Situation Calculus on A Dense Flow of Time

Akira Fusaoka

Department of Computer Science Ritsumeikan University
1916,Nojicho,Kusatsu-city,SIGA,525,JAPAN
fusaoka@cs.ritsumei.ac.jp

Abstract

In this paper, we attempt to reconstruct the situation calculus on a dense flow of time. The proposed system: ISC, which is formulated in the framework of S2S (the monoadic second-order theory of two successor functions), allows to deal with temporal properties of time duration such as the continuity of fluents. Also it incorporates an intensional feature into the situation calculus so that the inferential process itself can be represented in ISC. On the basis of this modification, we define a nonmonotonic schema called epistemological minimization which selects the preferable model with respect to the information order in the inferential process. This method of nonmonotonic reasoning is useful for a temporal explanation problem because a sequence of events is interpreted sometimes depending on the information order in the inferential process rather than the chronological order of the actual process.

INTRODUCTION

Situation calculus is well known as a standard framework in the formalization of the temporal reasoning, and especially, it has been providing a useful basis for the study of the frame problem since its initial formulation. However, the situation calculus has several limitations due to its simplicity [Lin & Shoham 1992]. In this paper, we forcus on a serious limitation among them. That is the inability to represent the time duration in the standard situation calculus so that the situation at the next time must be determined immediately by the current action. We try to overcome this limitation by introducing a dense time flow. In general, the flow of time is said to be dense if we can always find a new time instant between any two time points. This means that in every stage of temporal thinking we must presuppose the possibility that unexpected events may occur at any time and change the situation. Therefore, any interpretation and belief about temporal phenomenon is necessarily tentative because it is perfectly reasonable that the current belief will be changed via the discovery of an unknown event.

This uncertainty in the temporal reasoning on the dense time structure leads to about another problem into the standard situation calculus. That is the inability to reason directly about the inferential process itself because the situation calculus is not an intensional system. However, as shown in the following example, it is necessary for the formulation of temporal inference to allow the treatments of two time: the time in the domain of problems and the time in the mind of the reasoning agent herself, if we take the possibility of an unexpected event at any time in consideration.

In this paper, we reconstruct the situation calculus on the temporal ontology of the time interval. For the logical simplicity, we formulate this reconstruction on the classical S2S (the monoadic second-order theory of two successor functions [Rabin 1969] rather than the first-order logic and introduce a formal system called **ISC** : an intensional situation calculus.

The following example shows the reason why the intensional feature of **ISC** is required in the temporal reasoning.

Example1. A derivative of YSP

Let us assume the following events.
Event E1: It is observed that Teddy is alive and the gun is unloaded at a time $t0$.
Event E2: Teddy is shot at time t1 ($t0 \leq t1$).
Event E3: Teddy is observed to be dead at time t2 ($t1 \leq t2$).

The question is when Teddy is dead.

This problem is a derivative of the well known YSP(Yale Shooting Problem)[Hanks & McDermott 1987], but the answer depends on the order in which these events are informed.

Assume that these events are informed in one of the following sequences.
Case1. A strict sequence:
we observe E1 first, then E2 secondly and E3.
Case2. A sequence in the backward order:
we observe E3 first. After that we are informed E2 and informed E1 finally.
Case3. A random order:

We observe E1 first and then E3 secondly. After that we are informed E2 finally.

In the case1, we hold the belief after the first observation that Teddy is alive and the gun is unloaded, so that we infer the shooting at the time t1 does not change the state of Teddy. Consequently, Teddy is believed to be alive immediately before the event E3 which informs that he is dead. Then, what time structure do we build in the mind to explain this series of events consistently after we find the event E3? The most reasonable time structure, the solution1, is formed on the assumption that Teddy was killed in certain unexpected incident between t1 and t2. Another interpretation, the solution2, is an explanation on the assumption that the gun had been loaded by an unnoticed event between t0 and t1 so that the shooting at t1 caused Teddy to be dead. This time structure is possible but seems to be unnatural in this case.

