A Limitation of the Generalized Vickrey Auction in Electronic Commerce : Robustness against False-name Bids

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Abstract

Electronic Commerce (EC) has rapidly grown with the expansion of the Internet. Among these activities, auctions have recently achieved huge popularity, and have become a promising field for applying agent and Artificial Intelligence technologies. Although the Internet provides an infrastructure for much cheaper auctioning with many more sellers and buyers, we must consider the possibility of a new type of cheating, i.e., an agent tries to get some profit by submitting several bids under fictitious names (false-name bids). Although false-name bids are easier to execute than forming collusion, the vulnerability of auction protocols to false-name bids has not been discussed before.

In this paper, we examine the robustness of the generalized Vickrey auction (G.V.A.) against false-name bids. The G.V.A. has the best theoretical background among various auction mechanisms, i.e., it has proved to be incentive compatible and be able to achieve a Pareto efficient allocation. We show that false-name bids may be effective, i.e., the G.V.A. loses incentive compatibility under the possibility of false-name bids, when the marginal utility of an item increases or goods are complementary. Moreover, we prove that there exists no single-round sealed-bid auction protocol that simultaneously satisfies individual rationality, Pareto efficiency, and incentive compatibility in all cases if agents can submit false-name bids.

Introduction

Electronic commerce (EC) has made rapid progress in recent years. Internet auctions have become especially popular in EC. Commercial auction sites such as eBay (http://www.ebay.com/) and Onsale (http://www.onsale.com/) have been very successful and continue to expand. Computational agents are expected to work on behalf of humans in Internet auctions, e.g., to seek sellers or buyers and to negotiate the prices (Guttman, Moukas, & Maes 1998). Various theoretical and practical studies on Internet auctions have already been conducted. Sandholm pointed out several problems in applying traditional auction protocols to computational agent auctions (Sandholm 1996). Monderer presented a theoretical analysis of an upper bound on the seller's revenue (Monderer & Tennenholtz 1998). AuctionBot (http://auction.eecs.umich.edu/) is a configurable auction server that provides a tool for exploring auction mechanism designs (Wurman, Walsh, & Wellman 1998; Wurman, Wellman, & Walsh 1998). Moreover, auctions with multiple items or multiple units have been studied. Ausubel & Cramton investigated the effect of demand reduction lies on multiple unit auctions (Ausubel & Cramton 1998). Sandholm developed the *eMediator* prototype to support a variety of combinational auctions (Sandholm 1999). Auction techniques have also been applied to various fields, such as air conditioning control in building environments (Huberman & Clearwater 1995).

The Internet provides an excellent infrastructure for executing much cheaper auctions with many more sellers and buyers from all over the world. However, we must consider the possibility of new types of cheating. For example, an agent may try to profit by submitting a false bid under a fictitious name. Such an action is very difficult to detect since identifying each participant on the Internet is virtually impossible. We call a bid under a fictitious name a *false-name bid.*

As far as the authors know, the problem of false-name bids has not been previously addressed. On the other hand, the problems resulting from collusion have been discussed by many researchers (Milgrom 1998; Rasmusen 1989; Sandholm 1996). Compared with collusion, a false-name bid is easier to execute since it can be done alone, while a bidder has to seek out and persuade other bidders to join collusion.

In this paper, we examine the robustness of the Generalized Vickrey Auction (G.V.A.) against false-name bids. The G.V.A. is one instance of the Clarke-Groves mechanism (Mas-Colell, Whinston, & Green 1995; Varian 1995), and it is a generalized version of the well-known, widely advocated Vickrey auction (Vickrey 1961). The G.V.A. has proved to be incentive compatible, namely, the dominant strategies is for a bidder to bid his/her true valuation. In addition, the G.V.A.

achieves a Pareto efficient allocation. These characteristics are advantages of the G.V.A. compared to other auction mechanisms such as the simultaneous multiple round auction used in the FCC auction (McAfee & McMillan 1996; Milgrom 1998), where the free rider problem may cause inefficient allocations.

