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# Towards Synthesizing Optimal Coordination Modules for Distributed Agents

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## Abstract

In a discrete-event framework, we define the concept of a coordinable language and show that it is the necessary and sufficient existence condition of coordination modules for distributed agents to achieve conformance to a given inter-agent constraint language. We also present a synthesis algorithm to compute near optimal coordination modules.

## Background & Notation

We use the following notations from language and automata theory (Cassandras and Lafortune 1999):

- For an event set $\Sigma$, $\Sigma^*$ denotes the set of all finite strings over $\Sigma$, including the empty string $\varepsilon$; and for two event sets $\Sigma^1 \subseteq \Sigma^2$, $P_{\Sigma^2,\Sigma^1}$ denotes the natural projection from $(\Sigma^2)^*$ to $(\Sigma^1)^*$, erasing all $\sigma \in \Sigma^2 - \Sigma^1$ in $s \in (\Sigma^2)^*$.
- For a language $L$ over an event set $\Sigma$, i.e., $L \subseteq \Sigma^*$, $\bar{L}$ denotes the set of all prefixes of its strings. $L$ is said to be prefix-closed if $L = \bar{L}$.
- For an automaton $A = \left(X^A, \Sigma^A, \delta^A, x_0^A, X_m^A\right)$, $L(A)$ and $L_m(A)$ denote its generated prefix-closed and marked languages, respectively; and for $x \in X^A$, $s \in (\Sigma^A)^*$, $\delta^A(s,x)!$ denotes that $\delta^A(s,x)$ is defined; $Trim(A)$ denotes the procedure that computes and returns a nonblocking automaton which generates the same marked language as $A$; and $A = A_1 \parallel A_2$ denotes that automaton $A$ is the synchronous product of the two automata $A_1$ and $A_2$.

Consider a system modeled by an automaton $A$, with the event set $\Sigma^A$ partitioned into (i) $\Sigma^A = \Sigma_c^A \dot\cup \Sigma_{uc}^A$ and (ii) $\Sigma^A = \Sigma_o^A \dot\cup \Sigma_{uo}^A$, where $\Sigma_c^A$, $\Sigma_{uc}^A$, $\Sigma_o^A$ and $\Sigma_{uo}^A$ denote the sets of controllable, uncontrollable, observable and unobservable events of $A$, respectively. Let $K$ be a sublanguage of $L_m(A)$, i.e., $K \subseteq L_m(A)$. The statements '$K$ is controllable w.r.t $A$, $\Sigma_c^A$' and '$K$ is observable w.r.t $A$, $P_{\Sigma^A,\Sigma_o^A}$' refer, respectively, to the concepts of controllability (Ramadge and Wonham 1987) and observability (Lin and Wonham 1988) of a language $K$ in supervisory control theory. Finally, for an automaton $C$, the $Supcon(C, A, \Sigma_c^A)$ procedure (Wonham and Ramadge 1987) computes a nonblocking automaton $S$ such that $L_m(S)$ is the supremal controllable sublanguage (Ramadge and Wonham 1987) of $L_m(C) \cap L_m(A)$ w.r.t $A$ and $\Sigma_c^A$.



## Discrete-Event Agents & Coordination

Consider a system of two agents modeled by the respective automata $A_i = (X^{A_i}, \Sigma^{A_i}, \delta^{A_i}, x_0^{A_i}, X_m^{A_i})$ $(i \in \{1,2\})$, where $\Sigma^{A_1} \cap \Sigma^{A_2} = \emptyset$. The event set $\Sigma^{A_i}$ of agent $A_i$ is partitioned into the controllable set $\Sigma_c^{A_i}$ and the uncontrollable set $\Sigma_{uc}^{A_i}$. In enabling distributed agents to coordinate, each agent $A_i$ is equipped with a *coordination module* (CM) modeled by an automaton $S_i$ with the following properties:

1. $\Sigma^{S_i} = \Sigma^{A_i} \cup ComSet(S_i, A_j)$, where $ComSet(S_i, A_j) \subseteq \Sigma^{A_j}$, $(i,j \in \{1,2\}, i \neq j)$. $\Sigma^{S_i}$ is called the coordination event set for agent $A_i$, and $ComSet(S_i, A_j)$ is the set of events that agent $A_j$ needs to communicate to $A_i$ to synchronize $S_i$.
2. $S_i$ is $\Sigma_{uc}^{A_i}$-enabling, namely, $(\forall s \in (\Sigma^{S_i})^*)(\forall \sigma \in \Sigma_{uc}^{A_i})$ $((s \in L(S_i \parallel A_i)$ and $P_{\Sigma^{S_i},\Sigma^{A_i}}(s)\sigma \in L(A_i)) \Rightarrow (s\sigma \in L(S_i \parallel A_i)))$.
3. $S_i$ and $S_j$ are cooperative, namely, $(\forall s \in (\Sigma^A)^*)(\forall \sigma \in ComSet(S_i, A_j))$ $((P_{\Sigma^A,\Sigma^{A_j}}(s)\sigma \in L(A_j)$ and $P_{\Sigma^A,\Sigma^{S_j}}(s)\sigma \in L(S_j)) \Rightarrow (P_{\Sigma^A,\Sigma^{S_i}}(s)\sigma \in L(S_i)))$.