On the contrary, we prefer the solution2 to the solution1 for the case2. In this case, we initially believe from the first obsevation of E3 that Teddy was dead from t1 to t2. Because we have no knowledge about the state of the gun in this stage of inference, we have two preferable solutions. The first is the solution2, in which Teddy was dead as the result of the shooting. The second is the solution3 in which Teddy had been dead already before the shooting. However, the solution1 is unnatural in this case.

For the case3, all of the solutions give the comparably preferable interpretation.

This difference in the preference of the solutions is based on the simple reason that we usually avoid the annoying update of the already constructed belief in the inferential process.

This example reveals that the temporal structure and its interpretation for a sequence of events depends not only on the temporal order but also the epistemological order in which each event is recognized. However, this example brings about yet another frame problem because some difficulties arise in attempts to deal with this example in the usual nonmonotonic logics, even in the methods proposed to solve YSP [Hanks & McDermott 1987].

Intensional Situation Calculus : ISC

2.1 Time Structure

ISC is based upon a constructive time structure, which is built from an interval by iterating the interval division everytime new event is treated in the process of temporal inference. So that the time structure is represented with a binary tree of which node corresponds to an interval and the left and the right successors of the node correspond to the intervals before and after the occurrence of the event in that interval, respectively.

Definition 1. Temporal structure

Let U be the set of all finite sequences of 0's and 1's. Namely, $U = \{0,1\}^*$. U can be diagramed as a binary tree. We denote the parent node of x by $\circ x$, the successor nodes of x by $x0, x1$, and the length of x by $l(x)$.

We introduce two order relations in U.

(i) an epistemological order : Let a, b denote the elements of U. Define $a \sqsubseteq b$ iff a is an initial segment of b.

(ii) a temporal order : Let a, b denote the elements of U. A temporal order $a \preceq b$ is defined by $a \preceq b$ iff
$$[\exists z][z0 \sqsubseteq a \ \wedge \ z1 \sqsubseteq b]$$

A subset $V : V \subseteq U$ is called a subtree if and only if
$$(\forall x)[x \in V \rightarrow \circ x \in V] \wedge [x0 \in V \ iff \ x1 \in V]$$

A maximal linearly ordered subset of the tree U is called a branch of U. Since a branch is a decreasing sequence of intervals, it has a limitting point in $\{0, 1\}^\omega$, which corresponds to the time instant.

Definition 2. S2S

S2S (second-order theory of two successors) has the usual logical connectives and quantifiers for the variables $x, y, \ldots$ ranging over elements of $\{0, 1\}^*$. It contains also the set variables $X, Y, \ldots$ and their quantifier ranging over the subsets of $\{0, 1\}^*$ and the membership relation $\in$. Terms are obtained from the individual variables and a constant ϵ by application of the successor functions. Atomic formulas are of the form $ter1 = ter2, ter1 \sqsubseteq ter2, ter1 \in X$ where $ter1, ter2$ are terms and X is a set variable. The S2S-formula is generated from atomic formulas by the logical connectives and quantifiers ranging over both the individual and the set variables.

The most important feature of the S2S is decidability. Namely,

Rabin's Tree Theorem [Rabin 1969]

The (monoadic) second order theory of infinite binary tree is decidable.

The S2S theory has been widely used to study the dynamic logic and the temporal logic. Especially, the satisfiability of some tree based modal logics has been shown to be decidable based on Rabin's Tree Theorem [Thomas 1990]. Conversely, ISL can be formulated in the modal logic.

Definition 3. Temporal topology

Let x be an element of U. We define the subsets $\langle x$ and $x\rangle$ of U by
$$\langle x = \{z10^p \mid p \geq 0\}$$
$$\quad\quad \text{if } x = z10^q \text{ for some } z \in \{0,1\}^* \text{ and } q \geq 0$$
$$= \{0^p \mid p \geq 0\} \text{ elsewhere}$$

$$x\rangle = \{z01^p \mid p \geq 0\}$$
$$\quad\quad \text{if } x = z01^q \text{ for some } z \in \{0,1\}^* \text{ and } q \geq 0$$
$$= \{1^p \mid p \geq 0\} \text{ elsewhere}$$

$$[x] = \{y \mid x \sqsubseteq y\}$$

Note that these sets are definable by the S2S-formulas.