In this paper, we first introduce some preliminaries, and describe the G.V.A. in detail. Next, we first show simple cases where the G.V.A. is robust against false-name bids, and examine more general settings where the G.V.A. is vulnerable to false-name bids. Furthermore, we prove that there exists no single-round sealed-bid auction protocol that simultaneously satisfies individual rationality, Pareto efficiency, and incentive compatibility in all cases if agents can submit false-name bids.

Preliminaries

Auction protocol properties depend on each agent's utility structure. Auction protocols are divided into three classes according to the value of goods: private value auctions, common value auctions, and correlated value auctions (Rasmusen 1989). In this paper, we concentrate on private value auctions. Although an agent's valuation may be correlated with other agents' valuations, this restriction is reasonable for making a tractable analysis. In private value auctions, each agent knows its own preference, and its valuation is independent of the other agents' valuations. For example, an auction of antiques that will not be resold can be considered to be a private value auction. Furthermore, we assume that the value of goods is equivalent to their monetary value. We define an agent's utility as the difference between the true value of the allocated goods and the payment for the allocated goods. Each agent tries to maximize its own utility.

Different auction protocols have different properties. Although they are evaluated from various viewpoints, we primarily judge auction protocols by whether these protocols fulfill the three properties: incentive compatibility, Pareto efficiency, and individual rationality.

Incentive compatibility: An auction protocol is incentive compatible, if, for each agent, bidding its true private value is the best way to maximize its utility, i.e., lying does not benefit the agent. In computational settings, agents can deal with enormous amounts of data and infer other agents' preferences from the results of their bids. If knowing other agents' preferences is profitable, each agent tends to waste its resources in order to keep its preference secret and obtain the preferences of others. This situation can be avoided if bidding a true valuation becomes the dominant strategy.

Pareto efficiency: A Pareto efficient allocation means that the goods are allocated to bidders whose valuations are the highest, and that the sum of all participants' utilities (including the seller), namely, the social welfare, is maximized. In a more general setting, Pareto efficiency does not necessarily mean maximizing the social welfare. In an auction setting, since agents can transfer money among themselves, the sum of the utilities is always maximized in a Pareto efficient allocation.

Individual rationality: An auction protocol is individually rational if each auction participant does not suffer any loss, in other words, the payment never exceeds the valuation of the obtained goods. If an auction protocol is not individually rational, then some agents do not want to participate in the auction.

We say that auction protocols are robust against false-name bids, if each agent cannot obtain additional profit by submitting a false bid under a fictitious name. If the robustness is not satisfied, the auction mechanism loses incentive compatibility, in other words, participants try to manipulate the auction by submitting false-name bids. This may result in inefficient allocations.

The Generalized Vickrey Auction

Protocol

The generalized Vickrey auction is based on the Clarke-Groves mechanism (Milgrom 1998; Wurman, Walsh, & Wellman 1998). The Clarke-Groves mechanism is a mechanism that induces each agent to tell the true value of public goods (Clarke 1971; Groves & Loeb 1975; Vickrey 1961). The G.V.A. protocol can be applied to various auctions, including auctions for multiple items with interdependent values. Auction protocols that can deal with interdependent value goods are useful for auctions among computational agents (Sandholm 1996).

In addition to its wide applicability, the G.V.A. satisfies individual rationality, Pareto efficiency, and incentive compatibility. Furthermore, the required time for the G.V.A. is shorter than the simultaneous multiple round auction, which requires multiple rounds.

The G.V.A. protocol: Let G denote one possible allocation of goods.

1. Each agent declares a valuation function[1]. Let $v_i(G)$ denote agent i's valuation function for the allocation G.
2. The G.V.A. chooses the optimal allocation G^* that maximizes the sum of all the agents' declared valuations.
3. The G.V.A. announces winners and their payment p_i:

$$p_i = \sum_{j \neq i} v_j(G^*_{\sim i}) - \sum_{j \neq i} v_j(G^*). \qquad (1)$$

[1]The reported valuation function may or may not be the truth.

