A Probabilistic Interpretation for Lazy Nonmonotonic Reasoning

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Abstract

This paper presents a formal relationship for probability theory and a class of nonmonotonic reasoning which we call *lazy nonmonotonic reasoning*. In lazy nonmonotonic reasoning, nonmonotonicity emerges only when new added knowledge is contradictory to the previous belief.

In this paper, we consider nonmonotonic reasoning in terms of *consequence relation*. A consequence relation is a binary relation over formulas which expresses that a formula is derivable from another formula under inference rules of a considered system. A consequence relation which has lazy nonmonotonicity is called a *rational consequence relation* studied by Lehmann and Magidor (1988).

We provide a probabilistic semantics which characterizes a rational consequence relation exactly. Then, we show a relationship between propositional circumscription and consequence relation, and apply this semantics to a consequence relation defined by propositional circumscription which has lazy nonmonotonicity.

Introduction

This paper is concerned about a formal relationship between nonmonotonic reasoning and probability theory. Nonmonotonic reasoning is a formalization of reasoning when information is incomplete. If someone is forced to make a decision under incomplete information, he uses commonsense to supplement lack of information. Commonsense can be regarded as a collection of normal results. Those normal results are obtained because their probability is very near to certainty. So commonsense has a statistical or probabilistic property.

Although there are a lot of researches which simulate a behavior of nonmonotonic reasoning based on probability theory(see [Pearl 1989] for example), there is no formal relationship between nonmonotonic reasoning and probability theory, as Lifschitz (1989) pointed out.

In this paper, we consider nonmonotonic reasoning in terms of *consequence relation* (Gabbay 1985; Kraus, Lehmann, and Magidor 1988; Lehmann and Magidor 1988; Lehmann 1989). Consequence relation is a binary relation over formulas and expresses that a formula is derivable from another formula under inference rules of the considered system. The researchers consider desired properties in a consequence relation for nonmonotonic reasoning.

Gabbay (1985) was the first to consider nonmonotonic reasoning by a consequence relation and Kraus, Lehmann and Magidor (1988) give a semantics for a consequence relation of nonmonotonic reasoning called *preferential* consequence relation. The semantics is based on an order over possible states which is similar to an order over interpretations in circumscription (McCarthy 1980) or Shoham's preference logic (Shoham 1988).

Lehmann and Magidor (1988) define a more restricted consequence relation called *rational* consequence relation and shows that a consequence relation is rational if and only if it is defined by some *ranked* model. A model is ranked if a set of possible states is partitioned into a hierarchical structure, and in a rational consequence relation, the previous belief will be kept as long as the new knowledge does not contradict the previous belief. This nonmonotonicity can be said to be *lazy* because only contradictory knowledge can cause a belief revision.

Moreover, they investigate a relationship between Adams' logic (Adams 1975) (or equivalently, ε-semantics [Pearl 1988]) and rational entailment in which conditional assertion is followed by a set of conditional assertions. Although Adams' logic is based on probabilistic semantics, it only considers consistency and entailment for a set of conditional assertions and does not consider probabilistic semantics for a consequence relation. To give a probabilistic semantics to nonmonotonic reasoning, we have to go beyond

Adams' logic because most nonmonotonic reasoning systems define a consequence relation in the sense that the systems can define a derived result from a given set of axioms by the inference rules of those systems.

In this paper, we provide a probabilistic semantics which characterizes a rational consequence relation exactly. To do so, we define a *closed consequence relation in the limit*. This property means that there exists a probability function with positive parameter x such that a conditional probability of a pair of formulas in the consequence relation approaches 1, and a conditional probability of a pair of formulas not in the relation approaches α except 1 as x approaches 0.

Then, we can show that a consequence relation is closed in the limit if and only if the consequence relation is rational.

We apply this result to giving a probabilistic semantics for circumscription (McCarthy 1980) because circumscription has a similar semantics for a rational or preferential consequence relation and circumscription can define a consequence relation each pair of which consists of original axiom and derived result. Although we can show that every consequence relation defined by circumscription is a preferential consequence relation, it is not always rational. Especially, we can show that if there are some fixed propositions or if we minimize more than three propositions in parallel, then a consequence relation defined by this circumscription is always non-rational.