As known well, we can introduce a topology into the tree U by the definition that $[x]$ is a neighbor of x. A open set is also defined by the S2S formula. Namely, a set Q is open if and only if $(\forall x)[x \in Q \to [x] \subseteq Q]$

2.2 ISC

We present a formal treatment of **ISC** by using S2S, which is a reconstruction of the situation calculus on the interval ontology. First, we transform the elementaly objects of the situation calculus such as fluent, action and situation, into the subsets of U. Namely, a fluent p is represented by the set $P \subseteq U$ such that $(\forall x)[x \in P \to$ a fluent p holds in the interval $x]$ Note that $U - P$ also treated as a not-P fluent. Similarly, we use the set A instead of an action a where $(\forall x)[x \in A \to$ an action a is performed in the interval $x]$ A situation is simply corresponding to the set $\langle x$ or $x \rangle$.

Secondly we define a predicate $Hold$, which is also defined by;

$$Hold(P, \langle x) = (\forall y)[y \in \langle x \to (\exists z)[y \sqsubseteq z \wedge [z] \subseteq P]$$

$$Hold(P, x\rangle) = (\forall y)[y \in x\rangle \to (\exists z)[x \sqsubseteq z \wedge [z] \subseteq P]$$

The predicate $Hold(P, \langle x)$ means that the fluent P holds not only at the situation $\langle x$ but also in the neighbor of this situation.

The following property of $Hold$ is immediately follows from the above definition.

$$Hold(P, \langle x) \wedge Hold(Q, \langle x) \equiv Hold(P \cap Q, \langle x)$$

$$Hold(P, x\rangle) \wedge Hold(Q, x\rangle) \equiv Hold(P \cap Q, x\rangle)$$

Another set $Clip(P)$ is used in **ISC**, that is a set of the intervals in which the truth value of the fluent P changes from $true$ to $false$. $Clip(P)$ is defined by the the following S2S-formula.
$$Clip(P) = \{x \mid (\forall y)[y \in x0\rangle \to (\exists z)(z \in P \wedge y \sqsubseteq z)]\}$$
$$\wedge (\forall y)[y \in \langle x1 \to (\exists z)(z \in U - P \wedge y \sqsubseteq z)]$$
This definition means that $x \in Clip(P)$ if and only if P contains the infinite intervals in $x0\rangle$ and $U - P$ contains the infinite intervals in $\langle x1$. Note that $Clip(P)$ is defined as a close set. Therefore, we have
$$x \notin Clip(P) \to Hold(P, x0\rangle) \equiv Hold(P, \langle x1)$$
On the contrary,
$$x \in Clip(P) \to Hold(P, x0\rangle) \wedge Hold(U - P, \langle x1)$$
is not valid unless P is continuous.

Definition 4. ISC-system
Let $\Lambda = (P_1, P_2, ..., P_n, A_1, A_2, ..., A_m)$ be a set of the S2S formulas with the set-constants $P_1, P_2, ..., P_n, A_1, A_2, ..., A_m$. Λ is called **ISC**-system if and only if $A_i \cap A_j = \phi$ if $i \neq j$. This condition excludes the concurrent action.

For simplicity, we abbreviate $P_1, P_2, ..., P_n$ to $\tilde{P}$. Similarly, $\tilde{A}$ denotes $A_1, A_2, ..., A_m$ and $\tilde{CP}$ denotes $CP(P_1), ..., CP(P_n), CP(U - P_1), ..., CP(U - P_n)$.

A subtree $V \subseteq U$ is called a temporal domain of ISC-system if
$$V = \{\epsilon\} \cup \{x0, x1 \mid x \in A_i \text{ of } \tilde{A}\}$$
$$\cup \{x0, x1 \mid x \in CP_i \text{ of } \tilde{CP}\}$$
Note that an event (either a clipping of a fluent or an action) occurs only in the element of the subset V, that is,
$$\forall x \in U(x \notin V \to [x] \in P \vee [x] \in U - P)$$
for each fluent P.

