Approximation Reformulations

Daniel S. Weld*
Department of Computer Science and Engineering, FR-35
University of Washington
Seattle, WA 98195
weld@cs.washinton.edu

Abstract

Although computers are widely used to simulate complex physical systems, crafting the underlying models that enable computer analysis remains difficult. When a model is created for one task, it is often impossible to reuse the model for another purpose because each task requires a different set of simplifying assumptions. By representing modeling assumptions explicitly as approximation reformulations, we have developed qualitative techniques for switching between models. We assume that automated reasoning proceeds in three phases: 1) model selection, 2) quantitative analysis using the model, and 3) validation that the assumptions underlying the model were appropriate for the task at hand. If validation discovers a serious discrepancy between predicted and observed behavior, a new model must be chosen. We present a domain independent method for performing this model shift when the models are related by an approximation reformulation and describe a Common Lisp implementation of the theory.

Introduction

Although the bulk of work in model-based reasoning has focussed on problems of analysis in the framework of a single model, we believe that no single model can be adequate for a wide range of tasks. Imagine trying to diagnose a misbehaving auto with a flat molecular-level description of the whole system. Quantum mechanics may be the right level to reason about bond angle in impure hydrocarbon fuels, but it does not provide a useful model of the spark plugs or transmission.

To achieve robust performance when reasoning about complex systems, analytic programs must do what human experts do: switch between models, dynamically choosing perspectives and simplifying assumptions that are appropriate to the task at hand. Since different assumptions are warranted depending on the analytic question being answered, a program that uses multiple models should validate its choice in the context of the problem at hand. Thus the critical step is enabling the program to reason explicitly about modeling assumptions.

In this paper we address the problem of INTER-MODEL COMPARATIVE ANALYSIS: qualitatively computing the differences in the behaviors predicted by two different models. As input, inter-model comparative analysis takes two models and a mapping (called a reformulation) that links vocabulary terms in the two models. As output, inter-model comparative analysis predicts the how the time-varying behavior predicted by one model differs qualitatively from that predicted by the other. The next section explains how inter-model comparative analysis enables model-switching in tasks such as theory formation and innovative design.

Although inter-model comparative analysis is very difficult in general, we show that it can be efficiently solved when the mapping between models is of a restricted type (called an APPROXIMATION REFORMULATION). Intuitively, a simple model approximates a more complex model when the complex model has an exogenous parameter (called a FITTING PARAMETER) such that the quantitative behaviors predicted by the two models get arbitrarily close as the fitting parameter tends towards a limit. If models are related by approximation reformulations then the inter-model comparative analysis problem reduces to an *intra*-model comparative analysis problem and can be efficiently solved by the existing techniques of DQ analysis [Forbus, 1984, Weld, 1988a, Chiu and Kuipers, 1989] and exaggeration [Weld, 1990]. For example, consider the simple system of figure 1.

One can imagine many models of this system, each incorporating different simplifying assumptions, but for now consider two models that differ only in their treatment of the rope. Suppose model $\mathcal{A}$ considers the rope to be inelastic, while $\mathcal{B}$ models the stretching rope as a stiff spring (via Hooke's law $f = -kx$). One can show that $\mathcal{A}$ approximates $\mathcal{B}$, because $\mathcal{B}$'s parameter k is a fitting parameter. As the spring constant k tends towards infinity, the rope approaches the stretchless

*This research was funded in part by National Science Foundation Grants IRI-8902010 and IRI-8957302, and a cash donation from the Xerox corporation.

into a corresponding state in the ontology of the system system: $\Psi(\vec{q})$. The π_i projection functions extract the i-th parameter value from the two states and the difference is called the PDIFF.