However, in some cases, we can separate a set of interpretations into a hierarchy, and so, we can provide a probability function so that a consequence relation defined by the circumscription in those cases is equivalent to a consequence relation defined by the probability function.

Consequence Relations and Their Models

In this section, we briefly review a work on consequence relation by Lehmann, Kraus and Magidor (Kraus, Lehmann, and Magidor 1988; Lehmann and Magidor 1988). A summary of the work is found in (Lehmann 1989).

We use a propositional language L and consider a binary relation $\mid\!\sim$ over formulas in L called *consequence relation* which has some desired property in a considered reasoning system. Intuitively speaking, $A \mid\!\sim B$ means that if a state of knowledge is A, then B is derived from A as a belief by inference rules defined in a considered reasoning system.

Definition 1 *A consequence relation that satisfies all seven properties below is called a* rational *consequence relation.*

If $A \equiv B$ is a truth-functional tautology and $A \mid\!\sim C$, then $B \mid\!\sim C$. (1)

If $A \supset B$ is a truth-functional tautology and $C \mid\!\sim A$, then $C \mid\!\sim B$. (2)

$A \mid\!\sim A$. (3)

If $A \mid\!\sim B$ and $A \mid\!\sim C$, then $A \mid\!\sim B \wedge C$. (4)

If $A \mid\!\sim C$ and $B \mid\!\sim C$, then $A \vee B \mid\!\sim C$. (5)

If $A \mid\!\sim B$ and $A \mid\!\sim C$, then $A \wedge B \mid\!\sim C$. (6)

If $A \mid\!\sim C$ and $A \not\mid\!\sim \neg B$, then $A \wedge B \mid\!\sim C$. (7)

A consequence relation that satisfies the first six properties is called *preferential consequence relation*.

The property (7) is called *rational monotony* and proposed by Makinson as a desired property for nonmonotonic reasoning system (Lehmann and Magidor 1988) and corresponds with one of fundamental conditions for minimal change of belief proposed by Gärdenfors (1988). An intuitive meaning of rational monotony is that the previous conclusion stays in the new belief if the negation of the added information is not in the previous belief.

A semantics for a rational consequence relation called *ranked model* is also studied by Lehmann and Magidor (1988). This semantics is a restricted semantics for preferential consequence relation studied by Kraus, Lehmann and Magidor (1988) called *preferential model.*

Definition 2 *A* preferential *model W is a triple $\langle S, l, \prec\rangle$ where S is a set, the element of which is called states, l assigns a logical interpretation of formulas to each state and $\prec$ is a strict partial order on S (irreflexive and transitive relation) satisfying the following smoothness condition: for all $A \in L$, for all $t \in \widehat{A}(\stackrel{\text{def}}{=} \{s \mid s \in S, l(s) \models A\})$, either $\exists s$ minimal in $\widehat{A}$, such that $s \prec t$ or t is itself minimal in $\widehat{A}$.*

Definition 3 *A* ranked *model W is a preferential model $\langle S, l, \prec\rangle$ for which the strict partial order $\prec$ satisfies the following condition: for all s, t and u in S, if $s \prec t$ then either $s \prec u$ or $u \prec t$.*

This definition is different from the original definition but actually equivalent. In a ranked model, a set of states are divided into hierarchy so that if $s \prec t$ then s and t belong to different rank and if $\neg(s \prec t)$ and $\neg(t \prec s)$ then s and t belong to the same rank.
We can define consequence relations by the above models as follows.

Definition 4 *Let W be a preferential (or ranked) model $\langle S, l, \prec\rangle$ and A, B be formulas in L. The consequence relation defined by W will be denoted by $\mid\!\sim_W$ and is defined by: $A \mid\!\sim_W B$ if and only if for any s minimal in $\widehat{A}, l(s) \models B$.*

Kraus, Lehmann and Magidor (1988) show that a consequence relation is preferential if and only if it is the consequence relation defined by some preferential model, and Lehmann and Magidor (1988) show that a consequence relation is rational if and only if it is the consequence relation defined by some ranked model.

