On deciding consistency for CSPs of cyclic time intervals

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Abstract

We consider reasoning in an algebra of cyclic time intervals recently known in the literature; the algebra, $\mathcal{CIA}$, is somehow the cyclic time counterpart of Allen's algebra of linear time intervals, $\mathcal{LIA}$. A composition table has been built for $\mathcal{CIA}$; the table can be used by a path consistency algorithm, such as Allen's, to propagate knowledge expressed in the algebra as a constraint satisfaction problem (CSP). An important question which has not been answered so far is whether path consistency decides consistency for $\mathcal{CIA}$ atomic relations. We provide an example showing that the answer to the question is unfortunately negative: path consistency does not decide consistency for $\mathcal{CIA}$ atomic relations. We will then define an algebra of cyclic time points, $\mathcal{CPA}$, which, somehow, is for $\mathcal{CIA}$ what Vilain and Kautz's linear time point algebra, $\mathcal{LPA}$, is for $\mathcal{LIA}$. $\mathcal{CPA}$ is a subalgebra of $\mathcal{CYC}_t$, an algebra of ternary relations for cyclic ordering of 2D orientations also recently known in the literature. The pointisable part of $\mathcal{CIA}$, i.e., the part one can translate into $\mathcal{CPA}$, includes all atomic relations; from this, a complete solution search procedure for general $\mathcal{CIA}$ CSPs derives, which uses results known for the algebra $\mathcal{CYC}_t$.

Introduction

Reasoning explicitly about time and space is important for most Artificial Intelligence (AI) applications. We focus in this work on qualitative constraint-based temporal and spatial reasoning. Since Allen's (1983) influential work, which gave birth to an algebra of linear time intervals, $\mathcal{LIA}$, the field is gaining more and more importance.

We consider reasoning in an algebra of cyclic time intervals recently known in the literature (Balbiani & Osmani 2000; Hornsby, Egenhofer, & Hayes 1999), $\mathcal{CIA}$. Allen's constraint propagation algorithm (1983) can be used to propagate knowledge expressed in $\mathcal{CIA}$ as a CSP, thanks to a converse and a composition tables provided by Balbiani and Osmani (2000) for the algebra.

The first issue we will be considering is the question, left unanswered by previous work (Balbiani & Osmani 2000; Hornsby, Egenhofer, & Hayes 1999), of whether a path consistency algorithm, such as Allen's (1983), can decide consistency when the input is a $\mathcal{CIA}$ CSP of atomic relations. A positive answer to the question would mean that (1) complete knowledge can be checked for consistency in polynomial time, and (2) a general $\mathcal{CIA}$ CSP can be solved using Ladkin and Reinefeld's (1992) search procedure, which would (a) use path consistency in the preprocessing step and as the filtering method during the search, and (b) use the policy of instantiating an edge with an atomic relation at each node of the search tree. We provide an example showing that the answer to the question is unfortunately negative.

We will then consider the issue of how to get rid of this incompleteness result; specifically, the issue of finding a complete procedure for the atomic relations of the cyclic time interval algebra. We will first define an algebra of cyclic time points, $\mathcal{CPA}$, which will make explicit the link between cyclic time intervals and 2D orientations: (1) $\mathcal{CPA}$ is for $\mathcal{CIA}$ (Balbiani & Osmani 2000; Hornsby, Egenhofer, & Hayes 1999) what Vilain and Kautz's (1986) linear time point algebra, $\mathcal{LPA}$, is for $\mathcal{LIA}$ (Allen 1983); and (2) $\mathcal{CPA}$ is a subalgebra of $\mathcal{CYC}_t$, an algebra of ternary relations for cyclic ordering of 2D orientations also recently known in the literature (Isli & Cohn 2000). The pointisable part of $\mathcal{CIA}$, i.e., the part one can translate into $\mathcal{CPA}$, includes all atomic relations; from this, a complete solution search procedure for general $\mathcal{CIA}$ CSPs derives: (1) use path consistency in the preprocessing step and as the filtering method during the search, and keep the policy of instantiating an edge with an atomic relation at each node of the search tree; (2) then, when the search gets to a leaf of the search tree without detecting any inconsistency (all edges are then labelled with atomic relations and the CSP made path consistent), translate the scenario of the original CSP into $\mathcal{CPA}$ (thus into $\mathcal{CYC}_t$) and solve the resulting (ternary) CSP using a complete solution search procedure known for $\mathcal{CYC}_t$ CSPs (Isli & Cohn 2000).

