

# Investigating the Effect of Relevance and Reachability Constraints on SAT Encodings of Planning\*

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Recently, satisfiability (SAT) techniques have been shown to be more efficient at extracting solutions from a planning graph in Graphplan (Blum & Furst 1995) than the standard backward search (Kautz & Selman 1998). Graphplan gains efficiency from forward propagation and backward use of mutual exclusion constraints. The utility of SAT techniques for solution extraction raises two important questions: (a) Are the mutual exclusion constraints equally useful for solution extraction with SAT encodings? (b) If so, are there additional types of propagated constraints that can benefit them even more? Our ongoing research investigates these two questions.

The mutual exclusion relations (mutex) used in the standard Graphplan, that we shall refer to as *fmutex* constraints, are propagated by the following rules: Two facts  $P$  and  $Q$  are *fmutex* if all actions supporting  $P$  are pair-wise *fmutex* with all actions supporting  $Q$ . Two actions are *fmutex* if action  $A_1$  deletes another action  $A_2$ 's preconditions or effects, or if preconditions of  $A_1$  are pairwise *fmutex* with preconditions of  $A_2$ . *Fmutex* relations propagate forward from the initial state to provide "reachability" information to the backward search. Though they increase the size of the SAT encoding with a quadratic number of additional clauses, the overall encoding is simplified through unit-propagation based methods. E.g., *fmutexes* helps an 11 step/73 action logistics problem that takes 6 min 30 sec on Blackbox to solve in 7sec (60x speedup). It is also possible to propagate "relevance" relations starting from the goals. Such constraints would not be useful for standard Graphplan, unless it conducts the search in the forward direction.

In contrast, the search in SAT encodings is "direction-less" in that it neither goes exclusively backward from goals nor exclusively forward from the initial state. Any variable that has the best heuristic properties (smallest live domain (Tsang 1993), maximum unit propagation (Li & Anbulagan 1997)) is selected, irrespective of its position in the corresponding planning graph. This means that the solver can potentially exploit mutex relations based on reachability as well as relevance. With this insight, we developed two types of constraints that can be propagated in backward direction, and carry relevance information:

**Backward mutex (bmutex):** Bmutex constraints attempt to capture the idea that two actions (or propositions) are never relevant together. Facts in the goal level are not bmutex. If actions  $A_1$  and  $A_2$  give the same set of facts at the next level, then  $A_1$  is bmutex with  $A_2$ . Moreover, if all the

facts that  $A_1$  supports are pairwise bmutex with all the effects that  $A_2$  supports, then  $A_1$  is bmutex with  $A_2$ . Given two facts  $P$  and  $Q$ , if all the actions that consume  $P$  are pairwise bmutex with all the actions that consume  $Q$ , then  $P$  and  $Q$  are bmutex.

**Inseparability relations (insep):** Inseparability constraints capture the relation that a set of actions (propositions) must be either relevant all together, or none of them are relevant. The top level goals are inseparable in the goal level. If facts  $G_1$  and  $G_2$  are inseparable, let  $A(G_1)$  be set of actions supporting  $G_1$  and  $A(G_2)$  be the set of actions supporting  $G_2$ . If the actions in  $A(G_1)$  do not support any other relevant fact and so is also the case with  $A(G_2)$ , then the actions in  $A(G_1)$  are pair-wise inseparable with those in  $A(G_2)$ . A proposition  $P_1$  is inseparable from proposition  $P_2$  if every action that  $P_1$  supports is pairwise inseparable from every action that  $P_2$  supports. An action is inseparable with itself while a proposition is not so. If an action  $A_1$  is being considered to support proposition  $P$ , and  $Q$  is a proposition that is inseparable with  $P$ , then  $A_1$  cannot delete  $Q$ .

Our experience in adding these propagated constraints to SAT encodings has been somewhat mixed to-date. The bmutex constraints, as stated, do not seem to propagate well and taper off after some levels. This is largely due to the requirement that for any two actions to be bmutex, they must have exactly the same set of useful effects. In the case of inseparability relations, pair-wise relations miss the case where even if  $A(G_1)$  supports facts other than  $G_1$ , those facts may be inseparable with  $G_2$  (and vice-versa) making  $A(G_1)$  and  $A(G_2)$  inseparable. At present we are looking into variations of these constraints that have better propagation capabilities.

In summary, we have found that reachability based mutex propagation does improve the solvability of SAT encodings. Motivated by this, and the fact that SAT searchers are direction-less, we are investigating the utility of additional mutex relations based on relevance.

## References

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