Definition 5. Continuity of fluents
A fluent $P \subseteq U$ is said to be continuous if and only if every branch $B \subseteq U$ satisfies the following condition:
$$(\exists x)\{x \in B \to (\forall y)[x \sqsubseteq y \wedge y \in B \to y \in P]$$
$$\vee (\forall y)[x \sqsubseteq y \wedge y \in B \to y \in U - P]$$
Namely, P is continuous unless both P and $U - P$ contains the infinite elements of a branch, so that P is continuous if and only if $U - P$ is continuous.

A ISC-system $\Lambda = (\tilde{P}, \tilde{A})$ is called continuous if all fluents of Λ are continuous.

Theorem1. Totalness of the Hold
Let P be a continuous fluent. For, every $x \in U$,

$$Hold(P, \langle x) \vee Hold(U - P, \langle x)$$

$$Hold(P, x\rangle) \vee Hold(U - P, x\rangle)$$

Theorem2. Persistence of a fluent
We assume that a fluent P is continuous in the following.

(1) the persistence in the epistemological order:

   (a) $Hold(P, \langle x)$ *if and only if* $Hold(P, \langle x0)$
   (b) $Hold(P, x\rangle)$ *if and only if* $Hold(P, x1\rangle)$
   (c) $Hold(P, \langle x) \wedge (\forall y)[y \in U \wedge x \sqsubseteq y \to y \notin Clip(P)] \to Hold(P, x\rangle)$
   (d) $Hold(P, x\rangle) \wedge (\forall y)[y \in U \wedge x \sqsubseteq y \to y \notin Clip(P)] \to Hold(P, \langle x)$

(2) the persistence in the temporal order:
   Let $x, y \in U$ such that $x \preceq y$.

   (a) $Hold(P, \langle x) \wedge (\forall z)[z \in U \wedge x \preceq z \wedge z \preceq y \to z \notin Clip(P)] \to Hold(P, y\rangle)$
   (b) $Hold(P, y\rangle) \wedge (\forall z)[z \in U \wedge x \preceq z \wedge z \preceq y \to z \notin Clip(P)] \to Hold(P, \langle x)$

The proofs for the above theorems are trivial. However, notice that the continuity is an essential requirement for the persistence of a fluent. For example, assume that $\Lambda = (P, \{0, 1\}^*)$ is ISC-system, where $P = \{(10)^p(0)^q \mid p \geq 0, q \geq 0\}$. The fluent P is discontinuous because both P and $U - P$ contain the infinite elements of a branch $\{1, 10, 101, 1010, 10101, ...\}$. Clearly, we have $Hold(P, \langle 0)$ and $Hold(U - P, 1\rangle)$. But $Clip(P)$ is empty from the construction. This example reveals that for a discontinuous fluent we can not always specify the time instant (the division point of an interval) where a

fluent changes its value although the value apparently changes in the interval.

As is discussed later, the continuity condition is also necessary for the nonmonotonic reasoning in ISC-system to be decidable theory, but it restricts the expressiveness of **ISC**. For example, a super task with infinite actions such as Zeno's paradox can be described in **ISC**. For example, Thomson' lamp is given by the following causal rules;

$$x \in SW \rightarrow [Hold(On, x0)) \equiv Hold(U - On, \langle x1 \rangle]$$
$$\wedge [Hold(U - On, x0)) \equiv Hold(On, \langle x1 \rangle] \wedge x1 \in SW$$

where SW denotes SWithing which always changes the status of the lamp from on to off or off to on. However, by the following theorem, Thomson' lamp can not be continuous.

Theorem3. Finiteness of the Clip

Assume that a fluent P is continuous. Then $Clip(P)$ is finite.
Proof: For each branch B in V, $Clip(P)$ contains only finite elements of B. Therefore, we have a subtree V^+ in which all branchs are finite and $Clip(P) = Clip(P) \cap V^+$. From the campactness of tree, which is known as the fan theorem, V^+ itself is also finite. So that $Clip(P)$ is finite.

