Updating Knowledge Bases with Disjunctive Information

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Abstract

It is well known that the minimal change principle was widely used in knowledge base updates. However, recent research has shown that conventional minimal change methods, eg. the PMA (Winslett 1988), are generally problematic for updating knowledge bases with disjunctive information. In this paper, we propose two different approaches to deal with this problem – one is called the *minimal change with exceptions* (MCE), the other is called the *minimal change with maximal disjunctive inclusions* (MCD). The first method is syntax-based, while the second is model-theoretic. We show that these two approaches are equivalent for propositional knowledge base updates, and the second method is also appropriate for first order knowledge base updates. We then prove that our new update approaches still satisfy the standard Katsuno and Mendelzon's update postulates.

Introduction

The knowledge base update problem has been widely studied in AI. It generally addresses the following question: given a knowledge base (i.e. a set of logical formulas as a description of the world), what changes may be caused by an occurrence of new knowledge and how to specify the new knowledge base when the old one has changed? It is well known that the *minimal change principle* was employed in most formalizations of knowledge base updates (Baral 1994; Dalal 1988; Friedman & Halpern 1994; Katsuno & Mendelzon 1991a; Winslett 1988).

However, recent research has revealed that such minimal change is usually inappropriate for representing knowledge change with disjunctive information (Brewka & Hertzberg 1993; Kartha & Lifschitz 1994; Zhang & Foo 1995). This paper is concerned with the problem of updating knowledge bases with disjunctive information. In particular, we propose two different approaches to solve this problem – one is called the *minimal change with exceptions*, the other is called the *minimal change with maximal disjunctive inclusions*. The first method is syntax-dependent, while the second is model-based. We show that these two approaches are equivalent for propositional knowledge base updates, and the second method is also appropriate for first order knowledge base update. We then prove that our new approaches still satisfy the standard Katsuno-Mendelzon update postulates.

The paper is organized as follows. The next section first reviews the PMA – the classical minimal change approach for knowledge base updates, and shows how the PMA fails to deal with the update with disjunctive information. Section 3 proposes a new method called minimal change with exceptions, which overcomes the problem of the PMA but is syntax-based. Section 4 proposes an alternative method called minimal change with maximal disjunctive inclusions that is model-theoretic. It is observed that this approach is also suitable for the first order knowledge base updates. It is also shown that these two approaches are equivalent for propositional knowledge base updates. Section 5 investigates the relationship between our approaches and the standard Katsuno-Mendelzon update theory. Finally, section 6 discusses related work and concludes the paper.

The Minimal Change Approach: A Review

In this section we first introduce some preliminary concepts and review the PMA (*possible models approach* (Winslett 1988)) – a classical minimal change approach for update. Consider a finite propositional language $\mathcal{L}$[1]. We represent a *knowledge base* by a propositional formula ψ. A propositional formula ϕ is *complete* if ϕ is consistent and for any propositional formula μ, $\phi \models \mu$ or $\phi \models \neg\mu$. $Models(\psi)$ denotes the set of all models of ψ, i.e. all interpretations of $\mathcal{L}$ in which ψ is true. We also consider *state constraints* about the world. Let C be a satisfiable propositional formula that represents all state constraints about the world[2]. Thus, for any

[1] Note that the PMA was originally based on a first order language. Here we first consider propositional knowledge base update and will discuss the first order case in section 4.

[2] Usually, we use a set of formulas to represent state constraints. In this case, C can be viewed as a conjunction of all such formulas.

knowledge base ψ, we require $\psi \models C$. Let I be an interpretation of $\mathcal{L}$. We say that I is a *state* of the world if $I \models C$. For simplicity, we *fix* the universe of interpretations in language $\mathcal{L}$. That is, we restrict $|I_1| = |I_2|$ for any two interpretations I_1 and I_2 of $\mathcal{L}$. A knowledge base ψ can be treated as a *description* of the world, where $Models(\psi)$ is the set of all *possible states* of the world with respect to ψ.

Let ψ be the current knowledge base and μ a propositional formula which is regarded as new knowledge (information) about the world. Then, informally, the general question of updating ψ with μ is how to specify the new knowledge base after combining the new knowledge (information) μ into the current knowledge base ψ (we also call μ *update effect*). In the PMA, the knowledge base update is achieved by updating *every possible state* of the world with respect to ψ with μ, and such state update is constructed based on the *principle of minimal change* on models.