Probabilistic Semantics for Rational Consequence Relation

From this point, we assume the set of propositional symbols in L is always finite.

Definition 5 *Let L be a propositional language. Then probability function P_x on L with positive parameter x is a function from a set of formulas in L and positive real numbers to real numbers which satisfies the following conditions.*

1. *For any $A \in L$ and for any $x > 0$, $0 \leq P_x(A) \leq 1$.*
2. *For any $x > 0$, $P_x(\mathbf{T}) = 1$.*
3. *For any $A \in L$ and $B \in L$ and for any $x > 0$, if $A \wedge B$ is logically false then $P_x(A \vee B) = P_x(A) + P_x(B)$.*

If we ignore a parameter x, the above definition becomes the standard formulation for probability function on L (Gärdenfors 1988, p. 37). We introduce a parameter x to express the weight of the probability for every states. Spohn (1988) uses a similar probability function to relate his Natural Conditional Functions to probability theory.

Definition 6 *Let $A, B \in L$. We define the conditional probability of B under A, $P_x(B|A)$ as follows.*

$$P_x(B|A) = \begin{cases} 1 & \text{if } P_x(A) = 0 \\ \dfrac{P_x(A \wedge B)}{P_x(A)} & \text{otherwise} \end{cases}$$

Definition 7 *A probability function P_x on L with positive parameter x is said to be* convergent *if and only if for any $A \in L$, there exists α such that $\lim_{x \to 0} P_x(A) = \alpha$.*

Now, we define a consequence relation in terms of the above probability function P_x.

Definition 8 *A consequence relation $\mid\!\sim$ is said to be* closed in the limit *if and only if there exists convergent probability function P_x on L with positive parameter x such that for all $A \in L$ and $B \in L$,*

$$A \mid\!\sim B \text{ if and only if } \lim_{x \to 0} P_x(B|A) = 1.$$

Intuitively speaking, if a pair, $\langle A, B \rangle$ is included in the closed consequence relation in the limit, then we can let the conditional probability of B under A approach 1 as much as possible and if not, the conditional probability will approach some value except 1. This intuitive meaning will be justified later.

We can show the following relationship between closed consequence relation in the limit and rational consequence relation.

Theorem 1 *$\mid\!\sim$ is closed in the limit if and only if $\mid\!\sim$ is rational* [1].

Proof:
We can show only-if half by checking that every closed consequence relation in the limit satisfies all properties for rational consequence relation.
We show if-half. If $\mid\!\sim$ is rational, then there exists some ranked model $W = \langle S, l, \prec \rangle$ such that for every pair of formulas A and B, $A \mid\!\sim B$ if and only if $A \mid\!\sim_W B$ (Lehmann and Magidor 1988). Since the language is logically finite, there exists a finite ranked model with a finite number of ranks. Let the number of ranks be $n (n \geq 1)$. Let η_i be the number of states at the i-th rank (states which are higher in $\prec$ is in a higher rank).

Let a function P_x on L with positive parameter x be defined as follows:[2]

$$P_x(A) \stackrel{\text{def}}{=} \frac{\sum_{i=1}^{n} \eta_i^A * x^{i-1}}{\sum_{i=1}^{n} \eta_i * x^{i-1}}$$

where η_i^A is the number of states at the i-th rank that satisfies A.

Then, P_x is convergent and we can show the following consequence relation $\mid\!\sim'$ is equivalent to $\mid\!\sim_W$: $A \mid\!\sim' B$ if and only if $\lim_{x \to 0} P_x(B|A) = 1$ □

There is another probabilistic characterization for a rational consequence relation.

