

DELUSIONS OF OMNISCIENCE

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Abstract

Epistemic and doxastic modal logics (Hintikka [1962]), and the logics of theory change and belief revision (Alchourrón *et al.* [1985], Gärdenfors [1988]) are used for the representation of belief. Both kinds of logic are omniscient in various ways. We address these delusions of omniscience in this paper. We begin by defining three kinds of omniscience – logical, deductive, and factual. We will then discuss some of the strategies which have been used to dispose of or deflate omniscience. Our main concern will be for the *no worries* strategies, and the *other logics* strategy. We then comment on the prospects for omniscience free logics.

Introduction

There are two main kinds of logic used for the representation of belief. They are epistemic and doxastic modal logics (Hintikka [1962]), and the logics of theory change and belief revision (Alchourrón *et al.* [1985], Gärdenfors [1988]). Both kinds of logic behave in much the same way with respect to several features. Our concern is for the omniscience features. Both are logically omniscient. Both are deductively omniscient. Both lack factual omniscience. We address the delusions of omniscience in this paper.

We begin by defining the three kinds of omniscience mentioned above. We will then discuss some of the strategies which have been used to dispose of or deflate omniscience. Our main concern will be for the "no worries" strategies, and for the other logics strategy. We then consider the not unreasonable prospects for omniscience free logics.

Three Kinds of Omniscience

S5 is one of the most commonly used modal logics for knowledge representation. The main modal operator, \Box , is given an epistemic interpretation. It is interpreted to mean "It is known that". In most epistemic interpretations, the modal operator is indexed to epistemic agents. The \Box is multiplied to \Box_x , where x can be any one of a set of agents: a, b, c, d, \dots . For example, \Box_a is interpreted to mean " a knows that". \Box_a is often changed to K_a . The result is a multiply modal S5 system. (Rennie [1970]) D5 is also commonly used, but as a doxastic logic. Other weaker logics such as S4 or D4 (Hintikka [1962]), or S0.5 (Lemmon [1959]) or D0.5 are also used.

In this paper we will focus attention on epistemic rather than doxastic logics. But, most of the points to be made about omniscience in epistemic logic can be applied analogously to doxastic logic.

In all the modal logics mentioned above, the axiomatizations contain some sort of *Necessitation Rule* such as:

$$\vdash A \rightarrow \vdash \Box A$$

In S0.5, the antecedent of the Rule is restricted to standard non-modal propositional logic. No such restriction applies for S4 or S5. When this Rule is interpreted for epistemic logic, it follows that every knower automatically knows at least all tautologies. We define:

Where the knowledge agent automatically knows all the logical truths defined by some logic the agent is *logically omniscient*.

This kind of omniscience was viewed with some warmth by Rene Descartes. He says that there are some eternal truths which dwell in our minds:

To this class belong: *It is impossible that a given thing should at once be and not be*; ... and countless others. It would not be easy to enumerate them all; but one is not either likely to be ignorant of them when occasion arises to think of them and when we are

not blinded by prejudice. (pg 191)

We say that logics which make the agent logically omniscient for first order logic are *Cartesian* logics.

In standard belief revision theory (Gärdenfors [1988]) belief sets contain all the theorems of the base logic of our choice. The agents of AGM theory are *logically omniscient* in the Cartesian sense.

The second kind of omniscience is *deductive omniscience*. We define:

Where the knowledge agent automatically knows all the logical consequences of known propositions, the agent is *deductively omniscient*.

In all of the modal logics mentioned above, because of the **K** or *Distribution* axiom:

$$\Box(A \supset B) \supset (\Box A \supset \Box B)$$

and because every tautology is automatically known, all logical consequences are known.

Also, in AGM theory, any belief set, *K*, is closed under logical consequence:

$$K = \{A : K \vdash A\}$$

So, the agents of AGM theory are *deductively omniscient*.

Gärdenfors writes that:

An important feature of belief sets is that they need not be *maximal* in the sense that for every sentence *A* either *A* belongs to the belief set or $\neg A$ belongs to it. The epistemic interpretation of this is that an individual is normally not *omniscient*. (page 25 [1988])

This is a far too restrictive an approach to omniscience. It gives the misleading impression that there is no other kind of omniscience. The literature around this topic shows that there are other views. The kind of omniscience which Gärdenfors correctly claims is not a feature of AGM belief sets is our third kind of omniscience.