Formally, let I_1 and I_2 be two interpretations of $\mathcal{L}$. We say that I_1 and I_2 differs on a propositional letter l if l appears in exactly one of I_1 and I_2. $Diff(I_1, I_2)$ denotes the set of all different propositional letters between I_1 and I_2. Let I be an interpretation and $\mathcal{I}$ a set of interpretations. We define the set of all *minimal different interpretations* of $\mathcal{I}$ with respect to I as follows:

$$Min(I, \mathcal{I}) = \{I' \mid I' \in \mathcal{I}, \text{ and there does not exist other } I'' \in \mathcal{I} \text{ such that } Diff(I, I'') \subset Diff(I, I')\}.$$

Then we can present the formal definition of the state update in the PMA as follows.

Definition 1 *Let C be the state constraint, S a state of the world, i.e. $S \models C$, and μ a propositional formula. Then the* set of all possible states *of the world resulting from updating S with μ by the PMA, denoted as $Res(S, \mu)$, is defined as follows:*

$$Res(S, \mu) = Min(S, Models(C \wedge \mu)). \qquad (1)$$

Based on the definition of state update, we can then define the PMA update operator $\diamond_{pma}$ for knowledge bases.

Definition 2 *Let ψ be a knowledge base and μ a propositional formula. $\psi \diamond_{pma} \mu$ denotes the update of ψ with μ by the PMA*[3]*, where*

1. *If ψ entails μ or ψ is inconsistent, then $\psi \diamond_{pma} \mu \equiv \psi$, otherwise*
2. $Models(\psi \diamond_{pma} \mu) = \bigcup_{S \in Models(\psi)} Res(S, \mu)$.

In the above definition, condition 1 says that if ψ entails μ, then nothing is changed since the knowledge μ has been represented by knowledge base ψ; or if ψ is inconsistent, then any update can not change it into a consistent knowledge base (Katsuno & Mendelzon 1991a). Condition 2 says that if ψ is consistent and does not entail μ, then ψ should be changed, and this change is made by updating every model of ψ with μ as defined in Definition 1. It has been shown that under many circumstances, the PMA is powerful and effective for representing knowledge updates and reasoning about action (Winslett 1988; Katsuno & Mendelzon 1991a). However, recent research reveals that the PMA is problematic for update with disjunctive information. The following example illustrates such difficulty of the PMA.

Example 1. The dropping-box problem[4]. Suppose a table is painted with one part white and one part black. Therefore, a box on the table implies that it may be entirely within the white region, or within the black region, or touching the both regions. This constraint can be expressed by the following formula:

$$Ontable(Box) \supset Inwhite(Box) \vee Inblack(Box). \qquad (2)$$

Now suppose the current knowledge base is

$$\psi \equiv \neg Ontable(Box) \wedge \neg Inwhite(Box) \wedge \neg Inblack(Box) \wedge (2)$$

to express the fact that the box is not on the table. Consider the update of ψ with $\mu \equiv Ontable(Box)$ (i.e. the box is dropped on the table). Using the PMA, we have the result:

$$\psi \diamond_{pma} \mu \equiv Ontable(Box) \wedge (Inwhite(Box) \vee \neg Inblack(Box)) \wedge (\neg Inwhite(Box) \vee Inblack(Box)) \wedge (2),$$

which says that the box must be only within one of white or black region. Obviously, this result is not quite plausible from our intuition. □

In the above example, although the update has a simple effect $Ontable(Box)$, together with the constraint (2), it implies an indirect disjunctive effect $Inwhite(Box) \vee Inblack(Box)$. Therefore, according to the minimal change principle of the PMA, only $Inwhite(Box)$ or $Inblack(Box)$ should be true after this update, but not both, and this leads to an unintuitive solution.

MCE: A Syntax-based Approach

To overcome the problem with the PMA, in this section we propose an approach for update based on the principle of *Minimal Change with Exceptions*, which we abbreviate as the MCE. In fact, our approach is based on the PMA but with some modifications. The idea is described as follows. Consider the state update[5]. Generally, during the update, the truth value of any literal in the state changes minimally by default. But

[3] Here we only consider the *well-defined* update, that is, μ is consistent with the state constraint C.

[4] This example was suggested by Ray Reiter and discussed in (Kartha & Lifschitz 1994).

[5] Similar to the PMA, in our approach, updating a knowledge base is achieved by updating every possible model of the knowledge base.

if the truth value of a literal is *logically indefinite* with respect to the update, then this literal is treated as an *exception* to the minimal change principle. In this case, the change of this literal's truth value will not obey the rule of minimal change.