Definition 9 *Let L be a finite propositional language and $\mid\!\sim$ be a consequence relation. $\mid\!\sim$ is said to be ε-*definable *if and only if there exists a function $\lambda : L^2 \mapsto [0, 1]$ such that*

1. *for all $A, B \in L$, $A \mid\!\sim B$ if and only if $\lambda(A, B) = 1$.*
2. *for all $\varepsilon > 0$, there exists a proper probability function P such that for all $A, B \in L$,*
   $$|P(B|A) - \lambda(A, B)| < \varepsilon.$$

An ε-definable consequence relation fits our intuitive meaning stated above and is actually equivalent to a closed consequence relation in the limit and therefore, equivalent to a rational consequence relation.

Consequence Relation and Circumscription

Preferential Consequence Relation and Circumscription

Here, we refer circumscription to the following definition. This is a slightly modified version of generalized

[1] Independently, Morris, Pearl and Goldszmidt have obtained a similar result to this theorem, as have Lehmann and Magidor.

[2] This assignment is suggested in (Lehmann and Magidor 1988).

circumscription (Lifschitz 1984) as we use $<$ instead of $\leq$.

Definition 10 *Let A be a propositional formula and $\mathbf{P}$ be a tuple of propositions and $\mathbf{p}$ be a tuple of propositional variables. Then $Circum(A; <^{\mathbf{P}})$ is defined as follows:*

$$A(\mathbf{P}) \wedge \neg\exists \mathbf{p}(A(\mathbf{p}) \wedge \mathbf{p} <^{\mathbf{P}} \mathbf{P}),$$

where $A(\mathbf{p})$ is obtained by replacing every proposition of $\mathbf{P}$ in $A(\mathbf{P})$ by every corresponding propositional variable, and $\mathbf{p} <^{\mathbf{P}} \mathbf{P}$ is a binary relation over formulas which satisfies the following two conditions:

1. *For any $\mathbf{P}$, $\neg\mathbf{P} <^{\mathbf{P}} \mathbf{P}$*
2. *For any $\mathbf{P}$, $\mathbf{Q}$ and $\mathbf{R}$, if $\mathbf{P} <^{\mathbf{P}} \mathbf{Q}$ and $\mathbf{Q} <^{\mathbf{P}} \mathbf{R}$, then $\mathbf{P} <^{\mathbf{P}} \mathbf{R}$*

Then, we can define a consequence relation $\mid\!\sim_{<\mathbf{P}}$ as follows:

$$A \mid\!\sim_{<\mathbf{P}} B \text{ if and only if } Circum(A; <^{\mathbf{P}}) \models B.$$

Semantics for the above circumscription is based on the following order over interpretations: $I_1 <^{\mathbf{P}} I_2$ if and only if for every proposition P not in $\mathbf{P}$, $I_1[P] = I_2[P]$, and $p <^{\mathbf{P}} q$ is true if we replace $I_1[P]$ whose P is in $\mathbf{P}$ for p and $I_2[P]$ whose P is in $\mathbf{P}$ for q. Then, we can think of the following preferential model $W = \langle S, l, \prec\rangle$ where a set of logical interpretations is S, and l is an identity function and $\prec$ is a strict partial order $<^{\mathbf{P}}$ over those interpretations. We say the preferential model is *defined by* $<^{\mathbf{P}}$. As Kraus, Lehmann and Magidor (1988) pointed out, if S is finite, the smoothness condition is always satisfied. Here, we consider a finite set of interpretations, so the smoothness condition is always satisfied.

However, there are some differences between preferential consequence relation and circumscription. In propositional circumscription, for any satisfiable formula A, $A \not\mid\!\sim_{<\mathbf{P}} \mathbf{F}$ [3] (we say $\mid\!\sim$ is proper), but in preferential consequence relation this is not always the case.

And since we use an identity function for l in circumscription, there is a preferential consequence relation in a language which can not be represented by circumscription in the same language. For example, L contains only two propositions P and Q, then there is a proper preferential consequence relation such that $P \vee Q \mid\!\sim (\neg P \wedge Q) \vee (P \wedge \neg Q)$ and $P \not\mid\!\sim P \wedge \neg Q$ and $Q \not\mid\!\sim \neg P \wedge Q$, but, there is no consequence relation defined by circumscription equivalent to the preferential consequence relation. This is because two or more states are mapped to the same interpretation in a corresponding preferential model.