The third kind of omniscience is *factual omniscience*. We define:

Where the knowledge agent automatically knows, for any proposition *A*, whether *A* is true or not, the agent is *factually omniscient*.

This kind of omniscience includes the other two. There are no advocates of logics or theories which make this claim to epistemic divinity.

Dealing With Omniscience

We first consider two "no worries" strategies for deflating omniscience. These strategies rely upon re-interpreting logics such as **S5** or AGM theory. "No worries, the problem is not as you thought. We can continue with the formal system as is."

We will consider the *idealising* and *implicit/explicit* strategies. Space constraints mean that we will place greater emphasis on the analysis of the idealising strategy than on the implicit/explicit strategy. There are other strategies such as the *crash through* strategy, but we have no space to look at them here.

The *idealising* strategy was first suggested by Lemmon [1959]. He proposed that we accept omniscience as part of the idealisation of the agent. He made the suggestion that to interpret the epistemic logician's 'X knows that'

We may make a start, however, by treating *X* as a kind of logical fiction, the rational man. ... (A rational man knows (at least implicitly) the logical consequences of what he knows.)

Lemmon also writes, of his ideal rational man, that:

There are some queer consequences: *X* knows that *T*, let us say, where *T* is some very long tautology containing 396 propositional variables. But this is not to worry us ...

'The rational man' of Lemmon is, clearly, logically very knowledgeable (at least implicitly). This follows from the fact that the logical system which Lemmon is interpreting as an epistemic logic has as one of its rules:

If A is a tautology then $\Box A$ is a theorem

This rule is interpreted in epistemic logic as

If A is a tautology then X knows that A

Lemmon gives us a snapshot picture of his preferred rational man by looking directly at the axiomatisation of the logic, and drawing a *prima face* picture of the ideal epistemic agent from the axioms and rules of inference. It is no surprise that Lemmon does this. He is one of the great systematic axiomatisers of modal logics. This way of developing a *prima face* picture of the model to which the machine knower and believer conforms is clearly the *axiomatic* way of seeing the epistemic agent modelled by a logic.

The Idealised Agent

We begin by looking at the *axiomatic* way of seeing the epistemic agent modelled by several logics. In particular, we look at how logical and deductive omniscience vary from system to system. We will also make some cursory remarks on introspection and the extent to which the ideal has knowledge of its own ignorance.

To find out, we begin by looking at two *normal* modal logics: **S4** and **S5**, and Lemmon's preferred, *non-normal*, **S0.5**. From a logical point of view the three systems are related as follows (Let " $+++ \subseteq ***$ " be read as "All the theses of $+++$ are theses of $***$ "):

$$\mathbf{S0.5} \subseteq \mathbf{S4} \subseteq \mathbf{S5}$$

So S5 includes all the theses of all the others. We will start with S5 and work down through the systems, seeing what is lost at each point, and maybe lost to advantage.

The S5 Agent

Consider a Lemmon [1966] style axiomatisation for S5. The letters A and B are formula schema, and the usual definitions apply. The axiom schema of S5 are the axiom schema of any classical propositional logic together with:

1. $\Box(A \supset B) \supset (\Box A \supset \Box B)$ (Distribution)
2. $\Box A \supset A$ (Veridicality)
3. $\Box A \supset \Box \Box A$
(S4-axiom or *KK-thesis* or *Positive Introspection Thesis*)
4. $\sim A \supset \Box \sim \Box A$ (Strong S5-axiom)
5. $\Diamond A \supset \Box \Diamond A$
(Weak S5-axiom or *Negative Introspection Thesis*)

The Rules of inference are:

- R1. $\vdash A, \vdash(A \supset B) \rightarrow \vdash B$ (Modus Ponens)
- R2. $\vdash A \rightarrow \vdash \Box A$ (Necessitation)
- R3. $T \rightarrow \vdash \Box T$ (where T is any theorem of propositional logic) (Weak-necessitation)

5. and R3. are redundant for S5, but it is useful to note them for later comparisons.

We first note that the Veridicality Axiom is unchallenged in the literature and in philosophical debate, and sets out the universally agreed necessary condition for the truth of any knowledge claim.

We begin with the agent who conforms to epistemic S5. Consider first the Distribution Axiom and the Necessitation Rule. When they are taken together it becomes obvious that the ideal S5 agent is logically and deductively omniscient. We saw above that Lemmon's knower knew all the tautologies. But the S5 knower knows all the theses of epistemic logic as well. We have full, not weak, necessitation. Under epistemic interpretation we have:

If P is a thesis of this epistemic logic then X knows that P .