Informally, we say that the truth value of a literal is *logically indefinite* with respect to an update, if this literal occurs in a disjunction which is entailed by the constraint and the update effect and not satisfied in the initial knowledge base (or state). Consider the dropping-box example presented in last section where the constraint is (2) and the update effect is $Ontable(Box)$. As both $Inwhite(Box)$ and $Inblack(Box)$ are not true in the initial knowledge base but the disjunction $Inwhite(Box) \vee Inblack(Box)$ is entailed by (2) and $Ontable(Box)$, we know that $Inwhite(Box) \vee Inblack(Box)$ should be true in the resulting knowledge base but we can not determine the truth values of $Inwhite(Box)$ and $Inblack(Box)$ exactly. In this case, we say literals $Inwhite(Box)$ and $Inblack(Box)$ are *logically indefinite* with respect to the update. According to our idea described above, $Inwhite(Box)$ and $Inblack(Box)$ should be regarded as exceptions to the minimal change principle. Thus, $Inwhite(Box)$ and $Inblack(Box)$ are *not* forced to change minimally during the update, from which we get the desired solution including the case that both $Inwhite(Box)$ and $Inblack(Box)$ may be true after updating the knowledge base with $Ontable(Box)$.

Formally, let EXC be a set of propositional letters that we represent to be exceptional to the minimal change, I_1 and I_2 two interpretations. $Diff(I_1, I_2)^{EXC}$ denotes the set of all different propositional letters, which are *not* in EXC, between I_1 and I_2. That is, $l \in Diff(I_1, I_2)^{EXC}$ iff $[\neg]l \notin I_1 \cap I_2$ and $l \notin EXC$, where notation $[\neg]$ means that the negation sign $\neg$ may or may not occur. For example, let $I_1 = \{a, b, \neg c, \neg d\}$, $I_2 = \{\neg a, b, c, \neg d\}$ and $EXC = \{a, b\}$. Then $Diff(I_1, I_2)^{EXC} = \{c\}$. Let I be an interpretation and $\mathcal{I}$ a set of interpretations. We define the set of all minimal different interpretations of $\mathcal{I}$ with respect to I *with the exception* EXC as follows:

$$Min(I, \mathcal{I})^{EXC} = \{I' \mid I' \in \mathcal{I}, \text{ and there does not exist other } I'' \in \mathcal{I} \text{ such that } Diff(I, I'')^{EXC} \subset Diff(I, I')^{EXC}\}.$$

Let C be a propositional formula used to represent the state constraint and μ a propositional formula. We say a disjunction $\bigvee_{i=1}^{n}[\neg]l_i$ $(1 < n)$ satisfying $C \wedge \mu \models \bigvee_{i=1}^{n}[\neg]l_i$, where l_i is a propositional letter $(1 \leq i \leq n)$, is a *non-trivial* disjunction entailed by $C \wedge \mu$ if for any $M \subset \{1, \cdots, n\}$, $C \wedge \mu \not\models \bigvee_{j \in M}[\neg]l_j$. We denote the set of all non-trivial disjunctions entailed by $C \wedge \mu$ as $D(\mu)$. If $d \equiv \bigvee_{i=1}^{n}[\neg]l_i$ in $D(\mu)$, then we denote $|d| = \{l_1, \cdots, l_n\}$.

In the dropping-box example presented previously, as we have $(2) \wedge Ontable(Box) \models Inwhite(Box) \vee Inblack(Box)$, $(2) \wedge Ontable(Box) \not\models Inwhite(Box)$, and $(2) \wedge Ontable(Box) \not\models Inblack(Box)$, we then get $D(Ontable(Box)) = \{d\} = \{Inwhite(Box) \vee Inblack(Box)\}$, and $|d| = \{Inwhite(Box), Inblack(Box)\}$. Now we give the definition of state update in the MCE as follows.

Definition 3 *Let C be the state constraint, S a state of the world, i.e. $S \models C$, μ a propositional formula, and $D(\mu)$ the set of non-trivial disjunctions entailed by $C \wedge \mu$. We define the* exceptional letters *with respect to S and μ as follows:*

$$EXC(S, \mu) = \bigcup_{d \in D(\mu), S \not\models d} |d|. \tag{3}$$

Then the set of possible states resulting from updating S with μ by the MCE, denoted as $Res(S, \mu)^{EXC(S,\mu)}$, is defined as

$$Res(S, \mu)^{EXC(S,\mu)} = Min(S, Models(C \wedge \mu))^{EXC(S,\mu)}. \tag{4}$$

Let us examine Definition 3 in detail. Firstly, (3) defines a set of propositional letters that should be viewed as exceptions to the minimal change principle during the state update. If $d \in D(\mu)$ is already satisfied in S, then any letters which or whose negations occur in d will not be specified in $EXC(S, \mu)$, otherwise the letters should be in $EXC(S, \mu)$. For instance, suppose $S = \{\neg a, \neg b, c, \neg d\}$ and $D(\mu) = \{a \vee b, b \vee c\}$, then $EXC(S, \mu) = \{a, b\}$ while c is not in $EXC(S, \mu)$ as $S \models b \vee c$. Secondly, (4) defines the set of possible resulting states after updating S with μ. Note that any literals in S whose corresponding letters are in $EXC(S, \mu)$ will not obey the minimal change principle during the update. In the above example, if $C \equiv (d \supset a \vee b) \wedge (d \supset b \vee c)$ and $\mu \equiv d$[6], then we get

$Res(S, \mu)^{EXC(S,\mu)} = \{S_1, S_2, S_3\}$, where
$S_1 = \{a, \neg b, c, d\}$,
$S_2 = \{\neg a, b, c, d\}$, and
$S_3 = \{a, b, c, d\}$.