We say a formula A is *complete* if for every formula B in L, $A \models B$ or $A \models \neg B$. A complete formula corresponds with an interpretation. Then, the following property excludes a preferential consequence relation such that two or more states are mapped to the same interpretation in a corresponding preferential model.

If C is complete and $A \vee B \mid\!\sim \neg C$, then $A \mid\!\sim \neg C$ or $B \mid\!\sim \neg C$ [4].

Theorem 2 *$\mid\!\sim$ is a proper preferential consequence relation and satisfies the above property if and only if there is some $<^{\mathbf{P}}$ such that $\mid\!\sim_{<\mathbf{P}} = \mid\!\sim$*

Proof:
We can easily show that every consequence relation defined by a circumscription is a proper preferential consequence relation and satisfies the above property. We show the converse. Suppose $\mid\!\sim$ is a proper preferential consequence relation and satisfies the above property. Let $\alpha(\mathbf{P})$ and $\beta(\mathbf{P})$ be complete formulas. Define $\alpha(\mathbf{P}) \prec \beta(\mathbf{P})$ if and only if $\alpha(\mathbf{P}) \vee \beta(\mathbf{P}) \mid\!\sim \alpha(\mathbf{P})$ and $\alpha(\mathbf{P}) \neq \beta(\mathbf{P})$. Then $\prec$ is a irreflexive and transitive relation. Suppose we collect all pairs in $\prec$: $\alpha_1(\mathbf{P}) \prec \beta_1(\mathbf{P}) ... \alpha_n(\mathbf{P}) \prec \beta_n(\mathbf{P})$. Then, $\mathbf{p} <^{\mathbf{P}} \mathbf{P}$ is defined as follows: $(\alpha_1(\mathbf{p}) \wedge \beta_1(\mathbf{P})) \vee ...(\alpha_n(\mathbf{p}) \wedge \beta_n(\mathbf{P}))$. Then, we can show $\mid\!\sim_{<\mathbf{P}} = \mid\!\sim$. $\Box$

Rational Consequence Relation and Circumscription

Unfortunately, although a consequence relation defined by circumscription is always preferential, it is not always rational.

Theorem 3

1. *If a tuple of propositions, $\mathbf{P}$ does not contain all propositions in L and for any non-trivial partial order $<^{\mathbf{P}}$ (there are some interpretations, I and J such that $J <^{\mathbf{P}} I$), the consequence relation defined by $<^{\mathbf{P}}$ is always non-rational.*
2. *If $\mathbf{P}$ contains all propositions in L, then a consequence relation defined by minimizing one or two propositions in parallel is rational.*
3. *Even if $\mathbf{P}$ contains all propositions in L, a consequence relation defined by minimizing more than three propositions in parallel is always non-rational.*

Proof:

1. Since $<^{\mathbf{P}}$ is non-trivial, there exist some interpretations, I and J such that $J <^{\mathbf{P}} I$. And there exists some proposition P which is not in $\mathbf{P}$. Let K be a truth assignment which is the same as J except the assignment of P. Then since $J <^{\mathbf{P}} I$, the assignment of P in I is the same as in J from the definition of $<^{\mathbf{P}}$. Then, K is different both from J and from I in the assignment of P. We can show $\neg(J <^{\mathbf{P}} K)$ and $\neg(K <^{\mathbf{P}} I)$ and so, the preferential model defined by $<^{\mathbf{P}}$ is not ranked from Definition 3. Therefore, the consequence relation defined by $<^{\mathbf{P}}$ is not rational.