Example2. The derivative of YSP formulated in ISC

In the ISC-system, the example1 is described as follows.

(i)Rules of actions
$(\forall x)\{x \in SHOOT \rightarrow [Hold(Alive, x0))$
$\wedge Hold(Loaded, x0)) \rightarrow Hold(V - Alive, \langle x1 \rangle]\}$
$(\forall x)\{x \in LOAD \rightarrow [Hold(Loaded, \langle x1 \rangle]\}$

(ii)Rules of observation
$(\forall x)\{x \in OBL(P) \rightarrow [Hold(P, \langle x1 \rangle]\}$
$(\forall x)\{x \in OBR(P) \rightarrow [Hold(P, x0))]\}$

(iii) Spesification of events
Case1:
$OBL(Alive) = \{\epsilon\}, OBL(V - Loaded) = \{\epsilon\},$
$OBR(V - Alive) = \{11\}, SHOOT = \{1\}$
Case2:
$OBR(V - Alive) = \{\epsilon\}, OBL(Alive) = \{00\},$
$OBL(V - Loaded) = \{00\}, SHOOT = \{0\}$
Case3:
$OBL(Alive) = \{\epsilon\}, OBL(V - Loaded) = \{\epsilon\},$
$OBR(V - Alive) = \{1\}, SHOOT = \{10\}$

(iv) Solutions
Solution1 for Case1:
$V = \{\epsilon, 0, 1, 10, 11, 110, 111, 1100, 1101\},$
$Clip(Alive) = \{110\},$
Solution2 for Case1:
$V = \{\epsilon, 0, 1, 10, 11, 100, 101, 110, 111\},$
$Clip(V - Loaded) = \{10\}, Clip(Alive) = \{1\}$
Solution1 for Case2:
$V = \{\epsilon, 0, 1, 00, 01, 000, 001, 010, 011\},$
$Clip(Alive) = \{01\}$

Solution2 for Case2:
$V = \{\epsilon, 0, 1, 00, 01, 000, 001, 0010, 0011\},$
$Clip(V - Loaded) = \{001\}, Clip(Alive) = \{0\}$
Solution3 for Case2:
$V = \{\epsilon, 0, 1, 00, 01, 000, 001, 0010, 0011\},$
$Clip(Alive) = \{001\}$
Solution1 for Case3:
$V = \{\epsilon, 0, 1, 10, 11, 100, 101, 1010, 1011\},$
$Clip(Alive) = \{101\}$
Solution2 for Case3:
$V = \{\epsilon, 0, 1, 10, 11, 100, 101, 1000, 1001\},$
$Clip(V - Loaded) = \{100\}, Clip(Alive) = \{10\}$
Solution3 for Case3:
$V = \{\epsilon, 0, 1, 10, 11, 100, 101, 1000, 1001\},$
$Clip(Alive) = \{100\}$

NON-MONOTONICITY

In this paper, we focus on two aspects of the nonmonotonic temporal reasoning : the persistence and the frame problem (in the narrow sense) in the framework of **ISC**. The persistence means the principle of reasoning that any unnecessary clipping of fluents can be neglected in each stage of the inference. The frame problem is related to the stipulation that we can omit the specification about the behavior of the action in the case of no effect. Although the frame problem is often treated based on the persistence, the invariance of fluent irrelevant to the action is stronger requirment than the persistence of fluent on time flow. We present a new method of nonmonotonic reasoning, which selects the preferred model in two steps : first step, the selection related to the frame axiom and secondly, the slection related to the persistence. In the following, we present the description of this two-step minimization, together with the formulation of the other methods of nonmonotonic reasoning in the framework of S2S.

3.1 S2S formalization of existing theories

In general, the minimal model is defined through a preferential relation between two models. In the ISC-system, however, it can be specified by the circumscriptive formula [McCarthy 1980] called a C-form within the **ISC**.