Based on Definition 3, we define the knowledge base update in the MCE as follows.

Definition 4 *Let ψ be a knowledge base, μ a propositional formula. $\psi \diamond_{mce} \mu$ denotes the update of ψ with μ by the MCE, where*

1. *If ψ entails μ or ψ is inconsistent, then $\psi \diamond_{mce} \mu \equiv \psi$, otherwise*
2. $Models(\psi \diamond_{mce} \mu) = \bigcup_{S \in Models(\psi)} Res(S, \mu)^{EXC(S,\mu)}$.

Comparing with Definition 1 and 2, it is easy to see that the MCE is defined based on the PMA but with exception $EXC(S, \mu)$. Clearly, if $EXC(S, \mu) = \emptyset$ for any $S \in Models(\psi)$,

[6] This implies that $D(\mu) = \{a \vee b, b \vee c\}$.

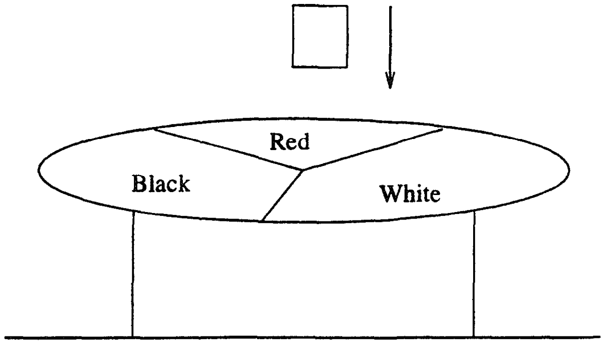

Figure 1: The extended dropping-box domain.

then the MCE reduces to the PMA. Consider the dropping-box example once again. As the knowledge base ψ corresponds to a unique state $S = \{\neg Ontable(Box), \neg Inwhite(Box), \neg Inblack(Box)\}$, we have $EXC(S, Ontable(Box)) = \{Inwhite(Box), Inblack(Box)\}$, then from Definition 3 and 4, we get the desired result:

$$\psi \diamond_{mce} \mu \equiv Ontable(Box) \wedge (Inwhite(Box) \vee Inblack(Box)) \wedge (2).$$

From the above discussion, we can see that the MCE overcomes the problem with the PMA of updating knowledge bases with disjunctive information. However, to specify the exceptional letters $EXC(S, \mu)$, we need to derive *every* non-trivial disjunction d from the constraint C and update effect μ, and then verify if $S \models d$ for each $S \in Models(\psi)$. Obviously, the first step is a syntactic procedure. Hence, we say that the MCE is *syntax-based*.

MCD: A Model-theoretic Approach

In this section, we propose an alternative method which solves the problem of updating knowledge bases with disjunctive information and is model-based.

The Approach

Our approach is based on the idea of the *Minimal Change with maximal Disjunctive inclusions*, that we call the MCD for short. To illustrate this idea simply, we first consider the following example.

Example 2. The extended dropping-box problem. Suppose a round table is painted with three equal parts of red color, white color and black color respectively. Intuitively, a box on the table implies that it may be entirely within one of these three regions, *or* touching any two of these three regions, *or* touching all of these three regions. This situation can be described by Figure 1. Also, a constraint to formalize this domain is specified as:

$$Ontable(Box) \supset Inred(Box) \vee Inwhite(Box) \vee Inblack(Box). \quad (5)$$

Now suppose the current knowledge base is

$$\psi \equiv \neg Ontable(Box) \wedge \neg Inred(Box) \wedge \neg Inwhite(Box) \wedge \neg Inblack(Box) \wedge (5),$$

which corresponds to a unique state:

$$S = \{\neg Ontable(Box), \neg Inred(Box), \neg Inwhite(Box), \neg Inblack(Box)\}.$$

Consider updating state S with $\mu \equiv Ontable(Box)$ (i.e. the box is dropped on the table). The question is: how can we get the desired possible states? Our idea is described as follows. Firstly, using the PMA we get the set of possible resulting states:

$Res(S, \mu) = \{S_1, S_2, S_3\}$, where
$S_1 = \{Ontable(Box), Inred(Box), \neg Inwhite(Box), \neg Inblack(Box)\}$,
$S_2 = \{Ontable(Box), \neg Inred(Box), Inwhite(Box), \neg Inblack(Box)\}$,
$S_3 = \{Ontable(Box), \neg Inred(Box), \neg Inwhite(Box), Inblack(Box)\}$.