Here, $G^*_{\sim i}$ is the allocation that maximizes the sum of all agents' valuations except agent i's valuation.

Agent i's utility after the payment is given by the following formula. Let $u_i(G^*)$ denote agent i's true valuation function for G^*.

$$u_i(G^*) - p_i = u_i(G^*) + \sum_{j \neq i} v_j(G^*) - \sum_{j \neq i} v_j(G^*_{\sim i}). \quad (2)$$

The reason why the G.V.A. is incentive compatible is as follows. The third term in formula (2), $(\sum_{j \neq i} v_j(G^*_{\sim i}))$, is independent of agent i's declaration. The optimal allocation G^* is chosen so that the sum of the agents' declared valuations are maximized, i.e.,

$$G^* = \arg\max_G (v_i(G) + \sum_{j \neq i} v_j(G)). \quad (3)$$

Agent i wants to maximize its utility represented as formula (2). Therefore, agent i can maximize its utility by submitting the true valuation, i.e., by setting $v_i(G) = u_i(G)$.

The Vickrey auction (second-price sealed-bid auction) is a well-known auction protocol (Vickrey 1961). The G.V.A. for a single item and a single unit is reduced to the Vickrey auction, where the highest bidder wins and pays the second highest bid.

The G.V.A. for multiple units of a single item, where each agent needs only a single unit, is reduced to the first rejected bid auction ($(M+1)$th-price auction) (Wurman, Wellman, & Walsh 1998). In the first rejected bid auction for M units ($M \geq 1$), winners are the highest bidders from the first to the Mth highest bid, and they pay a uniform price, the $(M+1)$th highest bid.

Example of the G.V.A.

We show how the G.V.A works with a simple example. Suppose that two agents denoted by agent 1 and agent 2 are bidding in the G.V.A. with two different items denoted by g_1 and g_2. An agent's bid is denoted by using a tuple: (a bid for g_1, a bid for g_2, a bid for a set $\{g_1, g_2\}$).

Suppose each agent bids as follows.

- agent 1's bid: (\$20, \$5, \$25)
- agent 2's bid: (\$10, \$15, \$30)

The G.V.A. allocates item g_1 to agent 1 and item g_2 to agent 2, respectively, since the allocation maximizes the sum of all agents' valuations. Agent 1's payment is calculated as follows. When agent 1 does not bid, both g_1 and g_2 are allocated to agent 2, and the valuation is \$30. When g_1 is allocated to agent 1 and g_2 is allocated to agent 2, agent 2's valuation of g_2 is \$15. Therefore, agent 1's payment is calculated as \$30 − \$15 = \$15 and its utility becomes \$20 − \$15 = \$5. Agent 2's payment is calculated as \$25 − \$20 = \$5 and its utility becomes \$15 − \$5 = \$10.

Features of the G.V.A.

Since the G.V.A. is an incentive compatible mechanism, it is robust against the free rider problem (Mas-Colell, Whinston, & Green 1995). In general, the free rider problem is that an agent makes unfair profit without paying the cost. The free rider problem can occur in other protocols, such as the simultaneous multiple round auction (McAfee & McMillan 1996; Milgrom 1998). This auction is designed to assign radio spectrum licenses, and it is currently used by the FCC (http://www.fcc.gov/wtb/auctions/).

The Simultaneous Multiple Round Auction: In the simultaneous multiple round auction, each agent submits one sealed-bid for the combination of items that it wants. Bidding occurs over rounds. The round result is announced before the next round starts. The auction is closed when no agent is willing to bid up from the previous round. The highest bidder for each item gets at the price of his/her bid. The agent has to pay a penalty to withdraw a bid.