[3] $\mathbf{F}$ is falsity

[4] This property corresponds with (R8) in (Katsuno and Mendelzon 1990)

2. We can easily check that a preferential model defined by minimizing one or two propositions is ranked.
3. Let $\mathbf{P}$ contain the following minimized propositions P, Q and R. And let the following three interpretations I, J and K satisfy the following conditions:
 (a) Every assignments are the same except assignments for P, Q and R.
 (b) $I \models \neg P \wedge \neg Q \wedge R$, $J \models P \wedge Q \wedge \neg R$ and $K \models \neg P \wedge Q \wedge R$.

 Then $I <^{\mathbf{P}} K$, but $\neg(I <^{\mathbf{P}} J)$ and $\neg(J <^{\mathbf{P}} K)$ and so, the preferential model defined by $<^{\mathbf{P}}$ is not ranked. Therefore, a consequence relation defined by minimizing more than three propositions is not rational. □

Although rational monotony corresponds with one of fundamental conditions for minimal change of belief proposed by Gärdenfors (1988), there are several examples in commonsense reasoning which correspond with the third case of the above theorem. A notable example is a *closed world assumption*(CWA). In CWA, we minimize all propositions and so, we do not have rational monotony if a number of propositions is more than three.

So, one may argue that a rational consequence relation is not practically *rational* in commonsense reasoning. However, what we would like to say here is *not* whether it is rational or not, but that circumscription in general does not have the probabilistic semantics which we have defined so far and that if an order defined by circumscription is ranked, then it has a probabilistic *rationale*.

Probabilistic Interpretation for Lazy Circumscription

In this subsection, we consider the following kind of circumscription.

Definition 11 *Circumscription $<^{\mathbf{P}}$ is* lazy *if the preferential model defined by $<^{\mathbf{P}}$ is ranked.*

We can show that a consequence relation $\mid\!\sim$ is proper and rational if and only if there is some $<^{\mathbf{P}}$ of lazy circumscription such that $\mid\!\sim_{<^{\mathbf{P}}} = \mid\!\sim$. And, if a circumscription is lazy, we can attach a probability function which defines an equivalent consequence relation to the consequence relation defined by $<^{\mathbf{P}}$.

For example, let a set of proposition be $\{P, Q\}$. Then, there are the following four interpretations:

$$\{\langle \neg P, \neg Q\rangle,\ \langle P, \neg Q\rangle,\ \langle \neg P, Q\rangle,\ \langle P, Q\rangle\}.$$

Suppose we minimize P and Q in parallel. We denote the strict partial order relation by this minimization as $<^{\langle P,Q\rangle}$. Then the consequence relation defined by $<^{\langle P,Q\rangle}$ is as follows:

$A(P,Q) \mid\!\sim_{<^{\langle P,Q\rangle}} B(P,Q)$ if and only if

$$A(P,Q) \wedge \neg\exists p \exists q (A(p,q) \wedge ((p,q) < (P,Q))) \models B(P,Q),$$

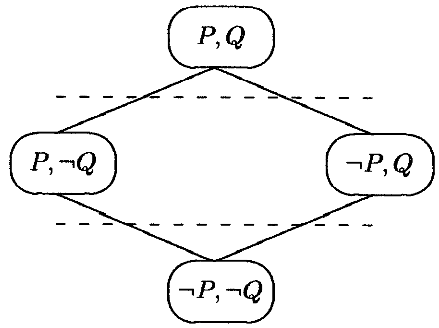

Figure 1: Partial Order by Minimizing P and Q.

where $(p,q) < (P,Q)$ is the following abbreviation:

$$(p,q) < (P,Q) \stackrel{\text{def}}{=} (p \supset P) \wedge (q \supset Q) \wedge \neg((P \supset p) \wedge (Q \supset q)).$$

The preferential model defined by $<^{\langle P,Q\rangle}$ is ranked (Figure 1). In the figure, a lower interpretation is more preferable than an upper interpretation. In probabilistic semantics, we regard this order as an order of probability. This means that a lower interpretation is more probable than an upper interpretation. Moreover, we make the probability function of an interpretation in $(i+1)$-th rank be x times as much as that of an interpretation in i-th rank so that we can ignore less probable interpretation as x approaches to 0. In this example, we attach the following probability function P_x with a positive parameter x to interpretations.