Definition 5 *Let $\mathcal{A}$ and $\mathcal{B}$ be models with* PARAM$(\mathcal{A})$ $= (P_1, \ldots, P_n)$. *Let Ψ be a reformulation such that $\mathcal{A} \preceq_\Psi \mathcal{B}$. Let $\vec{q}$ be an internal state of $\mathcal{B}$ representing a set of initial conditions. Let $\vec{p}$ be the internal state of $\mathcal{A}$ corresponding to the state $\Psi(\mathcal{B}_{\vec{q}}(0))$. Define the* BEHAVIOR DIFFERENCE BETWEEN *$\mathcal{A}$ and $\mathcal{B}$* USING Ψ OVER THE TIME INTERVAL $[t_s, t_f]$ GIVEN *$\vec{q}$ as* BDIFF$(\mathcal{A} \preceq_\Psi \mathcal{B}, \vec{q}, t_s, t_f) =$

$$\max_{1 \leq i \leq n} \left(\sup_{t \in [t_s, t_f]} |\text{PDIFF}(P_i, \Psi, \mathcal{A}_{\vec{p}}(t), \mathcal{B}_{\vec{q}}(t))| \right)$$

In other words, for each parameter in the simple model, we compare corresponding values in the complex model for all times and take the least upper bound of the absolute differences. The behavior difference is the maximum value of the suprema. It is important to recognize that while the PDIFF must be measured in the simpler model, the initial conditions must be specified in the more complex model to ensure that both models can be simulated.

Approximation Reformulations

Intuitively, one model approximates another when the behavior difference between them can be brought arbitrarily close to zero.

Definition 6 *Let $\mathcal{A}$ and $\mathcal{B}$ be models, and suppose there exists a reformulation Ψ such that $\mathcal{A} \preceq_\Psi \mathcal{B}$. Say that $\mathcal{A}$* APPROXIMATES *$\mathcal{B}$* UNDER *Ψ if there exists a parameter $Q_f \in$* INDEP$(\mathcal{B})$ *and an endpoint l of the closure of* RANGE(Q_f) *such that for all internal states $\vec{q}$ of $\mathcal{B}$, and forall times t,*

$$\lim_{\pi_f(\vec{q}) \to l} \text{BDIFF}(\mathcal{A} \preceq_\Psi \mathcal{B}, \vec{q}, 0, t) = 0$$

In this case, the parameter Q_f is called the FITTING PARAMETER *of Ψ and l is called its* APPROXIMATION LIMIT.

Since a fitting parameter is independent by definition, it is constant over time; this is why the definition refers to only its initial value $\pi_f(\vec{q})$. The idea behind the definition is that $\mathcal{A}$ approximates $\mathcal{B}$ if sending $\mathcal{B}$'s fitting parameter to a limit squeezes the behavior difference to zero. As a simple example, see figure 2.

At time zero the block is released at the top of the θ-degree inclined plane; under the force of gravity, it moves downward (and to the left, but both models ignore the horizontal component of movement). Let $\mathcal{A}$ be a model of this system with parameters θ, G, Y, V, A denoting angle, gravity, height, and the vertical components of velocity and acceleration respectively. Let $\mathcal{B}$ be a model with all these parameters plus an additional parameter, μ, denoting the coefficient of friction. Let Ψ be the projection function: $\Psi(\theta, g, y, v, a, \mu) = (\theta, g, y, v, a)$. Since Ψ is a reformulation, $\mathcal{A} \preceq_\Psi \mathcal{B}$.

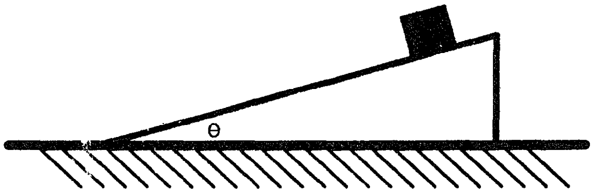

Figure 2: Block slides down an inclined plane.

Suppose INDEP$(\mathcal{A})$=$\{G, \theta\}$ and BOUND$(\mathcal{A}) = \{G, \theta, Y, V\}$; $\mathcal{A}$'s ODE's are: $(V = \frac{d}{dt}Y) \wedge (A = G\cos(\theta) = \frac{d}{dt}V)$ while $\mathcal{B}$ has an extra independent parameter μ and ODEs: $(V = \frac{d}{dt}Y) \wedge (A = G\cos(\theta) - \mu G\sin(\theta) = \frac{d}{dt}V)$. As the coefficient of friction μ tends to zero, the frictional force diminishes and the first equation of $\mathcal{B}$ gets arbitrarily close to the first equation of $\mathcal{A}$. When there is no friction then the equations are identical so it is clear that the behavior difference is zero. Thus, we can say that $\mathcal{A}$ approximates $\mathcal{B}$ with fitting parameter μ and approximation limit 0. But while this system provides a clear example of an approximation, it is a bit misleading. The case where the fitting parameter can actually take on the limiting value (i.e., where it is legal for $\mathcal{B}$ to have zero friction) is really a degenerate case of the approximation definition. In general, this is not the case, which is why the definition allows l to be in the closure of the parameter's range; this was illustrated with the elastic string example of figure 1.