Obviously, S_1, S_2 and S_3 are the desired possible resulting states. But, we know that the following states are also our desired resulting states:

$S_4 = \{Ontable(Box), Inred(Box), Inwhite(Box), \neg Inblack(Box)\}$,
$S_5 = \{Ontable(Box), Inred(Box), \neg Inwhite(Box), Inblack(Box)\}$,
$S_6 = \{Ontable(Box), \neg Inred(Box), Inwhite(Box), Inblack(Box)\}$,
$S_7 = \{Ontable(Box), Inred(Box), Inwhite(Box), Inblack(Box)\}$.

In fact, states S_4, S_5, S_6 and S_7 can be generated from $Res(S, \mu)$. Let $\mathcal{S}_P$ be any non-empty subset of $Res(S, \mu)$. Then, there always exists a model S' in $Models(Ontable(Box) \wedge (5))$[7], such that S' satisfies:

(i) for each $S_i \in \mathcal{S}_P$, $Diff(S, S_i) \subseteq Diff(S, S')$, and
(ii) there does not exist another S'' in $Models(Ontable(Box) \wedge (5))$ satisfying condition (i) but $Diff(S, S'') \subset Diff(S, S')$.

Now we claim that S' is also a desired resulting state. For instance, let $\mathcal{S}_P$ be $\{S_1\}$. Then from conditions (i) and (ii), the corresponding S' is S_1 itself. On the other hand, if $\mathcal{S}_P = \{S_1, S_2\}$, we get

$$S' = \{Ontable(Box), Inred(Box), Inwhite(Box), \neg Inblack(Box)\} = S_4.$$

Similarly, we can get S_5, S_6, and S_7 from $\{S_1, S_3\}$, $\{S_2, S_3\}$ and $\{S_1, S_2, S_3\}$ respectively. Therefore, every desired resulting state can be generated from the corresponding subset of $Res(S, \mu)$ by using the above procedure. □

Before we explain the above procedure in detail, we first introduce a useful concept. Let I_1 and I_2 be two interpretations, and $a_1 \vee \cdots \vee a_k$ a disjunction that is

[7] Clearly, $Models(Ontable(Box) \wedge (5))$ is the set of all possible states of the world in which $Ontable(Box)$ is true.

satisfied in I_1 and I_2. We say that I_1 *includes* I_2's interpretation for this disjunction iff for any disjunct a_i, $I_2 \models a_i$ implies $I_1 \models a_i$.

In the above example, as the disjunction $Inred(Box) \vee Inwhite(Box) \vee Inblack(Box)$ is a logical consequence of the update effect $Ontable(Box)$ and the constraint (5), different states in $Res(S, \mu)$ also represent different interpretations for this disjunction. But, because of the minimal change principle, $Res(S, \mu)$ may only include *partial* interpretations for the disjunction. For instance, using the PMA, we get the possible resulting states S_1, S_2 and S_3 which only represent three possible interpretations for this disjunction. To describe the update with disjunctive effect properly, as we have observed previously, we need to represent *every possible* interpretation of the disjunction but *without* losing the minimal change criterion for other information.

It is not hard to see that conditions (i) and (ii) above achieve this purpose. In particular, given a subset $\mathcal{S}_P$ of $Res(S, \mu)$, condition (i) states that for every state S_i in $\mathcal{S}_P$, S' is the state which *includes* S_i's interpretation for the disjunction, while condition (ii) restricts this S' to minimal change on other literals with respect to S. For every subset $\mathcal{S}_P$ of $Res(S, \mu)$, we can get the corresponding S'. So, we take all such S's to be the possible resulting states that include *maximal* possible interpretations of the disjunction without losing the minimal change criterion on other information. Therefore, this approach is so called the *minimal change with maximal disjunctive inclusions.*

Formal Descriptions

Based on the above discussion, we now develop our method formally. Similar to the previous presentation, we first define the state update in the MCD.