Let $\Lambda = (\tilde{P}, \tilde{A})$ be a continuous ISC-system. Assume that $\Phi(V, \tilde{P}, \tilde{A}, K)$ contains the description of a problem; that is, Φ consists of a logical conjunction of the domain independent axioms of Λ, the domain axioms (causal rules) and the description of the occurrence of events, where K represents a set to be minimized.
The C-form:
$$\exists(V, \tilde{P}, \tilde{A}, K)\{\Phi(V, \tilde{P}, \tilde{A}, K) \rightarrow$$
$$\forall(V', \tilde{P}', \tilde{A}', K')[\Phi(V', \tilde{P}', \tilde{A}', K') \rightarrow$$
$$PC(V, V', K, K')]\}$$
gives the minimal model with respect to the preferential condition $PC(V, V', K, K')$, if both $\Phi(V, \tilde{P}, \tilde{A})$ and $PC(V, V', K, K')$ are consistent S2S-foumulas. Also it

is decidable weather Φ has the minimal model from Rabin's Theorem.

The domain minimization

The first approach to the nonmonotonic reasoning in **ISC** is to construct a preferable model by removing the unnecessary intervals from the temporal structure. This is attained by circumscribing the temporal domain. The minimal model with respect to the domain V is given by the C-form with the following preferential condition;
$$PC_{domain}(V, V') \equiv (V' \subseteq V \to V \subseteq V')$$
Since this is a consistent S2S-formula, there exists the minimal model in the meaning that V is minimal with respect to the set inclusion. However, it is as weak as the standard nonmonotonic logics. For example, it is impossible to give a correct answer to the YSP.

The causal minimization

Haugh [Haugh 1987] presented one of the first solutions to YSP based upon the notion of the causal minimization, preferring the model which contains fewer potential causes. The potential cause is a set of belief about the possibility of clipping of a fluent, which is defined by the following S2S-formula.
$$RC = \{x \mid (\exists P)[x \in Clip(P)] \vee \\ (\exists A_i)[x \in A_i \wedge (\forall y)[y \in A_i \to \\ (\exists P)(\exists Q)[Hold(Q, y)) \to y \in Clip(P)]]]\}$$
Namely, RC is a set of intervals in which some fluent P is actually clipped or the action effects the clipping P if every precondition holds. The causal minimization is incorporated in **ISC** by the C-form with the following preferential condition;
$$PC_{cause}(V, V', RC, RC') \equiv (RC' \subseteq RC \to RC \subseteq RC') \wedge PC_{dmain}(V, V')$$
Although the causal minimization gives the correct answer for YSP, it doesn't work for the derivative of the YSP in the example1 because the set of potential causes is minimal for the following two solutions in case1;
Solution1:
$$V = \{\epsilon, 0, 1, 10, 11, 110, 111, 1100, 1101\},$$
$$RC = \{1, 110\}$$
Solution2:
$$V = \{\epsilon, 0, 1, 10, 11, 100, 101, 110, 111\}, RC = \{1, 11\}$$

The chronological minimization

The chronological minimization [Kautz 1986] is also successful approach to YSP which yields a particular preference to the model in which the clippings of fluents are delayed as long as possible. This preference condition can be represented by the S2S-formula such that;
$$\tilde{CP} \ll \tilde{CP}' \text{ if and only if}$$
$$\tilde{CP} \subseteq \tilde{CP}' \vee (\forall x)\{x \in \tilde{CP}' \wedge x \notin \tilde{CP} \to \\ (\exists z)[z \preceq x \wedge z \in \tilde{CP} \wedge z \notin \tilde{CP}']\}$$
The chronological minimization in **ISC** is described by using the preference condition.

$$PC_{chronology}(V, V', \tilde{CP}, \tilde{CP}') \equiv (\tilde{CP} \ll \tilde{CP}') \wedge \\ PC_{domain}(V, V')$$
Similar to the causal mimization, the chronological minimization is a S2S-definable and decidable theory but it also does not work for the example1.