This is logical omniscience. Thus are all the problems of epistemology solved. This S5 knower knows what knowledge is, quite explicitly. Here is Plato's philosopher guardian.

There is also the axiom which expands knowledge. It is the **K** axiom. This axiom, when put together with Necessitation, gives full strength to deductive omniscience. This axiom ensures that the agent does draw all the logical conclusions that follow from what is known.

We note in passing that S5 contains the strong S5 axiom -- sometimes called the Platonic axiom.

This is interpreted in epistemic logic as:

If not P then X knows that X does not know that P

The truly Socratic person is here exemplified, the person who knows just how ignorant they are. The full force of this axiom is often avoided by considering only the weak S5 axiom. The weaker axiom seems, in a sense, more reasonable. It is interpreted as:

If X does not know that P then X knows that X does not know that P

But we cannot have the weak without the strong in S5. We must pass by this whole area of idealisation.

We also note in passing the S4 axiom, the *KK-thesis* of many philosophical controversies (see Girle [1988, 1989] and Lenzen [1978]). This thesis is interpreted as:

If X knows that P , then X knows that X knows that P

We must also pass by this whole area of idealisation.

The ideal S5 agent is therefore, a fully aware knower who has immediate access to all the consequences of its knowledge, to all the theorems which constitute the logical structure of knowledge, and knows what it does not know. The S5 knower is logically and deductively omniscient.

The S4 Agent

The ideal S4 agent is the knower who conforms to Hintikka's epistemic logic. The S4 agent is almost the same as the S5 agent. The model contains both the S4 axiom (the *KK-thesis*), the Distribution axiom, and the Necessitation Rule. The difference is to be found in the absence of both of the S5 axioms.

The similarity between the S4 and S5 agents is great. The S4 agent is a fully aware knower who knows all the consequences of what it knows, and who knows the logical structure of knowledge. The S4 knower is logically and deductively omniscient.

The S0.5 Agent (Lemmon's ideal)

The S0.5 model contains the Distribution axiom, but it lacks the S5 and S4 axioms, and it lacks the full-blooded Necessitation Rule. It contains the weak Necessitation Rule:

If T is a theorem of PC then X knows that T

The S0.5 agent is not necessarily a fully aware knower. Indeed, there are no theorems in epistemic S0.5 of the form $K_x K_x P$. So, if being self-aware is to be represented by formulas of the form $K_x K_x P$, then self-awareness for the S0.5 agent is a purely contingent matter.

The S0.5 agent knows all the consequences of what it knows, but may not be fully aware of them. The S0.5 knower is logically omniscient in a restricted sense, and also deductively omniscient. We might note that the S0.5 agent could be said to be an ideal minimally Cartesian

the same as the test for the Validity of the formula which is the conclusion.

In the logic N the premises of arguments are indexed with (ρP) to show that they are in the usual starting world, ρ , for the root of the tree, and are premises as well. So, the start of a tree will be:

1.	$P_1 (nP)$	Premise 1
	⋮	⋮
n.	$P_n (nP)$	Premise n
n+1.	$\sim C(n)$	Negated Conclusion

The tree rules for N contain all the tree rules for propositional logic and the modal negation rules. The modal rules are:

(\diamond PN) $\diamond \alpha$ (ω)
 ⋮
 α (ν)
 where ν is NEW to this path of the tree.

(\square PN) α (ρP)
 ⋮
 α (ν)
 where ν is ANY index.

Note that the rule for \square applies only to the Premises, and to *any* formula in the premises. So, four things are clear. First, the premises are treated as if they and only they were necessarily true.

Second, if there are no premises, as in the test of the Validity of a formula, then the rule for \square will be inoperative. It follows that the Valid formulas will include all the tautologies of non-modal propositional logic. Their negations will always produce a closed tree.

Third, if τ is a tautology, then $\square \tau$ will be Valid. The closure will be in a world other than ρ .

Fourth, none of the "usual" modal formulas will be Valid. Formulas of the K form will certainly turn out invalid. Testing is left to the reader.

It soon becomes clear that the N agent is logically omniscient in the Cartesian sense, but is not deductively omniscient. Progress has been made. All that is now needed is to pare off the Cartesian omniscience, and there will be a completely omniscience free logic.