$$P_x(\langle \neg P, \neg Q\rangle) \stackrel{\text{def}}{=} \frac{1}{1+2x+x^2}$$

$$P_x(\langle P, \neg Q\rangle) \stackrel{\text{def}}{=} \frac{x}{1+2x+x^2}$$

$$P_x(\langle \neg P, Q\rangle) \stackrel{\text{def}}{=} \frac{x}{1+2x+x^2}$$

$$P_x(\langle P, Q\rangle) \stackrel{\text{def}}{=} \frac{x^2}{1+2x+x^2}$$

Then, probability of formula A is defined by a sum of interpretations which satisfies A.

$$P_x(A) \stackrel{\text{def}}{=} \sum_{I \models A} P_x(I)$$

Let $\mid\!\sim$ be a consequence relation as follows.

$$A \mid\!\sim B \text{ if and only if } \lim_{x \to 0} P_x(B|A) = 1$$

Intuitively, making x approach to 0 means that we consider only the most probable interpretations which satisfy A and the fact that $P_x(B|A)$ approaches to 1 means that in all the most probable interpretations which satisfy A, B is extremely probable. This is a probabilistic semantics for lazy circumscription.

Let us check if $P \vee Q \mid\!\sim \neg P \vee \neg Q$. Since $\langle P, \neg Q\rangle$, $\langle \neg P, Q\rangle$ and $\langle P, Q\rangle$ satisfy $P \vee Q$,

$$P_x(P \vee Q) = P_x(\langle P, \neg Q\rangle) + P_x(\langle \neg P, Q\rangle) + P_x(\langle P, Q\rangle) = \frac{2x + x^2}{1 + 2x + x^2}$$

Similarly, $P_x((P \vee Q) \wedge (\neg P \vee \neg Q)) = \dfrac{2x}{1 + 2x + x^2}$

Then,

$$\lim_{x\to 0} P_x(\neg P \vee \neg Q | P \vee Q) = \lim_{x\to 0} \frac{P_x((P \vee Q) \wedge (\neg P \vee \neg Q))}{P_x(P \vee Q)} = \lim_{x\to 0} \frac{2x}{2x + x^2} = 1$$

Therefore, $P \vee Q \mid\!\sim \neg P \vee \neg Q$. This means that in all the most probable interpretations which satisfy $P \vee Q$, $\neg P \vee \neg Q$ is extremely probable and this corresponds with the result of $P \vee Q \mid\!\sim_{<\langle P,Q\rangle} \neg P \vee \neg Q$.

And suppose we check if $P \vee Q \mid\!\sim P \wedge \neg Q$.

$$\lim_{x\to 0} P_x(P \wedge \neg Q | P \vee Q) = \lim_{x\to 0} \frac{P_x((P \vee Q) \wedge (P \wedge \neg Q))}{P_x(P \vee Q)} = \lim_{x\to 0} \frac{x}{2x + x^2} \neq 1$$

Therefore, $P \vee Q \not\mid\!\sim P \wedge \neg Q$ and this corresponds with the result of $P \vee Q \not\mid\!\sim_{<\langle P,Q\rangle} P \wedge \neg Q$. In the same way, we can show that for every $A, B \in L$, $A \mid\!\sim B$ if and only if $A \mid\!\sim_{<\langle P,Q\rangle} B$.

Conclusion

We propose a probabilistic semantics called closed consequence relation in the limit for lazy nonmonotonic reasoning and show that a consequence relation is closed in the limit if and only if it is rational. Then, we apply our result to giving a probabilistic semantics for a class of circumscription which has lazy nonmonotonicity.

We think we need to do the following research.

1. We would like to know a probabilistic semantics which characterizes a consequence relation defined by whole class of circumscription exactly.
2. We can not apply our result to Default Logic(Reiter 1980) or Autoepistemic Logic(Moore 1984) because a consequence relation defined by those logics is not even preferential. We must extend our result to apply those logics.

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