3.2 Epistemological Minimization

As we discussed in the beginning of this section, the two level selection of the model is necessary for the nonmonotonic reasoning in **ISC**-system. First, the selection of the time structure, which is as faithful as possible to the frame axiom, is attained by using the causal minimization.
Let
$$RF(R) = \{x \mid (x \in Clip(R) \wedge x \in \tilde{A}) \vee \\ (\exists A_i)[x \in A_i \wedge (\forall y)(y \in A_i \to (\exists Q)[Hold(Q, y)) \to y \in \\ Clip(R)]]\}$$
where $R = P_1, ..., P_n, V - P_1, ..., V - P_n$.
The set RF represents the potential cause similarly to the causal minimization. However, RF is different from the RC at the point that $x \in A_i$ whenever $x \in RF(P)$. We use a relation $RF \ll RF'$ to abbreviate the following series of set inclusions
$$RF(P_1) \subseteq RF(P_1') \wedge ... \wedge RF(P_n) \subseteq RF(P_n') \wedge ... \wedge \\ RF(V - P_n) \subseteq RF(V' - P_n').$$
The preference condition PC_{frame} is defined by
$$PC_{frame} \equiv RF \ll RF'.$$
This minimization of RF precludes the time structure which contains the action with the unexpected occurrence of clipping.

Secondly, the selection of the model which satisfies the principle of the persistence, is attained by the devising of the new minimization schema called an epistemological minimization, in which the clipping of a fluent is postponed as long as possible with respect to the epistemological order rather than the chronological order.

The preference condition for the epistemological minimization is given by the following formula;
$$PC_{persistence}(V, V', \tilde{RE}, \tilde{RE}') \equiv \\ (V' \subseteq V \to V \subseteq V') \wedge (\tilde{RE} \ll \tilde{RE}')$$
where
$$RE = \{x \mid (\exists P)x \in Clip(P) \wedge x \notin A_i \forall A_i \in \tilde{A}\}$$
Note that RE counts the intervals without action but some fluent P is clipped. The relation of preferential condition is defined as follows.
$$\tilde{RE} \ll \tilde{RE}' \text{ if and only if}$$
$$\tilde{RE} \subseteq \tilde{RE}' \vee (\forall x)\{x \in \tilde{RE}' \wedge x \notin \tilde{RE} \to (\exists z)[l(x) < \\ l(z) \wedge z \in \tilde{RE} \wedge z \notin \tilde{RE}']\}$$
The epistemological minimization is given in the C-form by using this preference condition. However, this is not a S2S-formula because the inequi-level predicate $l(x) < l(z)$ is not S2S-definable. The S2S-formula augmented with the inequi-level predicate is known to be undecidable. However, the epistemological minimization theory is decidable because RE is not only finite but also bounded if all fluents are continuous.

The nonmonotonic reasoning in **ISC** is accomplished by using these prefernce conditions in two steps : first PC_{frame} secondly $PC_{persistence}$. Namely, if $\Phi(V, \tilde{P}, \tilde{A}), \tilde{RF}, \tilde{RE}$, is consistent then there exists the minimal model which satisfies the C-form.
$$\exists(V, \tilde{P}, \tilde{A}, \tilde{RF}, \tilde{RE})\{\Phi(V, \tilde{P}, \tilde{A}, \tilde{CP}) \to$$
$$\forall(V', \tilde{P}', \tilde{A}', \tilde{RF}', \tilde{RE}')[\Phi(V', \tilde{P}', \tilde{A}', \tilde{RF}', \tilde{RE}')$$
$$\to PC_{frame} \to PC_{persistence}]\}$$