Conclusion

There is hope for omniscience free formal models of epistemic agents, but this hope requires a more critical and creative approach to logical systems by the researchers working in this area.

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agent.

Finally, having looked at the three epistemic logics, we can see that all of these epistemic logics, seen as theoretical models for ideal epistemic agents, have two things in common. It is that all of the ideal agents modelled by these systems are at least logically omniscient in the Cartesian sense outlined above, and are deductively omniscient.

It is possible to go to weaker epistemic logics, logics which lack some of these features, but for the moment we will go no further. We turn to evaluation, to judgement.

The important question for the idealising accounts is whether they are *too* ideal. Michie and Johnston [1985], point out that:

It is the task of knowledge engineering to design and construct ... conceptual interfaces to allow people ... and machines ... to understand each other. (pg 65)

and

In order for any beings, human or machine, to talk to each other, they must share the same mental structures. (pg 72)

If they are anywhere near correct about this then it is important to decide whether machine knowledge and machine belief are to be more or less like human knowledge and belief.

The ideal agents we have looked at are all logically omniscient in some sense. If we were to become Cartesian, then we might accept the **S0.5** ideal. The **S4** and **S5** ideals are far too strong. But even the **S0.5** ideal retains a deductive omniscience which is too strong.

It is clear that logics with the **K** axiom, are almost certainly going to give an ideal which will be deductively omniscient. Can we be rid of the **K** axiom, or can we have a less powerful deductive apparatus? We turn to this below.

Implicit and Explicit

The second strategy is the *implicit/explicit* strategy. The idea is that rational believers are *committed* to the logical consequences of what they explicitly believe. So, at the weakest, an AGM style belief set is a *commitment* set. If it can be shown that any of the beliefs to which one is committed is false, then one must revise one's explicit and implicit belief sets.

This view has been advocated from early in the history of modern epistemic logic (e.g. Wu [1972]). It has been supported in an interesting way by Andre Fuhrmann [1988].

This strategy is quite acceptable, but it leaves open the problem of just where the division is between explicit and implicit, particularly when it comes to the division between

explicitly known and implicitly known consequences of one's knowledge. Is this to be completely *ad hoc*?

Those who see this problem as crucial, and who find the strategies either unsuccessful or *ad hoc*, count this against the major logics and revision systems for knowledge representation. They can follow either the line that epistemic logic and revision logic are fatally flawed, or the line that other logics should be sought.

We will not give up easily. We turn now to other logics.

Other Logics

We acknowledge one response and consider one response in favour of other logics.

First, we acknowledge that there are a set of logics (Lemmon [1966]) which are known as the **E** logics. These logics have no theorems of the form $\Box A$. That means that there is no automatic knowledge of logical theorems. These logics are non-Cartesian. Nevertheless, these logics model agents who are deductively omniscient.

Second, we consider the logic **N** (Fitting *et. al.* [1992]), the Pure Logic of Necessitation. This is a logic which does not contain **K**, and it does not contain any theorems of the form $\Box A$. This logic promises a better model of the epistemic agent.

At first sight, the logic **N** is a normal modal logic. But, normal modal logics have four features, according to Fitting. They are:

1. If α is a tautology then $\Box \alpha$ is Valid.
2. If α is Valid, then so is $\Box \alpha$
3. Any formula of the form:
($\Box(A \supset B) \supset (\Box A \supset \Box B)$) is Valid.
4. *Modus Ponens* is a Valid argument form.

Any logic which fails to have any one of these features is sub-normal. We have seen, for example, that **S0.5** has 1, 3, and 4, but fails to have 2. $\Box(p \supset p)$ is Valid, but $\Box \Box(p \supset p)$ is not.

N fails to have the third feature, but has all the rest. So it lacks the **K** axiom. In fact, **N** is "the weakest modal logic containing propositional calculus and closed under modus ponens and necessitation." (page 350, Fitting *et. al.* [1992]). The most astonishing feature of this logic is that the rule of necessitation can be applied "to all formulas and not only to axioms." (ibid.)

We move to setting out truth-tree rules for **N**. We will use *world indexed* formulas. For example, $A(\omega)$, is understood to mean that *A is true in world ω* .

But first we should note that when an argument is tested for Validity in a truth-tree, the tree begins with the premises and the negation of the conclusion. The test of the Validity of an empty premise set argument will be exactly