Exploiting Approximations

We seek a qualitative characterization of the difference in behavior that two models predict. By assuming one model as 'current' and considering a shift in models, we can phrase this question as comparative analysis: "What is the effect on predicted behavior of shifting from the current model to a different one?" Since this comparison is based on a switch in models, rather than a perturbation to the boundary parameters of a single model, we call it *inter*-model comparative analysis, rather than the *intra*-model case that has been studied in the past [Weld, 1988a]. Space considerations preclude precise definitions of the following terms; see [Weld, 1989]:

- A behavior TRANSITIONS whenever a parameter moves to or from a LANDMARK VALUE. A model's TIME-FUNCTION, $\mathcal{T}$, maps from transitions (the i-th being written γ_i) to the time when they occur.
- An input to intra-model comparative analysis is a perturbation δ to the initial values of boundary parameters. Since δ can be thought of as a vector of qualitative values, $\pi_f(\delta) = [+]$ means that the perturbation specifies an increase in the fitting parameter.

$1 \leq i \leq k$. *A* STATE *of $\mathcal{A}$ is an n-tuple such that the values $p_1, \ldots, p_k$ are an internal state, $p_{k+1}, \ldots, p_n$ are in their ranges, and $p_1, \ldots, p_n$ satisfy the model's quantitative constraints. A set of* INITIAL CONDITIONS *for $\mathcal{A}$ is an internal state of $\mathcal{A}$. The* BEHAVIOR *of $\mathcal{A}$ given initial conditions $\vec{p}_0$ is the unique function*

$$\mathcal{A}_{\vec{p}_0} : \Re \rightarrow \text{RANGE}(P_1) \times \ldots \times \text{RANGE}(P_n)$$

defined by $\mathcal{A}_{\vec{p}_0}(t) = (P_1(t), \ldots, P_n(t))$ where the P_i are closed-form solutions to the model's ordinary differential equations given the boundary values $\vec{p}_0$.

Thus a model $\mathcal{A}$ is an abstract description of a system, a state is a snapshot of the values of all the model's parameters at a given time, and an internal state is a compact representation of a state. Combining a model $\mathcal{A}$ and a set of initial conditions $\vec{p}$ specifies a behavior $\mathcal{A}_{\vec{p}}$ that maps from times to states. Given a behavior or a state, one can use a projection function to isolate the parameter or parameter value of interest. For example, to extract the closed-form solution i-th parameter from the $\mathcal{A}_{\vec{p}}$ behavior, one would write $\pi_i(\mathcal{A}_{\vec{p}})$. If mnemonic names are used then the parameter name may be substituted in place of the index. For example, to determine the velocity (parameter V) specified by a state $\vec{p}_i$, one would write $\pi_V(\vec{p}_i)$.

The definition above describes the relationship between a model, initial conditions and the resulting behavior but it deliberately does not say anything about how to compute the behavior; our objective is a general theory of model shifting that is independent of particular simulation, symbolic solution, or numeric approximation methods.

Reformulations

Since two models may describe a physical system using different parameters, some work is necessary to enable behavioral comparison. In this section, we introduce REFORMULATION FUNCTIONS to define a correspondence between the ontologies of different models. Next we discuss how to measure the difference in the predicted behavior of two models that are connected by a reformulation. Finally, we consider a restricted class of reformulations, called APPROXIMATIONS, that have useful properties regarding this behavior difference. In the next section we show that inter-model comparative analysis can be efficiently solved if one of the models approximates the other.