Definition 5 *Let C be the state constraint, S a state of the world, i.e., $S \models C$, and μ a propositional formula. Then the set of all possible states of the world resulting from updating S with μ by the MCD, denoted as $Res(S, \mu)^{mcd}$, is defined as follows:*

$$Res(S, \mu)^{mcd} = \bigcup_{\mathcal{S}_P \in 2^{Res(S,\mu)}} Dis(S, \mathcal{S}_P)^{8}, \qquad (6)$$

where

$Dis(S, \mathcal{S}_P) = \{S' \mid S' \in Models(C \wedge \mu)$ *such that (i) for each S_i in $\mathcal{S}_P$, $Diff(S, S_i) \subseteq Diff(S, S')$, and (ii) there does not exist other S'' in $Models(C \wedge \mu)$ satisfying (i) but $Diff(S, S'') \subset Diff(S, S')\}$.*

Note that in Definition 5, we consider the power set of $Res(S, \mu)$, so that any element $\mathcal{S}_P$ of $2^{Res(S,\mu)}$ is a subset of $Res(S, \mu)$. $Dis(S, \mathcal{S}_P)$ represents the set of states that include the interpretations of the disjunctive information represented by all states of $\mathcal{S}_P$ without losing the minimal change criterion on other information. Clearly, if there is only one element S' in $\mathcal{S}_P$, then $Dis(S, \mathcal{S}_P) = \mathcal{S}_P = \{S'\}$.

After specifying the state update, we then define the MCD update operator $\diamond_{mcd}$ for knowledge bases as follows.

Definition 6 *Let ψ be a knowledge base and μ a propositional formula. $\psi \diamond_{mcd} \mu$ denotes the update of ψ with μ by the MCD, where*

1. *If ψ entails μ or ψ is inconsistent, then $\psi \diamond_{mcd} \mu \equiv \psi$, otherwise*
2. *$Models(\psi \diamond_{mcd} \mu) = \bigcup_{S \in Models(\psi)} Res(S, \mu)^{mcd}$.*

Example 3. Continue considering the extended dropping-box example. The initial knowledge base is

$$\psi \equiv \neg Ontable(Box) \wedge \neg Inred(Box) \wedge \neg Inwhite(Box) \wedge \neg Inblack(Box) \wedge (5).$$

Now, we update ψ with $\mu \equiv Ontable(Box)$. From Definition 5, we have

$$\bigcup_{S \in Models(\psi)} Res(S, \mu)^{mcd} = \{S_1, S_2, S_3, S_4, S_5, S_6, S_7\},$$

where $S_1, \cdots, S_7$ have been given in 4.1 previously. Finally, from Definition 6, we get the desired resulting knowledge base

$$\psi \diamond_{mcd} \mu \equiv Ontable(Box) \wedge (Inred(Box) \vee Inwhite(Box) \vee Inblack(Box)) \wedge (5).$$

□

Existence as Disjunctive Information

So far, we have only considered the propositional knowledge base update, while a disjunctive information related to the update is simply a propositional disjunction. If we consider the first order knowledge base update, on the other hand, disjunctive information may be also represented by a formula with an existential quantifier. For instance, a formula

$$Occupied(Table) \equiv \exists x. On(x, Table), \qquad (7)$$

represents a constraint that the table is occupied iff there exists (exist) some object (objects) on the table, but we can not determine how many objects there are and what the exact object (objects) is (are). A natural question is: can the MCD also deal with this kind of disjunctive information?

Consider a first order language $\mathcal{L}_F$ with equality. We can extend the MCD in the following way. A knowledge base ψ_F is represented by a closed first order formula. Let C_F and μ_F be two closed first order formulas, where C_F represents the state constraints (C_F can be viewed as the conjunction of a set of closed first order formulas)[9], and μ_F represents a new knowledge (information). Then the update of ψ_F with μ_F is defined exactly the same as the propositional knowledge

[8] Recall that $Res(S, \mu)$ is defined by (1) in section 2.

[9] Note that ψ_F should satisfy the condition $\psi_F \models C_F$.

base update as defined in Definition 5 and 6, except that propositional formulas C, ψ and μ in Definition 5 and 6 are replaced by formulas C_F, ψ_F and μ_F respectively.

Suppose a knowledge base is $\psi_F \equiv \neg Occupied(Table) \wedge (7)$. Then, updating ψ_F with $\mu_F \equiv Occupied(table)$ implies an effect $\exists x.On(x, Table)$, i.e. $(7) \wedge \mu_F \models \exists x.On(x, Table)$. Using the extended MCD as described above, the resulting knowledge base $\psi_F \diamond_{mcd} \mu_F$ will imply a consequence that there may be one *or* more objects on the table. That is, *every possible* interpretation of $\exists x.On(x, Table)$ will be represented by $\psi_F \diamond_{mcd} \mu_F$. If we use the PMA, on the other hand, the solution implies that there is only one object on the table, which seems too restricted.

Equivalence between MCE and MCD

Restricting the MCD to propositional case, we can show that the MCD and MCE are equivalent for propositional knowledge base updates.

Theorem 1 *Let ψ be a propositional knowledge base and μ a propositional formula. Then for any propositional formula ϕ, $\psi \diamond_{mce} \mu \models \phi$ iff $\psi \diamond_{mcd} \mu \models \phi$.*

Since the MCD is model-theoretic while the MCE is syntax-based, the MCD in fact provides a semantics for the MCE. The following result further shows the relationship between the MCD (MCE) and the PMA.