Reasoning about 2D orientations is one of the privileged domains of QSR[1]; its applications include robot navigation and shape description (see Cohn's (1997) survey article on QSR techniques and applications). Applications of reasoning about cyclic intervals, on the other hand, cover reasoning about cyclic events and cyclic tasks (see (Hornsby, Egen-

[1] Qualitative Spatial Reasoning.

hofer, & Hayes 1999) for a example on land use changes).

Constraint satisfaction problems

A constraint satisfaction problem (CSP) of order n consists of the following: (1) a finite set of n variables $x_1, \ldots, x_n$; (2) a set U (called the universe of the problem); and (3) a set of constraints on values from U which may be assigned to the variables. The problem is solvable if the constraints can be satisfied by some assignement of values $a_1, \ldots, a_n \in U$ to the variables $x_1, \ldots, x_n$, in which case the sequence $(a_1, \ldots, a_n)$ is called a solution. Two problems are equivalent if they have the same set of solutions.

An m-ary constraint is of the form $R(x_{i_1}, \cdots, x_{i_m})$, and asserts that the m-tuple of values assigned to the variables $x_{i_1}, \cdots, x_{i_m}$ must lie in the m-ary relation R (an m-ary relation over the universe U is any subset of U^m). An m-ary CSP is one of which the constraints are m-ary constraints. We will be considering exclusively binary CSPs and ternary CSPs.

Operations on binary and ternary relations. For any two binary relations R and S, $R \cap S$ is the intersection of R and S, $R \cup S$ is the union of R and S, $R \circ S$ is the composition of R and S, and $R^{\smile}$ is the converse of R; the definitions of these are well-known. For ternary relations, we have additionally the rotation of a relation R, denoted by $R^{\frown}$. The definitions of the operations for ternary relations are as follows (see (Isli & Cohn 2000) for details): $R \cap S = \{(a,b,c) : (a,b,c) \in R \text{ and } (a,b,c) \in S\}$, $R \cup S = \{(a,b,c) : (a,b,c) \in R \text{ or } (a,b,c) \in S\}$, $R \circ S = \{(a,b,c) : \text{for some } d, (a,b,d) \in R \text{ and } (a,d,c) \in S\}$, $R^{\smile} = \{(a,b,c) : (a,c,b) \in R\}$, $R^{\frown} = \{(a,b,c) : (c,a,b) \in R\}$.

Constraint matrices. A binary CSP P of order n over a universe U can be associated with the following binary constraint matrix, denoted M^P: (1) initialise all entries to the universal relation: $(\forall i,j \leq n)((M^P)_{ij} \leftarrow \top_U^b)$; (2) initialise the diagonal elements to the identity relation: $(\forall i \leq n)((M^P)_{ii} \leftarrow \mathcal{I}_U^b)$; and (3) for all pairs (x_i, x_j) of variables on which a constraint $(x_i, x_j) \in R$ is specified: $(M^P)_{ij} \leftarrow (M^P)_{ij} \cap R, (M^P)_{ji} \leftarrow ((M^P)_{ij})^{\smile}$.

A ternary CSP P of order n over a universe U can be associated with the following ternary constraint matrix, denoted M^P: (1) initialise all entries to the universal relation: $(\forall i,j,k \leq n)((M^P)_{ijk} \leftarrow \top_U^t)$; (2) initialise the diagonal elements to the identity relation: $(\forall i \leq n)((M^P)_{iii} \leftarrow \mathcal{I}_U^t)$; and (3) for all triples (x_i, x_j, x_k) of variables on which a constraint $(x_i, x_j, x_k) \in R$ is specified:

$$(M^P)_{ijk} \leftarrow (M^P)_{ijk} \cap R, \quad (M^P)_{ikj} \leftarrow ((M^P)_{ijk})^{\smile},$$
$$(M^P)_{jki} \leftarrow ((M^P)_{ijk})^{\frown}, \quad (M^P)_{jik} \leftarrow ((M^P)_{jki})^{\smile},$$
$$(M^P)_{kij} \leftarrow ((M^P)_{jki})^{\frown}, \quad (M^P)_{kji} \leftarrow ((M^P)_{kij})^{\smile}.$$

We make the assumption that, unless explicitly specified otherwise, a CSP is given as a constraint matrix.

The constraint graph of a binary CSP. The constraint graph, $\mathcal{G}_P$, of a binary CSP P on n variables $x_1, \ldots, x_n$ is defined in the standard way: (1) the vertices of $\mathcal{G}_P$ are the variables of P; (2) there exists an adge of $\mathcal{G}_P$ from vertex x_i to vertex x_j if and only if P contains a constraint on the pair (x_i, x_j); and (3) the label on edge (x_i, x_j) is the relation R such that the constraint of P on the pair (x_i, x_j) is $R(x_i, x_j)$.