Let $A = A_1 \parallel A_2$ and $S_{12} = (S_1, S_2)$ denote the CM pair of $S_1$ and $S_2$. Write $S_{12}/A$ for the system of two agents $A_1$ and $A_2$ coordinating through their respective CM's.

**Definition 1.** Coordinated Behaviors

*1.* Prefix-closed coordinated behavior $L(S_{12}/A)$

*(a)* $\varepsilon \in L(S_{12}/A)$.

*(b)* $(\forall s \in L(S_{12}/A))(\forall \sigma \in \Sigma^{A_i})$ $(s\sigma \in L(S_{12}/A) \Leftrightarrow$ $(s\sigma \in L(A)$ *and* $(P_{\Sigma^A,\Sigma^{S_i}}(s)\sigma \in L(S_i))))$.

*2.* Marked coordinated behavior $L_m(S_{12}/A)$

$$L_m(S_{12}/A) = L(S_{12}/A) \cap L_m(A) \cap L_m(S_1) \cap L_m(S_2).$$

CM pair $S_{12}$ is nonblocking if $\overline{L_m(S_{12}/A)} = L(S_{12}/A)$.

**Definition 2.** Coordinable Language: *Let* $\Sigma_{com} \subseteq \Sigma^A$. *A language* $K \subseteq L_m(A)$ *is* coordinable *w.r.t* $A$ *and* $\Sigma_{com}$ *if*

*1.* *$K$ is controllable w.r.t $A$ and $\Sigma_c^A = \Sigma_c^{A_1} \cup \Sigma_c^{A_2}$; and*

*2.* *$K$ is observable w.r.t $A$ and $P_{\Sigma^A,\Sigma^{A_i} \cup \Sigma_{com}}$ $(i \in \{1,2\})$.*

**Theorem 1.** *Let $\emptyset \neq K \subseteq L_m(A)$ and $\Sigma_{com} \subseteq \Sigma^A$. Then, there exists a nonblocking CM pair $S_{12} = (S_1, S_2)$, with CM $S_i$ for $A_i$, such that $L_m(S_{12}/A) = K$ and $\Sigma_{com} = ComSet(S_1, A_2) \cup ComSet(S_2, A_1)$, if and only if $K$ is coordinable w.r.t $A$ and $\Sigma_{com}$.*

## Problem Statement and Solution Properties

**Problem.** Multiagent Coordination Problem (MCP): *Given an inter-agent constraint automaton $C$ over $\Sigma^A$, construct a nonblocking CM pair $S_{12} = (S_1, S_2)$ such that $L_m(S_{12}/A) \subseteq L_m(A) \cap L_m(C)$.*

When solving MCP, it is desirable to synthesize optimal CM's, i.e., CM's with the following properties: 1) *Minimal Intervention* - the coordination does not unnecessarily disable controllable events; 2) *Minimal Communication* - the number of events to be communicated between the agents is minimal; and 3) *Efficient Implementation* - each CM is of minimal state size (among all CM's satisfying the first two properties).

Let $S = Supcon(C, A, \Sigma_c^A)$, where $\Sigma_c^A = \Sigma_c^{A_1} \cup \Sigma_c^{A_2}$. Then *minimal intervention* can be guaranteed by synthesizing CM's $S_1$ and $S_2$ such that $L_m(S_{12}/A) = L_m(S)$. Procedure $CM$ below computes CM's $S_i$ given $S$, system event set $\Sigma^A$, and event set $\Sigma^{CM_i} \subseteq \Sigma^A$ for $\Sigma^{S_i} = \Sigma^{CM_i}$. By the constructive proof of Theorem 1 presented elsewhere, it can be shown that if $L_m(S)$ is coordinable w.r.t $A$ and $\Sigma^{CM_1} \cup \Sigma^{CM_2}$ and $S_i = CM(S, A, \Sigma^{CM_i})$ ($i \in \{1, 2\}$), the CM pair $(S_1, S_2)$ is nonblocking and $L_m(S_{12}/A) = L_m(S)$.

**Procedure** $CM$ $(S, \Sigma^A, \Sigma^{CM_i})$

**begin**

- Let $\pi : X_p \to 2^{X^S} - \{\emptyset\}$ be a bijective mapping;
- Compute automaton $S'_i = (\Sigma^{CM_i}, X_p, \delta^{S'_i}, x_0^{S'_i}, X_m^{S'_i})$:
  - $x_0^{S'_i} \in X_p$ with $\pi(x_0^{S'_i}) = \{\delta^S(s, x_0^S) \mid P_{\Sigma^A, \Sigma^{CM_i}}(s) = \varepsilon\}$;
  - $X_m^{S'_i} = \{x_p \in X_p \mid (\exists s \in L_m(S))\delta^S(s, x_0^S) \in \pi(x_p)\}$;
  - $(\forall \sigma \in \Sigma^{CM_i})(\forall x_p \in X_p)$ $(\delta^{S'_i}(\sigma, x_p)!$ if and only if $(\exists s\sigma \in L(S))\delta^S(s, x_0^S) \in \pi(x_p))$;
    When defined, $\delta^{S'_i}(\sigma, x_p) = x'_p$ with $\pi(x'_p) = \{\delta^S(s', x) \mid x \in \pi(x_p), P_{\Sigma^A, \Sigma^{CM_i}}(s') = \sigma\}$;
- Return $S_i = Trim(S'_i)$;