Theorem 2 *Let ψ be a propositional knowledge base and μ a propositional formula. Then for any propositional formula ϕ, $\psi \diamond_{mcd} \mu \models \phi$ (or $\psi \diamond_{mce} \mu \models \phi$) implies $\psi \diamond_{pma} \mu \models \phi$.*

Relationship with KM Update Postulates

In this section, we discuss how our update approaches relate to Katsuno and Mendelzon's update theory (Katsuno & Mendelzon 1991a; 1991b). As Theorem 1 states that the MCE and MCD are equivalent for propositional knowledge base updates, we need only address the relationship between the MCD and the Katsuno and Mendelzon's update theory.

The motivation of Katsuno and Mendelzon's proposal for update is an observation on the difference between revision and update. Basically, revision is intended to represent changes of an agent's belief state reflecting new information about the static world, while update is intended to represent changes of agent's belief in response to a dynamic world. Based on this observation, differently from the AGM postulates for revision (Gardenfors 1988), Katsuno and Mendelzon proposed the following alternative postulates for any update operator $\diamond$ (Katsuno & Mendelzon 1991a).

(U1) $\psi \diamond \mu$ implies μ.
(U2) If ψ implies μ then $\psi \diamond \mu \equiv \psi$.
(U3) If both ψ and μ are satisfiable then $\psi \diamond \mu$ is also satisfiable.
(U4) If $\psi_1 \equiv \psi_2$ and $\mu_1 \equiv \mu_2$ then $\psi_1 \diamond \mu_1 \equiv \psi_2 \diamond \mu_2$.
(U5) $(\psi \diamond \mu) \wedge \phi$ implies $\psi \diamond (\mu \wedge \phi)$.
(U6) If $\psi \diamond \mu_1$ implies μ_2 and $\psi \diamond \mu_2$ implies μ_1 then $\psi \diamond \mu_1 \equiv \psi \diamond \mu_2$.
(U7) If ψ is complete then $(\psi \diamond \mu_1) \wedge (\psi \diamond \mu_2)$ implies $\psi \diamond (\mu_1 \vee \mu_2)$.
(U8) $(\psi_1 \vee \psi_2) \diamond \mu \equiv (\psi_1 \diamond \mu) \vee (\psi_2 \diamond \mu)$.

In fact, the Katsuno-Mendelzon's update postulates characterize the update semantics for a class of update operators that are based on the principle of minimal change. For instance, the PMA update operator $\diamond_{pma}$ satisfies all postulates (U1) – (U8).

In order to investigate the relationship between the MCD and Katsuno-Mendelzon's update postulates, we first introduce the following useful notations and concepts. Let $\mathcal{I}$ be the set of all interpretations of the propositional language $\mathcal{L}$. A *preordering* $\leq$ over $\mathcal{I}$ is reflexive and transitive relation on $\mathcal{I}$. $\leq$ is called a *partial ordering* if it is an antisymmetric preordering. Let $\mathcal{M} \subseteq \mathcal{I}$. We denote $min(\mathcal{M}, \leq)$ to be the set of all interpretations in $\mathcal{M}$ that are minimal with respect to $\leq$.

Definition 7 *Let $\mathcal{I}$ be the set of all interpretations of $\mathcal{L}$, $\mathcal{I}_1 \subseteq \mathcal{I}$, and S an interpretation of $\mathcal{L}$. We define an ordering over $\mathcal{I}$ with respect to $\mathcal{I}_1$ and S as follows. For any two $S_1, S_2 \in \mathcal{I}$, $S_1 \leq_{\mathcal{I}_1;S} S_2$ iff $S_1 = S_2$; or*
(i) For all $S_i \in \mathcal{I}_1$, $Diff(S, S_i) \subseteq Diff(S, S_1)$, and
(ii) If for all $S_i \in \mathcal{I}_1$, $Diff(S, S_i) \subseteq Diff(S, S_2)$, then $Diff(S, S_1) \subseteq Diff(S, S_2)$.

Lemma 1 *$\leq_{\mathcal{I}_1;S}$ is a partial ordering.*

Intuitively, $\leq_{\mathcal{I}_1;S}$ represents a measure on $Diff(S, S')$ for any state S' in $\mathcal{I}$ with respect to $\mathcal{I}_1$. In particular, if $S_1 \leq_{\mathcal{I}_1;S} S_2$, it means that (i) $Diff(S, S_i)$ is an upper bond for the set $\{Diff(S, S_i) \mid S_i \in \mathcal{I}_1\}$ with respect to set inclusion $\subseteq$, while $Diff(S, S_2)$ is not an upper bond for this set; *or* (ii) both $Diff(S, S_1)$ and $Diff(S, S_2)$ are upper bonds for set $\{Diff(S, S_i) \mid S_i \in \mathcal{I}_1\}$, but $Diff(S, S_1)$ is a smaller one than $Diff(S, S_2)$ is (i.e. $Diff(S, S_1) \subseteq Diff(S, S_2)$). We can extend the ordering $\leq_{\mathcal{I}_1;S}$ to the general case.