Example3. The derivative of YSP: epistemological minimization

We have the following $\tilde{RF}, \tilde{RE}$ set from the description in the example2.
Case1;solution1
$Clip(Alive) = \{110\}, \tilde{RF} = \{1\}, \tilde{RE} = \{110\}$,
Case1;solution2
$Clip(V - Loaded) = \{10\}, Clip(Alive) = \{1\}$,
$\tilde{RF} = \{1\}, \tilde{RE} = \{10\}$,
Case2;solution1
$Clip(Alive) = \{01\}, \tilde{RF} = \{0\}, \tilde{RE} = \{01\}$,
Case2;solution2
$Clip(V - Loaded) = \{001\}, Clip(Alive) = \{0\}$,
$\tilde{RF} = \{0\}, \tilde{RE} = \{001\}$,
Case2;solution3
$Clip(Alive) = \{001\}, \tilde{RF} = \{0\}, \tilde{RE} = \{001\}$,
Case3;solution1
$Clip(Alive) = \{101\}, \tilde{RF} = \{10\}, \tilde{RE} = \{101\}$,
Case3;solution2
$Clip(V - Loaded) = \{100\}, Clip(Alive) = \{10\}$,
$\tilde{RF} = \{10\}, \tilde{RE} = \{100\}$,
Case3;solution3
$Clip(Alive) = \{100\}, \tilde{RF} = \{10\}, \tilde{RE} = \{100\}$
According to the epistemological minimization, we select the solution1 for the case1, either the solution2 or the solution3 for the case2, and one of all solutions for case3.

Concluding Remarks

We extend the situation calculus on a dense flow of time and give a new schema for nonmonotonic temporal reasoning in the modified situation calculus. The epistemological minimization proposed in this paper satisfies the criterion of the epistemologically complete theory [Lin & Shoham 1992]. Also it satisfies the AGM postulates in the belief revision theory[Alchourrón, Gärdenfors & Makinson 1985].

An extension to the continuous time is necessary to deal with the real-valued fluent. Sandewall [Sandewall 1989] studied extensively the reasoning about action on the continuous time by explicitly introducing the real-valued time parameter. However, in the situation calculus, time is a constructive object built from the action and fluent, so that the different (intuitionistic) definition of the continuous time is required. For example, we can extend **ISC** by adding the following postulate based on the intermediate value theorem.

For every real-valued fluent P,
$$\forall x[P(\langle x) = a \wedge P(\rangle x) = b \to$$
$$\exists y(x \sqsubseteq y \wedge P(\langle y) = P(\rangle y) = \tfrac{a+b}{2})].$$

References

Alchourrón,C.E.; Gärdenfors, P.; Makinson, D. 1985. On the logic of theory change: partial meet functions for contraction and revision. Journal of Symbolic Logic, 50:510-530.

Baker,A.B. 1991. Nonmonotonic reasoning in the framework of situation calculus.*Artificial Intelligence.* vol 49:5-23.

Benthem,J.V. 1991. *The Logic of Time : A model-theoretic investigation into the varieties of temporal ontology and temporal discourse.* Kluwer Academic Publishers.

Friedman,N.; Halpern,J.Y. 1994. A Knowledge-Based Framework for Belief Change Part I: Foundation. In Proceedings 5th Conf. on Theoretical Aspects of Reasoning About Knowledge.

Hanks,S.; McDermott,D. 1987. Nonmonotonic Logic and Temporal Projection. *Artificial Intelligence.* 33:379-412.

Haugh,B.A. 1987. Simple causal minimization for temporal persistence and projection. In Proceedings 5th National Conf. on Artificial Intelligence.

Isozaki,H.; Shoham,Y, 1992. A Mechanism for Reasoning about Time and Belief. In Proceedings International Conf. on Fifth Generation Computer System.

Kautz,H.A. 1986. The logic of Persistence. In Proceedings 4th National Conf. on Artificial Intelligence.

Lin,F.;Shoham,Y. Shoham,Y. 1992. Concurrent Actions in the Situation Calculus. In Proceedings 10th National Conf. on Artificial Intelligence.

McCarthy, J. 1980. Circumscription - a form of nonmonotonic reasoning. *Artificial Intelligence.* 13:27-39.

Rabin,M.O. 1969. Decidability of Second-order Theories and Automata on Infinite Trees. Trans. Amer. Math. Soc. 141, 1969.

Sandewall, E. 1989. Filter Preferential Entailmant for the Logic of Action in Almost Continuous Worlds. In Proceedings of International Joint Conf. on Artificial Intelligence.

Shoham Y. 1988. *Reasonong About Change.* The MIT Press.

Thomas, W. 1990. Automata on Infinite Objects. *Handbook of Theoretical Computer Science,*J.V.Leeuwen eds. volume B, The MIT Press.