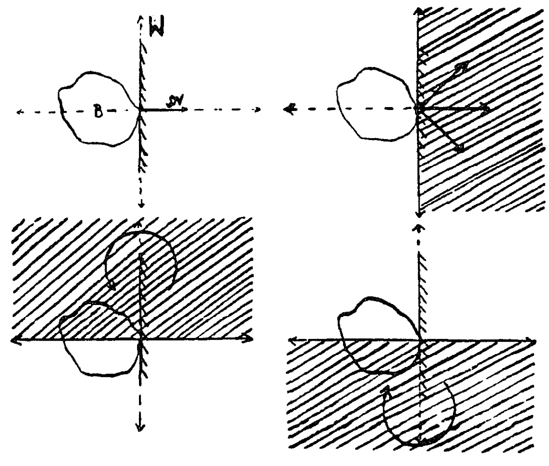

Figure 2: Constraints imposed by surface contact

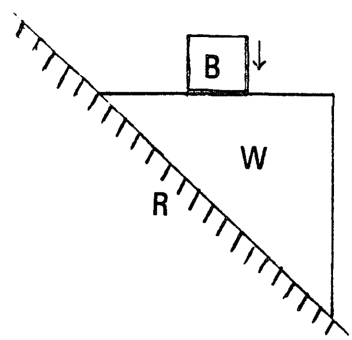

Figure 3: Block on wedge on ramp

A stack of blocks is a simple example of partially constrained motions. When blocks are stacked, any block in the stack is prevented from moving in any *downward* direction because it has contact along a surface with the surface normal in the *down* direction and the block (or table) it is in contact with is constrained in all *downward* directions.

A block resting on a wedge on a ramp (Fig. 3) is free to move in any *downward* direction because the obstacle is not sufficiently constrained. The block may move *downward* by pushing the wedge *down-right*, but if the wedge could not be pushed to the side (perhaps a catch on the ramp), the wedge could not move in any of the directions required by the shape of the surface between the block and the wedge, and consequently the block would be constrained from moving *downward*.

3.2 Constraints Imposed at Corners

When one surface slides off another there will be an instant when contact between two convex corners can occur. In this case the surface tangent is not clearly defined at the point or line

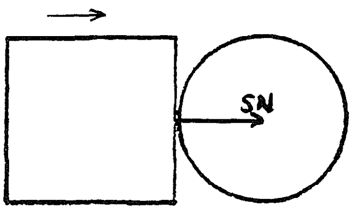

Figure 4: Block pushing a boulder

$$
\begin{aligned}
&(\forall\ obj,\ obst,\ p,\ q,\ sn)\\
&\quad \{RigidBody(body) \wedge RigidBody(obj) \wedge\\
&\quad\ Surface(body,\ p) \wedge Surface(obj,\ q) \wedge\\
&\quad\ Contact(p,\ q) \wedge Surface\text{-}Normal(q,\ sn) \wedge\\
&\quad\ Origin\text{-}Dir(p,\ o_1) \wedge Origin\text{-}Dir(q,\ o_2) \wedge\\
&\quad\ \{(\exists d_1)\ [Half\text{-}Plane(\text{-}sn,\ d_1)\\
&\qquad\quad \wedge\ TransMotion(body,\ d_1)] \vee\\
&\quad\ \ (\exists r_1 x_1)\ [Rotate\text{-}90(\text{-}sn,\ x_1, r_1)\\
&\qquad\qquad \wedge\ Half\text{-}Plane(x_1, o_1)\\
&\qquad\qquad \wedge\ RotMotion(body,\ r_1)]\ \}\}\\
&\quad \Longrightarrow\\
&\quad \{(\exists d_2)\ [Half\text{-}Plane(sn,\ d_2)\\
&\qquad\quad \wedge\ TransMotion(obj,\ d_2)] \vee\\
&\quad\ (\exists r_2 x_2)\ [Rotate\text{-}90(sn,\ x_2, r_2)\\
&\qquad\qquad \wedge\ Half\text{-}Plane(x_2, o_2)\\
&\qquad\qquad \wedge\ RotMotion(obj,\ r_2)]\ \}
\end{aligned}
$$

Figure 5: The law of motion transfer

of contact.[1] To determine what motions cannot occur in this case, we must first determine the contacts which are possible between each adjacent surface and the corners.