The basic idea behind reformulations is that a complex model $\mathcal{B}$ can be compared to a simpler one $\mathcal{A}$ if an internal state of $\mathcal{B}$ allows us to construct a complete description of an internal state of $\mathcal{A}$. Although this notion is very general (almost any continuous function, meaningful or not, is a reformulation), it provides a useful foundation. Later, we refine the idea to a useful class of reformulations called approximations.

Definition 3 *Let $\mathcal{A}$ and $\mathcal{B}$ be models with n and m parameters such that* $\text{BOUND}(\mathcal{A}) = \{P_1 \ldots P_k\}$ *and* $\text{BOUND}(\mathcal{B}) = \{Q_1 \ldots Q_l\}$. *If there exists a continuous function Ψ' from* $\text{RANGE}(Q_1) \times \ldots \times \text{RANGE}(Q_l)$ *onto* $\text{RANGE}(P_1) \times \ldots \times \text{RANGE}(P_k)$ *then say that Ψ* COMPARES *$\mathcal{B}$ to $\mathcal{A}$ (written $\mathcal{A} \preceq_{\Psi} \mathcal{B}$) where Ψ is an extension of Ψ' that maps from states (rather than internal states) of $\mathcal{B}$ to states of $\mathcal{A}$ in the obvious way. Ψ is called a* REFORMULATION FUNCTION *from $\mathcal{B}$ to $\mathcal{A}$. For any state $\vec{q}$ of $\mathcal{B}$, if $\vec{p} = \Psi(\vec{q})$ then $\vec{p}$ is said to be the* CORRESPONDING STATE *of $\vec{q}$.*

For example, let $\mathcal{B}$ be a model of the two dimensional motion of a billiard ball using polar coordinates and let $\mathcal{A}$ be a model of the same system using rectangular coordinates. In this case $\mathcal{A} \preceq_{\Psi} \mathcal{B}$ because a reformulation function exists. Let Ψ be defined from $\{\theta\} \times \{R\}$ to $\{X\} \times \{Y\}$ with $\Psi(\theta, R) = (R\cos\theta,\ R\sin\theta)$

For the rest of this paper, however, we assume that all reformulation functions are defined in terms of simple arithmetic operations (addition, subtraction, multiplication, and division). In fact, for many examples it suffices to specify trivial reformulations that equate parameters pairwise in the two models.

Proposition 1 *Let $\mathcal{A}$ and $\mathcal{B}$ be models with k and l boundary parameters respectively. $\mathcal{A} \preceq_{\Psi} \mathcal{B}$ iff $k \leq l$.*

Proof: This is an easy corollary of the Borsuk-Ulam theorem [Massey, 1967, p170]. □

Proposition 2 *The compared-to relation $\preceq_{\Psi}$ is reflexive and transitive but not symmetric.*

Proof: See [Weld, 1989]. □

Intuitively this means that one can compare a "large" model to a "smaller" one but not vice versa. The lack of symmetry results from a reformulation function being onto, but not necessarily invertible. Note that our definition allows many possible reformulations between two nonempty models, most of which are uninteresting or irrelevant. Meaningful comparison between two models requires a good choice of Ψ, hence much of this paper is concerned with characterizing useful classes of reformulations.

Behavior Difference

To perform model switching we are interested in a qualitative measure of behavior difference. However, a quantitative measure also proves useful. In both cases we define difference in terms of the parameters of the simpler of the two models (i.e., in terms of $\mathcal{A}$ if $\mathcal{A} \preceq_{\Psi} \mathcal{B}$) because of the inherent asymmetry of reformulations.

Definition 4 *Let $\mathcal{A}$ and $\mathcal{B}$ be models with* $\text{PARAM}(\mathcal{A}) = (P_1, \ldots, P_n)$. *Let $\vec{p}$ be a state of $\mathcal{A}$ and $\vec{q}$ be a state of $\mathcal{B}$. Suppose that Ψ is a reformulation such that $\mathcal{A} \preceq_{\Psi} \mathcal{B}$. Define the* DIFFERENCE IN P_i BETWEEN $\vec{p}$ AND $\vec{q}$ USING Ψ *as*

$$\text{PDIFF}(P_i, \Psi, \vec{p}, \vec{q}) = \pi_i(\Psi(\vec{q})) - \pi_i(\vec{p})$$