We illustrate the free rider problem in the simultaneous multiple round auction. Suppose that agent 1, agent 2, and agent 3 are bidding for two different goods denoted by g_1 and g_2. Agent 1 bids \$5 for g_1 (where the true valuation is \$7), agent 2 bids \$5 for g_2 (where the true valuation is \$7), and agent 3 bids \$11 for a set of g_1 and g_2 (where the true valuation is \$11). After the first round, they learn each other's bid. In the second round, both agent 1 and agent 2 have to make the decision whether to raise the bid or not. Each agent hopes that the other agent raises the bid, so it can get the good without increasing the payment, i.e., it can get a free ride. If neither agent raises the bid (hoping to get a free ride), a Pareto efficient allocation cannot be achieved.

Although the G.V.A. is not widely used, it has the potential to be used in the Internet auctions aided by agents since it is theoretically well-founded as described so far.

Robustness of the G.V.A. in Simple Situations

First, we show the cases where the G.V.A. is robust against false-name bids in simple auction settings.

In an auction of a single item and a single unit, the G.V.A. is reduced to the normal Vickrey auction. It is robust against false-name bids for the following reason. If an agent can win the auction without a false-name bid, submitting a false-name bid only results in increasing its payment. If the agent cannot win the auction without a false-name bid, although the agent may win the auction by submitting a false-name bid, it has to pay more than its true valuation. As a result, submitting a false-name bid does not increase its utility in the Vickrey auction.

In an auction of a single item with multiple units, where each bidder needs only a single item, the G.V.A.

is reduced to the first rejected bid auction. The first rejected bid auction is robust against false-name bids. The reason for this is similar to that in the Vickrey auction.

In the following, we examine the robustness of the G.V.A. in more general settings.

Robustness of the G.V.A. in Single Item, Multiple Unit, Multiple Requirement Auctions

This section discusses the G.V.A. in an auction of a single item with multiple units, where each bidder may desire multiple units. In this situation, the key to deciding whether the G.V.A is robust/vulnerable is the agents' marginal utilities. First, we examine the robustness using some examples.

Example

Example 1 [vulnerable] Suppose that two agents denoted by agent 1 and agent 2 are bidding for a single item with two units.

- agent 1's bid: (\$6, \$6)
  Agent 1 bids \$6 for the first unit and \$6 for the second unit, a total of \$12 for both units.
- agent 2's bid: (\$3, \$5)
  Agent 2 bids \$3 for the first unit and \$5 for the second unit, a total of \$8 for both units.

The G.V.A. allocates the two units to agent 1. Agent 1 pays \$8 and its utility is \$12 – \$8 = \$4.

Now, suppose that instead of bidding (\$6, \$6), agent 1 submits a bid (\$6, \$0), and then submits a false-name bid (\$6, \$0) using the identity of agent 3.

- agent 1's bid: (\$6, \$0)
- agent 2's bid: (\$3, \$5)
- agent 3's bid: (\$6, \$0)

The G.V.A. allocates a single unit to agent 1 and a single unit to agent 3. Agent 1's payment is \$9 – \$6 = \$3 and agent 3's payment is \$9 – \$6 = \$3. It turns out that agent 1 can get both units and its utility is \$12 – \$6 = \$6, since agent 3 is a fictitious name of agent 1.

The difference between agent 1's utility with a false-name bid and the truthful bid is \$6 – \$4 = \$2. Therefore, submitting a false-bid is profitable for agent 1.

Example 2 [robust] Let us assume that two agents denoted by agent 1 and agent 2 are bidding for two units of a single item.

- agent 1's bid: (\$5, \$5)
  agent 1 bids \$5 for the first unit, and \$5 for the second unit, a total of \$10 for both units.
- agent 2's bid: (\$4, \$2)

The G.V.A. allocates the two units to agent 1. The payment is \$6–\$0 = \$6 and the utility is \$10–\$6 = \$4.

In this case, if agent 1 submits a false-name bid using the identity of agent 3, agent 1's utility does not increase. Suppose that agent 1 submits a false-name bid (using the identity of agent 3) by separating its original bid.