Strong k-consistency, refinement. Let P be a CSP of order n, V its set of variables and U its universe. An instantiation of P is any n-tuple $(a_1, a_2, \ldots, a_n)$ of U^n, representing an assignment of a value to each variable. A consistent instantiation is an instantiation $(a_1, a_2, \ldots, a_n)$ which is a solution; i.e., an instantiation satisfying all the constraints. P is consistent if it has at least one solution; it is inconsistent otherwise. The consistency problem of P is the problem of verifying whether P is consistent.

Let $V' = \{x_{i_1}, \ldots, x_{i_j}\}$ be a subset of V. The sub-CSP of P generated by V', denoted $P_{|V'}$, is the CSP with set of variables V' and whose constraint matrix is obtained by projecting the constraint matrix of P onto V'. P is k-consistent (Freuder 1982) if for any subset V' of V containing $k-1$ variables, and for any variable $X \in V$, every solution to $P_{|V'}$ can be extended to a solution to $P_{|V' \cup \{X\}}$. P is strongly k-consistent if it is j-consistent, for all $j \leq k$. 1-consistency, 2-consistency and 3-consistency correspond to node-consistency, arc-consistency and path-consistency, respectively (Mackworth 1977; Montanari 1974). Strong n-consistency of P corresponds to what is called global consistency in (Dechter 1992). Global consistency facilitates the important task of searching for a solution, which can be done, when the property is met, without backtracking (Freuder 1982). A refinement of P is a CSP P' with the same set of variables and such that: (1) $(\forall i,j)((M^{P'})_{ij} \subseteq (M^P)_{ij})$, in the case of P being binary; and (2) $(\forall i,j,k)((M^{P'})_{ijk} \subseteq (M^P)_{ijk})$, in the case of P being ternary.

The algebra of cyclic time intervals

The algebra of cyclic time intervals (Balbiani & Osmani 2000; Hornsby, Egenhofer, & Hayes 1999), $\mathcal{CIA}$, is the cyclic time counterpart of Allen's (1983) well-known algebra of linear time intervals, $\mathcal{LIA}$. Throughout the rest of the paper, we make the assumption that the 2D space is associated with a reference system (O, x, y), and refer to the circle centred at O and of unit radius as $\mathcal{C}_{O,1}$. In order to describe $\mathcal{CIA}$, we can consider any fixed circle as the model of cyclic time; in particular, we can, and do, consider that cyclic time is modelled by circle $\mathcal{C}_{O,1}$. Cyclic time intervals, or c-intervals for short, are in this way arcs of $\mathcal{C}_{O,1}$. A c-interval I will be represented as $I = (I-, I+)$, where $I-$ is the starting endpoint of I and $I+$ is the finishing endpoint of I. The two endpoints of a c-interval are supposed distinct, i.e., a c-interval is durative (not of null length), and cannot cover the entire cyclic time; furthermore, we assume that a c-interval is directed in an anticlockwise direction, i.e., cyclic time is supposed to flow in an anticlockwise direction. Summarised, a c-interval $I = (I-, I+)$ satisfies the following: (1) $I- \neq I+$; and (2) we move in an anticlockwise direction when we scan the c-interval from its starting endpoint to its finishing endpoint; in other words, using the unique relation, *cyc*, of the CYCORD theory (Megiddo 1976; Röhrig 1994), which we will see shortly, if P is a point strictly inside I then we have $cyc(I+, P, I-)$.

We refer to the set of all c-intervals, or, equivalently, to the set of all (anticlockwisely) directed arcs of $\mathcal{C}_{O,1}$, as cI. Two intervals of linear time can stand in one and only one of 13 possible qualitative configurations; these 13 possible configurations correspond to the 13 atomic relations of Allen's (1983) algebra $\mathcal{LIA}$. In the case of cyclic time, there are 16 possible qualitative configurations of two c-intervals, corresponding to the 16 atomic relations of the cyclic time interval algebra $\mathcal{CIA}$ (Balbiani & Osmani 2000; Hornsby, Egenhofer, & Hayes 1999). These 16 atomic relations are *dc* (disconnected), *m* (meets), *o* (overlaps), *fi* (finished-by), *di* (contains), *s* (starts), *eq* (equals), *si* (started-by), *d* (during), *f* (finishes), *oi* (overlapped-by), *moi* (meets and overlapped-by), *ooi* (overlaps and overlapped-by), *mi* (met-by), *mmi* (meets and met-by), *omi* (overlaps and met-by). If c-ntervals I and J are such that I overlaps J, we represent the configuration as $o(I,J)$ or as $(I,J) \in o$; generally, if I is related to J by the atomic relation r, we represent the situation as $r(I,J)$ or as $(I,J) \in r$. An illustration of the 16 atomic relations of $\mathcal{CIA}$ can be found in (Balbiani & Osmani 2000; Hornsby, Egenhofer, & Hayes 1999). A (general) relation of $\mathcal{CIA}$, say R, is any subset of the set $\mathcal{CIA}$-at of all $\mathcal{CIA}$ atomic relations: $(\forall I,J)(R(I,J) \Leftrightarrow \bigvee_{r \in R} r(I,J))$. The $\mathcal{CIA}$ converse table and composition table can be found in (Balbiani & Osmani 2000).