In other words the difference in the value of a parameter in the two states is calculated by using the reformulation Ψ to convert the complex-system state $\vec{q}$

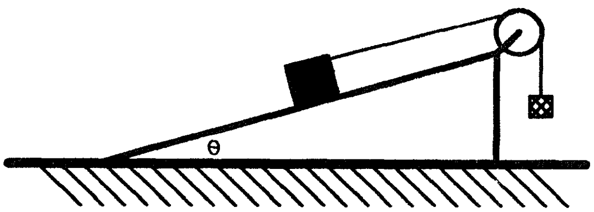

Figure 1: A pulley, two weights and an inclined plane.

ideal and the difference in the time-varying behavior predicted by the two models vanishes.

Since $\mathcal{A}$ approximates $\mathcal{B}$, one can solve an inter-model comparative analysis problem such as "Will model $\mathcal{B}$ predict a higher terminal velocity than $\mathcal{A}$?" by solving an intra-model comparative analysis problem about the fitting parameter in model $\mathcal{B}$, i.e. "Will terminal velocity increase if k decreases?" This reduction from an inter-model to an intra-model comparative analysis problem means that the well-studied techniques of DQ analysis and exaggeration may be used to solve model switching problems.

Motivation

Inter-model comparative analysis allows one to use behavioral discrepancies to guide shifts in modeling detail. We expect this to have application to the problems of (among others) theory formation and the evaluation of design modifications — each of these problems can be thought of as improving a given model or determining that no better model exists.

- The goal of theory revison is to improve a theory that fails to account for all observations. This fits into our paradigm by considering a theory of the world as a model and the failure as a discrepancy between observed and predicted values. Inter-model comparative analysis allows the learner to compare alternate theories to see if they account for the discrepancy.
- If a proposed design fails to meet a behavioral specification (e.g. the power consumption is too great), then a new design must be found that alleviates the discrepancy. If one considers the two designs as models, then inter-model comparative analysis evaluates the effect of the proposed change.

The rest of the paper is couched within the objectives of the first application. Given a set of discrepancies between the predictions of a model and observations of the actual system, determine if a model exists that will predict a behavior which is in closer agreement to the observations. We assume reasoning proceeds in three phases: choice of a model, analysis of the model, and validation that the model is appropriate. If validation instead shows that the model was an inappropriate choice then a new model must be selected. We assume that the analysis performed on each model is quantitative, behavioral prediction (i.e., numerical simulation), but our model-switching technique is qualitative; it works for approximation reformulations.

Models and Behaviors

We consider a model to be a description of a physical system in terms of one or more PARAMETERS, continuous, continuously-differentiable functions from an interval of $\Re$ into an interval of $\Re$ that have only a finite number of points where the derivative crosses zero in any bounded interval [Kuipers, 1986]. To specify the interdependence between parameters in a physical system, models contain qualitative and quantitative constraints. By quantitative constraints we mean simply a system of ordinary differential equations (ODEs). A model's qualitative constraints are a finite set of instantiations of the six constraints used by QSIM — ADD, MINUS, MULT, M^+, M^-, and $\frac{d}{dt}$ — see [Kuipers, 1986] for the details. Naturally, it is important that the quantitative and qualitative descriptions are consistent. We say that a set of qualitative constraints AGREES with a set of ordinary differential equations (ODEs) iff every solution to the ODEs satisfies the constraints.

Definition 1 *Let $(P_1, \ldots, P_n)$ be an ordered list of parameters. Let* C *be a set of qualitative constraints defined over $\{P_i\}$. Let* D *be a set of ordinary differential equations over $\{P_i\}$. Say that $\mathcal{A} = ((P_1, \ldots, P_n), \mathrm{C}, \mathrm{D})$ is a* MODEL *if* C *agrees with* D *and* D *specifies a unique (closed form) solution. Let* PARAM *be a function taking a model to the list of parameters for the model. Let* BOUND *be a function taking a model to the sublist $(P_1, \ldots, P_l)$ of boundary parameters. Let* INDEP *be a function taking a model to the sublist $(P_1, \ldots, P_k)$ of independent parameters, where $1 \leq k \leq l \leq n$.*

Parameters whose values must be known for all time to determine the model's behavior are called independent; we assume that they are constant. In addition to these independent parameters, many models have dependent parameters whose value must be known at at least one time point to specify a unique behavior; the union of these and the independents are called boundary parameters by analogy to the boundary conditions of an ODE. We use calligraphic letters to denote models, lower case letters to denote real numbers, and capitals to denote parameters. All parameters are numbered so we will frequently talk about the i-th parameter of a model as P_i, but when discussing a particular model we may use mnemonic names like V for velocity.