- agent 1's bid: (\$5, \$0)
- agent 2's bid: (\$4, \$2)
- agent 3's bid: (\$5, \$0)

The G.V.A. allocates a single unit to agent 1 and a single unit to agent 3, respectively. Agent 1's payment is \$9 – \$5 = \$4 and agent 3's payment is \$9 – \$5 = \$4. As a result, agent 1's utility is \$10 – \$8 = \$2. Submitting a false-name bid is not profitable for agent 1.

Marginal utility

As we have seen in the previous subsection, the G.V.A. is robust in some situations, and vulnerable in other situations. We find that the robustness of the G.V.A. for a single item with multiple units depends on the marginal utility of a single item. The marginal utility of an item means an increase in the agent's utility as a result of obtaining one additional unit. For example, when we buy a CD or a book, the marginal utility usually diminishes, since having a CD or a book is enough, multiple units of the same CD or book are wasteful. One example where the marginal utility increases is an all-or-nothing situation, where an agent needs a certain number of units, otherwise the good is useless (one sock, glove, etc.).

The following theorem shows one sufficient condition where the G.V.A. is robust against false-name bids.

Theorem 1 *The G.V.A. is robust, i.e., submitting false-name bids is not profitable, if the declared marginal utility of each agent is constant/diminishes*[2].

Proof: Let us assume that an agent is submitting false-name bids, i.e., submitting bids using multiple identities. We show that if the agent merges these bids under a single identity, the same allocation as in the original case is attained, and the payment of the agent never increases.

Suppose that there are n units of a single item. Let A denote the set of all buyer agents, and $b_{i,j}$ denote agent i's bid for jth units of the item. Next, let $B(A)$ denote the set of bids $\{b_{i,j} \mid i \in A, 1 \leq j \leq n\}$, $nth(1, B(A))$ denote the largest bid in $B(A)$, $nth(2, B(A))$ denote the second largest bid in $B(A)$, and so on.

The inequality $b_{i,j} \geq b_{i,j+1}$ holds for all i, j according to the assumption that the declared marginal utility is constant/diminishes. In this case, the sum of the declared valuations is maximized by allocating units for the bids $nth(1, B(A))$, $nth(2, B(A))$, ...,

[2] Even if the true marginal utility of each agent is constant/diminishes, there is a chance that an agent exists whose declared marginal utility increases, i.e., the agent declares a false valuation. In such a case, submitting false-name bids might be profitable.

$nth(n, B(A))$. If the declared marginal utility increases, this property cannot be satisfied.

Suppose that agent x submits false-name bids using two identities, agent y and agent z, and obtains l items under the identity y, and m items under the identity z (where $l + m = k$). For simplicity, let us assume that $b_{y,j} = 0$ for $l < j \leq n$ and $b_{z,j} = 0$ for $m < j \leq n$ [3].

Agent y's payment P_y is represented by the sum of the bids $nth(n+1, B(A)), nth(n+2, B(A)), \ldots, nth(n+l, B(A))$ [4]. Similarly, agent z's payment P_z is also calculated by the sum of the bids $nth(n+1, B(A)), \ldots, nth(n+m, B(A))$.

Then, let us assume that agent x merges these bids and submits them under a single identity x. For simplicity, let us assume $b_{x,j} = 0$ for $k < j \leq n$. This assumption does not affect the allocation result and x's payment. By submitting these bids, agent x can still obtain k units, and its payment P_x becomes equal to the sum of the bids $nth(n+1, B(A)), nth(n+2, B(A)), \ldots, nth(n+k, B(A))$.

From these facts, it is obvious that $P_y + P_z \geq P_x$ holds. In other words, the payment of an agent becomes smaller (or equal) when the agent merges the false-name bids and submits them using a single identity. Therefore, the G.V.A. is robust against false-name bids, as long as the declared marginal utility of each agent is constant/diminishes. □

In Example 1, agent 2's marginal utility increases. On the other hand, the marginal utility of each agent is constant/diminishes in Example 2.