Definition 13 (Convex Corner) *A corner is* **convex** *if the angle between the two adjacent surfaces is greater than 180 degrees. The constraints imposed by contact between two convex corners are the* intersection *of those imposed by contact with the adjacent surfaces, provided each half plane of the obstacle corresponding to the regions to be constrained is itself constrained.*

Definition 14 (Concave Corner) *A corner is* **concave** *if the angle between the two adjacent surfaces is less than 180 degrees. The constraints imposed by contact between two concave corners are the* union *of those imposed by contact with the adjacent surfaces, provided each half plane of the obstacle corresponding to the regions to be constrained is itself constrained.*

3.3 Pushing

We have seen how a fixed body will prevent motion. Now we will explore how a moving body will transfer motion. Again there are two considerations. Given contact between an object and a moving body:

1. How will the motion of the body affect the object?
2. What motions of the body will affect the object?

The law describing the motions an object must undergo when in contact with a moving body are given in figure 5. This says that if a body is moving "into" an object, the object must move in at least one of the following ways, and if none of these motions are possible the body cannot move:

- translational motion into the open half plane centered on the body's surface normal at the point of contact,
- rotational motion clockwise about any axis which lies in the open half plane centered ninety degrees clockwise from the body's surface normal at the point of contact, or
- rotational motion counter-clockwise about any axis which lies in the open half plane centered ninety degrees counter-clockwise from the body's surface normal at the point of contact.

[1]When a corner contacts a surface it is sufficient to know the surface normal of the surface because the surface normal of the corner (for our purposes) is the negation of this

The body is moving into the object if it has any of the following motions:

- translational motion into the open half plane centered on the body's surface normal at the point of contact,
- rotational motion clockwise about any axis which lies in the open half plane centered ninety degrees clockwise from the body's surface normal at the point of contact, or
- rotational motion counter-clockwise about any axis which lies in the open half plane centered ninety degrees counter-clockwise from the body's surface normal at the point of contact.

Imagine the block moving right in figure 4. **SN** indicates the surface normal of the block. Because the direction of motion of the block is into the boulder, the boulder must move in one of the following directions:

- translationally right
- translationally down right
- translationally up right
- clockwise about an axis down from the contact
- clockwise about an axis down left from the contact
- clockwise about an axis down right from the contact
- counter-clockwise about an axis up from the contact
- counter-clockwise about an axis up left from the contact
- counter-clockwise about an axis up right from the contact

The way we handle external forces such as gravity, friction, and magnetism is to create an imaginary body pushing on an (possibly) imaginary surface of the object. For example, a dropped brick would be pushed downward by gravity at its center of mass.

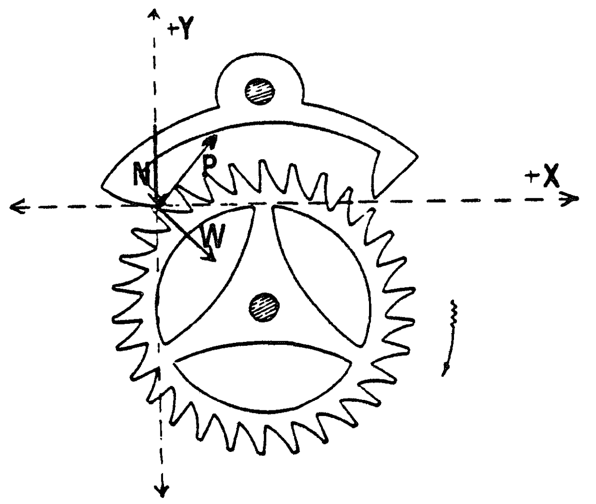

Figure 6: Recoil Escapement

4 Sample Mechanism

The principles of blocking and pushing discussed here allow us to determine the instantaneous behavior of a mechanism.[2] Typically lower pair recognition [Gelsey, 1987] will leave at most one degree of freedom for each component of a mechanism and the only significant forces transmitted through contact. To determine the instantaneous behavior of the mechanism from any given configuration we first compute the constrained motions. After the impossible motions have been eliminated, the intersection of the free directions and the motions transferred by pushing should yield a single set of consistent motions for each part of the mechanism. If there is no consistent set the mechanism cannot move.

For example, consider the recoil escapement shown in figure 6. The direction of the surface normal of the pallet tooth is **N**, the direction to the center of rotation of the wheel is **W**, and the direction to the center of rotation of the pallet is **P**. In this type of escapement when the pallet arm contacts the scape wheel, the arm is not sufficiently constrained by the wheel. (Counter-clockwise rotation is possible about an axis counter-clockwise of the pallet's surface normal.) As a result the pallet's continued swing drives the wheel backward (motion transferred in the counter-clockwise direction), causing the entire clock mechanism to move backward (recoil).

5 Discussion

We have presented three aspects of the kinematic analysis of a rigid body device; the constraint of motion, the transfer of motion, and the propagation of external forces. In order to do this we developed a logical theory of rigid body interactions; which provides a symbolic framework for geometric descriptions and laws describing mechanical constraint and motion transfer.

Other work [Nielsen, 1988a] combines this result with a set of all possible configurations of the objects in the form of a place vocabulary [Forbus, 1981; Faltings, 1987; Nielsen, 1988b] and dynamical information (as produced by [Forbus, 1984]) to provide an envisionment which qualitatively describes all possible behaviors of such devices as a mechanical clock.

[2] Any ambiguity arises from the qualitative representation of spatial directions. The other laws and definitions suffice for more detailed representations.

6 Acknowledgements

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