A behavior describes a model's changing state over time. Both qualitative and quantitative descriptions are necessary. We use the QSIM representation [Kuipers, 1986] as a qualitative description; the quantitative behavioral representation is defined below.

Definition 2 *Let $\mathcal{A}$ be a model with parameters $P_1, \ldots, P_n$ of which the first k are boundary parameters. An* INTERNAL STATE *of $\mathcal{A}$ is a k-tuple $\vec{p} = (p_1, \ldots, p_k)$ such that $p_i \in$* RANGE*(P_i) forall i such that*

- The output of intra-model comparative analysis is an array of relative change values for all parameters for all transitions. $\textsc{rcs}(\mathcal{B}_{\vec{q}}, \delta, \gamma_i)$ denotes a vector of relative change values for all parameters in model $\mathcal{B}$, given initial condition $\vec{q}$ and perturbation δ, formed by slicing through this array at transition γ_i.
- A reformulation Ψ, can be trivially extended to map qualitative values as well as real numbers. Thus if $\mathcal{A} \preceq_\Psi \mathcal{B}$, then $\pi_j(\Psi(\textsc{rcs}(\mathcal{B}_{\vec{q}}, \delta, \gamma_i)))$ denotes the relative change value of the j-th paramter of $\mathcal{A}$, which corresponds to the predicted change of $\mathcal{B}$ at γ_i given perturbation δ.
- The *inter*-model statement $\textsc{rc}(P, \mathcal{A} \preceq_\Psi \mathcal{B}, \vec{q}, \gamma_i) = [+]$ means that a model switch along the reformulation Ψ (i.e. from model $\mathcal{A}$ to $\mathcal{B}$) given initial conditions $\Psi(\vec{q})$ and $\vec{q}$ respectively, will cause P to have a higher predicted value at transition γ_i. This can be abbreviated ${}^{\vec{q}}_{\Psi}P{\Uparrow}_i$.

Proposition 3 *(CA Reduction Theorem). Let $\mathcal{A}$ and $\mathcal{B}$ be models such that* $\textsc{param}(\mathcal{A}) = (P_1, \ldots, P_n)$, $\textsc{param}(\mathcal{B}) = (Q_1, \ldots, Q_m)$*, and $\mathcal{B}$ has k boundary parameters. Let Ψ be a reformulation such that $\mathcal{A} \preceq_\Psi \mathcal{B}$. Suppose that $\mathcal{A}$ approximates $\mathcal{B}$ under Ψ with fitting parameter Q_f and approximation limit l where l is the greatest lower bound of* $\textsc{range}(Q_f)$*. Suppose the two time functions are equal: ${}_{\mathcal{A}}\mathcal{T} = {}_{\mathcal{B}}\mathcal{T}$. Let δ denote a perturbation that increases the fitting parameter, $\pi_f(\delta) = [+]$, and holds all other boundary parameters constant, $\forall i, 1 \leq i \leq k \wedge i \neq f \rightarrow \pi_i(\delta) = [0]$. Let $\vec{q}$ denote an internal state of $\mathcal{B}$ such that $\pi_f(\vec{q}) = l$, and let $\vec{p}$ be the internal state of $\mathcal{A}$ corresponding to $\vec{q}$. For any parameter $P_j \in$* $\textsc{param}(\mathcal{A})$ *and for any transition γ_i,*

$$\pi_j(\Psi(\textsc{rcs}(\mathcal{B}_{\vec{q}}, \delta, \gamma_i))) = \textsc{rc}(P_j, \mathcal{A} \preceq_\Psi \mathcal{B}, \vec{q}, \gamma_i)$$