Robustness of the G.V.A. in Multiple Item Auctions

This section discusses the robustness of the G.V.A. in multiple item auctions. In this situation, the key to decide whether the G.V.A is robust/vulnerable is the utility structure of an agent. The structure is represented by introducing economic notions, i.e., substitutional/complementary.

Example We present an example where the G.V.A. is vulnerable to false-name bids. Suppose that there are different items denoted by g_1 and g_2 and two agents denoted by agent 1 and agent 2. We denote an agent's bid using a tuple: (a bid for g_1, a bid for g_2, a bid for set$\{g_1, g_2\}$).

- agent 1's bid: (\$25, \$5, \$30)
- agent 2's bid: (\$0, \$0, \$40)

Agent 2 wins a set $\{g_1, g_2\}$ at \$30 and its utility is \$10, while agent 1's utility is \$0.

Next, we suppose agent 1 submits a false-name bid under the identity of agent 3.

- agent 1's bid: (\$25, \$5, \$30)
- agent 2's bid: (\$0, \$0, \$40)
- agent 3's bid: (\$0, \$30, \$30)

The item g_1 goes to agent 1 and g_2 goes to agent 3. The payment for agent 1 is \$40 − \$30 = \$10 and the payment for agent 3 is \$40 − \$25 = \$15. Namely, agent 1 can obtain the two items with \$25, so its utility is \$30 − \$25 = \$5. This means a false-name bid is effective.

Substitutional/Complementary

Since the robustness of the G.V.A. for a single item with multiple units depends on the agents' marginal utilities, we find that the robustness of the G.V.A. with multiple items depends on whether the goods are complementary/substitutional[5].

Suppose that there are two different items denoted by A and B. We define A and B are complementary, if the sum of the utility of only having A and the utility of only having B is lower than the sum of the utility of simultaneously having A and B, i.e.,

$$u_i(A) + u_i(B) < u_i(\{A, B\}).$$

Here, let u_i denote a valuation function of an item or a set of items for an agent i.

We define A and B are substitutional, if the sum of the utility of only having A and the utility of only having B is higher than (or equal to) the utility of simultaneously having A and B, i.e.,

$$u_i(A) + u_i(B) \geq u_i(\{A, B\}).$$

For example, we can consider tea and sugar to be complementary and tea and coffee to be substitutional.

The following theorem shows one sufficient condition where the G.V.A. is robust against false-name bids.

Theorem 2 *The G.V.A. is robust, i.e., submitting false-name bids is not profitable, if all items are substitutional for all agents according to the declared valuations of agents.*

The proof can be given in a way similar to that of Theorem 1.

In the previous example, the items are complementary for agent 2. Therefore, there is a chance that submitting a false-name can be profitable.

Table 1 summarizes the obtained results.

[3] Without this assumption, the payments could be larger, since agent z's bids $\{b_{z,j} \mid m < j \leq n\}$ might be used to calculate the payment of agent y.

[4] In general, P_y is calculated by the sum of the bids $nth(n-l+1, B(A-\{y\})), nth(n-l+2, B(A-\{y\})), \ldots, nth(n, B(A-\{y\}))$. We can obtain the above result since the set $\{b_{y,j} \mid 1 \leq j \leq l\}$ is included in the winning bids, and $b_{y,j} = 0$ for $l < j \leq n$.

[5] In microeconomic studies, the definition that item A and item B are complementary is as follows: if the price of item B increases, the demand of item A decreases, and vice versa. This definition is more strict than our definition.

Table 1: Robustness against false-name bids

number of items	number of units	number of requirements	property	robustness
single	single	single		○
single	multiple	single		○
single	multiple	multiple	marginal utility is constant/diminishes	○
			marginal utility increases	×
multiple	multiple	multiple	substitutional	○
			complementary	×

Non-Existence of Desirable Protocols

So far, we have investigated the robustness of the G.V.A., and clarified the circumstances where submitting a false-name bid is effective in the G.V.A. The next question is whether any auction protocol exists that is robust against false-name bids or not. In this section, we show a negative result, i.e., we show proof that there exists no single-round sealed-bid auction protocol that simultaneously satisfies individual rationality, Pareto efficiency, and incentive compatibility in all cases if agents can submit false-name bids.