Definition 8 *Let Π be a class of sets of interpretations. We define $S_1 \leq_{\Pi;S} S_2$ iff there exists some $\mathcal{I}_k \in \Pi$ such that $S_1 \leq_{\mathcal{I}_k;S} S_2$ or there exists some S' such that $S_1 \leq_{\Pi;S} S' \leq_{\Pi;S} S_2$.*

Clearly, the ordering $\leq_{\Pi;S}$ is the union of $\leq_{\mathcal{I}_k;S}$ for every $\mathcal{I}_k$ in Π. For instance, if $\Pi = \{\mathcal{I}_1, \mathcal{I}_2\}$, then $S_1 \leq_{\Pi;S} S_2$ if one of the following cases holds: (i) $S_1 \leq_{\mathcal{I}_1;S} S_2$; (ii) $S_1 \leq_{\mathcal{I}_2;S} S_2$; or (iii) there is S' such that $S_1 \leq_{\mathcal{I}_1;S} S'$ (or $S_1 \leq_{\mathcal{I}_2;S} S'$) and $S' \leq_{\mathcal{I}_2;S} S_2$ (or $S' \leq_{\mathcal{I}_1;S} S_2$).

Lemma 2 *$\leq_{\Pi;S}$ is a partial ordering.*

Theorem 3 *Let ψ be a propositional knowledge base, where C is the state constraint, and μ a propositional formula. Then*

$$Models(\psi \diamond_{mcd} \mu) = \bigcup_{S \in Models(\psi)} min(Models(C \wedge \mu), \leq_{2^{Res(S,\mu)}; S}).$$

The above theorem shows that our MCD (and the same as the MCE) can be characterized by an alternative minimal change criterion. Therefore, it is not surprising that we have the following solution.

Theorem 4 *The MCD update operator $\diamond_{mcd}$ satisfies Katsuno-Mendelzon's postulates (U1) – (U8).*

Theorem 4 reveals an important fact that Katsuno-Mendelzon's postulates (U1) – (U8) are not necessarily contradictious with the semantics of updating knowledge base with disjunctive information.

Concluding Remarks

In this paper, we proposed two different approaches, called the MCE and MCD respectively, to deal with the problem of updating knowledge bases with disjunctive information.

In fact, the problem of dealing with the incomplete (disjunctive) information in dynamic systems has been studied by many researchers recently (eg. (Brewka & Hertzberg 1993; Kartha & Lifschitz 1994; Lin 1996; Zhang & Foo 1995)). However, most of their work concentrated on reasoning about action. Restricted by the formalizations, their methods, therefore, seemed not directly applicable for general knowledge base updates as we discussed in this paper. On the other hand, some of their approaches only dealt with the direct disjunctive effect (Brewka & Hertzberg 1993). We did extend our previous work to deal with knowledge base updates (Zhang & Foo 1995). However, as our previous method is syntax-based, there is no proper semantics to support the theory.

There are some features of our approaches represented in this paper. First, our approaches can deal with both direct and indirect disjunctive effects of updates. For instance, in the dropping-box problem (i.e. Example 1), the update effect $\mu \equiv Ontable(Box)$ is definite. Together with the constraint (2), however, it implies an indirect disjunctive effect $Inwhite(Box) \vee Inblack(Box)$. In this case, our approaches produce the desired result. Second, due to the equivalence between the MCE and MCD for propositional knowledge base updates, the MCD provides a proper semantics for the MCE. Moreover, the MCD is also suitable for first order knowledge base updates. Third, as the MCD and MCE satisfy Katsuno and Mendelzon's postulates (U1) - (U8), we can see that in general, the principle of minimal change can be independent from the update with disjunctive information.

Many related problems remain open. One is the computational tractability of our approaches. Another is the extension of our ideas to solve the problem of reasoning about actions with nondeterministic effects. An obvious difference between update and action is that the change caused by an action will not obey KM update postulate (U2) if the action implies a nondeterministic effect. For instance, tossing a coin. In this case, our current approaches can not derive desired solutions. A third is the connection of the principles of minimal change with exceptions and minimal change with maximal disjunctive inclusions to other nonmonotonic mechanisms such as default logic and circumscription. These issues are being addressed in our current work.

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