For example, if $\pi_j(\Psi(\textsc{rcs}(\mathcal{B}_{\vec{q}}, \delta, \gamma_i))) = [+]$ then switching from $\mathcal{A}$ to $\mathcal{B}$ will increase the predicted value of P_j at γ_i: ${}^{\vec{p}}_{\Psi}P_j{\Uparrow}_i$. Thus while inter-model comparative analysis appears quite difficult in general, the previous theorem shows that it can be performed easily in certain cases. If the reformulation linking the two models is an approximation, then inter-model comparative analysis reduces to an intra-model comparative analysis problem in the more complex model, with an initial RC of the fitting parameter away from the approximation limit.

Implementation

To test the ideas of model shifting with approximation reformulations, we have implemented a Common Lisp program, SAM, that embodies our theory of inter-model comparative analysis. As input SAM is given a GRAPH OF MODELS (GoM) [Penberthy, 1987, Addanki *et al.*, 1989]. Each edge in the GoM is an approximation reformulation labeled with the fitting parameter and approximation limit. For simplicity, SAM's GoM representation only allows reformulations that can be expressed as projection functions.

SAM solves an analysis task by reasoning in three phases. First a model is chosen; by default, the simplest model is used. The user enters data values for some number of parameters at an observed event. Next, the model's quantitative ODE description is integrated using a fourth-order Runge-Kutta algorithm with adaptive stepsize control [Press *et al.*, 1986]. Once quantitative simulation is complete, SAM compares the predictions with observations and computes discrepancies. If any discrepancies exceed the user-specified threshold, SAM seeks to switch models. It does this by generating a list of GoM models that differ from the current model by the elimination of one assumption. For each candidate model, SAM performs inter-model comparative analysis as follows.

1. The candidate model is simulated using the QSIM qualitative simulator [Kuipers, 1986]. If more than one qualitative behavior results, the user is consulted to disambiguate.
2. An initial perturbation, δ, is created by choosing a change for the fitting parameter that is in the direction away from the approximation limit, and constraining all other boundary parameters not to change.
3. DQ analysis is applied to determine the relative changes resulting from the perturbation.
4. The GoM's reformulation function converts the intra-model RC values to inter-model RC values.

If the RC values predicted by inter-model comparative analysis match the observed discrepancy, the candidate model is accepted and the Runge-Kutta integrator is called on this model. Otherwise, the next candidate is considered. Since DQ analysis is incomplete [Weld, 1988a], it is not guaranteed to deduce all RC values for every parameter. As a result, it is critical that SAM's matcher considers both agreement (RC predicted matches discrepancy) and agnosticism (no RC predicted) as an acceptable match. The current implementation warns when a switch is taken as a result of DQ agnosticism; an alternative approach for future investigation would be to select the candidate model with the best match (least agnosticism) rather than the first acceptable match.

If none of the candidate models matches the discrepancy, SAM complains that the observations were noisy. This is the likely cause, but other problems are also possible; for example, the GoM could be incomplete. Since DQ analysis is sound [Weld, 1988a] and since the matcher accepts agnosticism, the intra-model comparative analysis routine will never miss a valid model switch. However, SAM uses a generalization of proposition 3 that weakens the constraints on fitting parameter value and time function equality. Since this generalization has not been proven sound, the problem could be that intra-model comparative analysis was performed too far from the approximation limit resulting in an incorrect value for the inter-model RC value [Weld, 1989]. Although theoretically possible, we have been unable to construct an example which would cause this error. Thus we feel that the conclusion of noisy data is a reasonable one.

Since this control algorithm causes each model switch to move upwards in the $\preceq_{\Psi}$ lattice, SAM's analysis is guaranteed to terminate either by producing an acceptable prediction or by failing at the most complex model. This does not mean, however, that the optimal model (in the sense of an accuracy vs computational cost tradeoff) will be found.

SAM has been tested on about five problems, all using a simple (2 model) GoM corresponding to the example in figure 2.