Theorem 3 *In auctions for multiple units of a single item and multiple requirements of agents, there exists no single-round sealed-bid auction protocol that simultaneously satisfies individual rationality, Pareto efficiency, and incentive compatibility in all cases if agents can submit false-name bids.*

Proof: It is sufficient to show one instance where no auction protocol satisfies the prerequisites.

Let us assume that there are two units of a single item, and three agents denoted by agent 1, agent 2, and agent 3.

- agent 1's bid: $(a, 0)$
  agent 1 bids a for the first unit and 0 for the second unit, total of a for both units.
- agent 2's bid: (b, a)
- agent 3's bid: $(a, 0)$

Let us assume $a > b$. According to Pareto efficiency, agent 1 and agent 3 get one unit. Let P_a denote the payment of agent 1.

When agent 2 and agent 3 reveal their true valuations, if agent 1 submits a bid, $a' = b+\epsilon$, the allocation does not change. Let $P_{a'}$ denote agent 1's payment in this situation. According to individual rationality, the inequality $P_{a'} \leq a'$ should hold. Furthermore, according to incentive compatibility, $P_a \leq P_{a'}$ should hold. These assumptions lead to $P_a \leq b + \epsilon$. The condition for agent 3's payment is identical to that for agent 1's payment.

Next, we assume another case with two agents denoted by agent 1 and agent 2.

- agent 1's bid: (a, a)
- agent 2's bid: (b, a)

According to Pareto efficiency, the two units go to agent 1. Let us denote the payment of agent 1 $P_{(a,a)}$. If agent 1 submits a false-name bid using the identity of agent 3, the same result as in the previous case can be obtained. According to incentive compatibility, the following inequality must hold, otherwise, agent 1 can profit by submitting a false-name bid: $P_{(a,a)} \leq 2 \times P_a \leq 2b + 2\epsilon$.

On the other hand, let us consider the case when there are two agents.

- agent 1's bid: (c, c)
- agent 2's bid: (b, a)

Let us assume $b+\epsilon < c < a$, and $a+b > 2c$. According to Pareto efficiency, the two units go to agent 2. So, agent 1 cannot gain any utility. However, if agent 1 replaces the bid (c, c) with (a, a), both units go to agent 1 and the payment is $P_{(a,a)} \leq 2b + 2\epsilon$, which is smaller than $2c$, i.e., agent 1's true value of these two units. Therefore, agent 1 can increase the utility by submitting a false bid (over-bidding its true valuation).

Thus, in auctions for multiple units of a single item and multiple requirements of agents, there exists no auction protocol that simultaneously satisfies individual rationality, Pareto efficiency, and incentive compatibility in all cases if agents can submit false-name bids. □

Theorem 4 *In auctions with multiple items, there exists no single-round sealed-bid auction protocol that simultaneously satisfies individual rationality, Pareto efficiency, and incentive compatibility in all cases if agents can submit false-name bids.*

The proof can be given in a way similar to that of Theorem 3.

Conclusions

We have discussed the robustness of the generalized Vickrey auction (G.V.A.) against false-name bids. Although, to our knowledge, this problem has not been previously addressed, it can be a serious problem in Internet auctions. We have clarified the circumstances where submitting false-name bids is profitable. More specifically, we have shown that the robustness of the G.V.A. depends on the utility structure of each agent. Moreover, we have proved that there exists no single-round sealed-bid auction protocol that simultaneously satisfies individual rationality, Pareto efficiency, and incentive compatibility in all cases if agents can submit false-name bids.

We obtained a rather negative result in the problem of false-name bids. However, there are many situations where obtaining the optimal allocation is not necessary. In such a situation, it is enough to design an auction mechanism simultaneously satisfying individual rationality and incentive compatibility. Our future goal is to find an auction mechanism that can obtain reasonably good (but not Pareto efficient) allocations.

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