**end**

**Definition 3.** *Let $L \subseteq L_m(A)$. A subset of $\Sigma^{A_j}$ is a minimal (cardinality) communication set of agent $A_i$ from $A_j$ w.r.t $L$, and denoted by $MinComSet(L, A_i, A_j)$, if*

1. *$L$ is observable w.r.t $A$, $P_{\Sigma^A, \Sigma^{A_i} \cup MinComSet(L, A_i, A_j)}$;*
2. *$(\forall \Sigma_{com}^{A_j} \subseteq \Sigma^{A_j})$($L$ is observable w.r.t $A$ and $P_{\Sigma^A, \Sigma^{A_i} \cup \Sigma_{com}^{A_j}}$)$\Rightarrow |MinComSet(L, A_i, A_j)| \leq |\Sigma_{com}^{A_j}|$.*

The event set $MinComSet(L, A_i, A_j)$ could be computed by adapting the minimal sensor-selection algorithm (Haji-Valizadeh and Loparo 1996). To guarantee *minimal communication* between two agents coordinating to achieve $L_m(S)$, CM's $S_1$ and $S_2$ can be computed such that $\Sigma^{S_i} = \Sigma^{A_i} \cup MinComSet(L_m(S), A_i, A_j)$.

Finally, the reduction procedure $Supreduce$ presented in (Su and Wonham 2004) could be modified as procedure $CMreduce$, in attempting to address the *efficient implementation* of CM's $S_i$. Procedure $CMreduce(S_i, A, \Sigma_c^{A_i})$ can often return a greatly state-size reduced CM for agent $A_i$ achieving the same behavior of $S_i \parallel A$.

## Coordination Module Synthesis

In what follows, the discrete-event techniques of control and sensor selection can be adapted and utilized to address MCP as in the algorithm below. The CM's returned by the algorithm are minimally interventive and entail minimal communication, but have a relatively small state size that is not necessarily minimal; hence they are said to be near optimal.

**Algorithm:** Coordination Module Synthesis

**Input**: Agents $A_1$, $A_2$ and constraint $C$ where $\Sigma^C = \Sigma^{A_1} \cup \Sigma^{A_2}$ and $\Sigma^{A_1} \cap \Sigma^{A_2} = \emptyset$

**Output**: A near optimal nonblocking CM pair $S_{12} = (S_1, S_2)$ such that $L_m(S_{12}/A) \subseteq L_m(A) \cap L_m(C)$

**begin**

- Compute automaton $A$ and controllable set $\Sigma_c^A$
  $A \leftarrow A_1 \parallel A_2$; $\Sigma_c^A \leftarrow \Sigma_c^{A_1} \cup \Sigma_c^{A_2}$;
- Compute a nonblocking supervisor $S$
  $S \leftarrow Supcon(C, A, \Sigma_c^A)$;
- Compute coordination event sets $\Sigma^{CM_1}$, $\Sigma^{CM_2}$
  $\Sigma^{CM_i} \leftarrow \Sigma^{A_i} \cup MinComSet(L_m(S), A_i, A_j)$, $(i, j \in \{1, 2\})$;
- Compute CM's $S_1$, $S_2$
  $S_i \leftarrow CM(S, A, \Sigma^{CM_i})$, $(i \in \{1, 2\})$;
- Reduce state size of CM's
  $S_i \leftarrow CMreduce(S_i, A, \Sigma_c^{A_i})$, $(i \in \{1, 2\})$;
- Return CM pair $S_{12} = (S_1, S_2)$;

**end**

## Conclusion

The contributions of this paper to discrete-event multiagent coordination include the existence condition (Theorem 1) of CM's and a synthesis algorithm for near optimal CM design.

## References


Cassandras, C. G., and Lafortune, S. 1999. *Introduction to Discrete Event Systems*. Kluwer Academic Publishers, MA, USA.

Haji-Valizadeh, A., and Loparo, K. A. 1996. Minimizing the cardinality of an events set for supervisors of discrete-event dynamical systems. *IEEE Trans. Autom. Control* 41(11):1579–1593.

Lin, F., and Wonham, W. M. 1988. On observability of discrete event systems. *Inf. Sci.* 44:173–198.

Ramadge, P. J., and Wonham, W. M. 1987. Supervisory control of a class of discrete event processes. *SIAM J. Control Optim.* 25(1):206–230.

Su, R., and Wonham, W. M. 2004. Supervisor reduction for discrete-event systems. *Discret. Event Dyn. Syst.-Theory Appl.* 14(1):31–53.

Wonham, W. M., and Ramadge, P. J. 1987. On the supremal controllable sublanguage of a given language. *SIAM J. Control Optim.* 25(3):637–659.