Conclusion

We view the theory presented in this paper as a promising first step towards automating the types of model switching that occur during problem solving. Since our technique only works when models are related by an approximation reformulation, an important question is whether these relations are common. While we have no general answer to this question, we did enumerate as many assumptions for the simple mechanics domain as we could (17). Of these seventeen, all but the assumption of unbreakable rope could be expressed as approximation reformulations. This suggests the generality of our approach. However, we have yet to demonstrate that our theory can be extended to handle systems with multiple operating regions.

We also hope to increase the speed of model switching. The approach we are taking is to combine our ideas with Falkenhainer and Forbus's scheme for compositional modeling [Falkenhainer and Forbus, 1988]. The goal here is to switch models based on intra-model comparative analysis as performed in the complex model of a single component rather than of the system as a whole. In a sense, this means switching the modeling focus to enable a change of modeling detail.

Another idea is to perform explanation-based generalization on the results of inter-model comparative analysis to build up a library of parameter-change rules (as used by PROMPT [Addanki *et al.*, 1989]).

Our general agenda is to develop a comprehensive approach to automated model management that facilitates integrated reasoning with models of differing focus [Falkenhainer and Forbus, 1988], ontology [Collins and Forbus, 1987, Amador and Weld, 1990], and temporal granularity [Kuipers, 1987, Weld, 1986] — as well as varying accuracy. This paper supplies the mathematical foundation for part of this theory.

Acknowledgements

This work benefited from conversations with Olivier Raiman, J. Scott Penberthy, Dorothy Neville, Ben Kuipers, Rich Keller, Leo Joskowicz, Walter Hamscher, Steve Hanks, Paul Beame, Tony Barrett and Franz Amador. Elisha Sacks kindly provided the Common Lisp code for the Runge-Kutta algorithm, and Ben Kuipers supplied QSIM; both of these are used in SAM.

References

[Addanki *et al.*, 1989] S. Addanki, R. Cremonini, and J. S. Penberthy. Reasoning about Assumptions in Graphs of Models. In *Proceedings of IJCAI-89*, August 1989.

[Amador and Weld, 1990] F. Amador and D. Weld. Multi-Level Reasoning about Populations. In *Proceedings of the 4th Qualitative Physics Workshop*, July 1990.

[Chiu and Kuipers, 1989] C. Chiu and B. Kuipers. Comparative Analysis and Qualitative Integral Representations. In *Proceedings of the 3rd Qualitative Physics Workshop*, August 1989.

[Collins and Forbus, 1987] J. Collins and K. Forbus. Reasoning About Fluids Via Molecular Collections. In *Proceedings of AAAI-87*, July 1987.

[Falkenhainer and Forbus, 1988] B. Falkenhainer and K. Forbus. Setting up Large Scale Qualitative Models. In *Proceedings of AAAI-88*, August 1988.

[Forbus, 1984] K. Forbus. Qualitative Process Theory. *Artificial Intelligence*, 24, December 1984.

[Kuipers, 1986] B. Kuipers. Qualitative Simulation. *Artificial Intelligence*, 29, September 1986.

[Kuipers, 1987] B. Kuipers. Abstraction by Time-Scale in Qualitative Simulation. In *Proceedings of AAAI-87*, July 1987.

[Massey, 1967] W. Massey. *Algebraic Topology: An Introduction.* Springer-Verlag, 1967.

[Penberthy, 1987] J.S. Penberthy. Incremental Analysis and the Graph of Models: A First Step towards Analysis in the Plumber's World. MS Thesis, MIT Laboratory for Computer Science, January 1987.

[Press *et al.*, 1986] W. Press, B. Flannery, S. Teukolsky, and W. Vetterling, editors. *Numerical Recipes.* Cambridge University Press, Cambridge, England, 1986.

[Weld, 1986] D. Weld. The Use of Aggregation in Causal Simulation. *Artificial Intelligence*, 30(1), October 1986.

[Weld, 1988] D. Weld. Comparative Analysis. *Artificial Intelligence*, 36(3), October 1988.

[Weld, 1989] D. Weld. Automated Model Switching: Discrepancy Driven Selection of Approximation Reformulations. Technical Report 89-08-01, University of Washington, Department of Computer Science and Engineering, October 1989.

[Weld, 1990] D. Weld. Exaggeration. *Artificial Intelligence*, 43(2), 1990.