The algebra of 2D orientations

We refer to the set of 2D orientations as $2D\mathcal{O}$. Two natural isomorphisms will be of use in the rest of the paper. In order to facilitate their definitions, we refer to the set of all directed lines containing O as $d\mathcal{L}_O$.

Definition 1 *The isomorphisms $\mathcal{I}_1$ and $\mathcal{I}_2$ are defined as follows:*

1. $\mathcal{I}_1 : 2D\mathcal{O} \to \mathcal{C}_{O,1}$; *$\mathcal{I}_1(z)$ is the point $P_z \in \mathcal{C}_{O,1}$ such that the orientation of the vector $\overrightarrow{OP_z}$ is z.*
2. $\mathcal{I}_2 : 2D\mathcal{O} \to d\mathcal{L}_O$; *$\mathcal{I}_2(z)$ is the line $\ell_{O,z} \in d\mathcal{L}_O$ of orientation z.*

Definition 2 *The angle determined by two directed lines D_1 and D_2, denoted (D_1, D_2), is the one corresponding to the move in an anticlockwise direction from D_1 to D_2. The angle (z_1, z_2) determined by orientations z_1 and z_2 is the angle $(\ell_{O,z_1}, \ell_{O,z_2})$, where $\ell_{O,z_1} = \mathcal{I}_2(z_1)$ and $\ell_{O,z_2} = \mathcal{I}_2(z_2)$.*

The set $2D\mathcal{O}$ can thus be viewed as the set of points of $\mathcal{C}_{O,1}$ (or of any fixed circle), or as the set of directed lines containing O (or any fixed point).

Isli and Cohn (2000) have defined two algebras of orientations of the 2-dimensional space: one of binary relations, the other of ternary relations. The binary algebra, $\mathcal{CYC}_b$, contains four atomic relations: e (equal), l (left), o (opposite) and r (right); these are interpreted as follows: $(\forall x, y \in 2D\mathcal{O})(e(y,x) \Leftrightarrow (x,y) = 0)$; $(\forall x, y \in 2D\mathcal{O})(l(y,x) \Leftrightarrow (x,y) \in (0,\pi))$; $(\forall x, y \in 2D\mathcal{O})(o(y,x) \Leftrightarrow (x,y) = \pi)$; $(\forall x, y \in 2D\mathcal{O})(r(y,x) \Leftrightarrow (x,y) \in (\pi, 2\pi))$. A (general) relation of $\mathcal{CYC}_b$ is any subset, say R, of the set $\mathcal{CYC}_b$-at of all $\mathcal{CYC}_b$ atomic relations: $(\forall x, y \in 2D\mathcal{O})(R(y,x) \Leftrightarrow \bigvee_{r \in R} r(y,x))$. The algebra $\mathcal{CYC}_b$ is very similar in structure to Allen's (Allen 1983) algebra, $\mathcal{LIA}$, and, indeed, to the cyclic time interval algebra (Balbiani & Osmani 2000; Hornsby, Egenhofer, & Hayes 1999), $\mathcal{CIA}$: for each of the three, Allen's (1983) well-known propagation algorithm can be used to propagate knowledge expressed in the algebra as a CSP. Allen's algorithm decides consistency for $\mathcal{LIA}$ CSPs of atomic relations (van Beek 1992; van Beek & Cohen 1990). As shown in (Isli & Cohn 2000), path consistency does not decide consistency for $\mathcal{CYC}_b$ CSPs of atomic relations. Finally, as we show in the present work, path consistency does not decide consistency for $\mathcal{CIA}$ CSPs of atomic relations.

The algebra $\mathcal{CYC}_b$ cannot express the relation *cyc* of the CYCORD theory (Megiddo 1976; Röhrig 1994), known to be important for robot navigation, one of the privileged application domains of QSR (Qualitative Spatial Reasoning): the relation *cyc* can be used to represent the panorama of a robot. Based on $\mathcal{CYC}_b$, Isli and Cohn (2000) have built an algebra of ternary relations, $\mathcal{CYC}_t$, for cyclic ordering of 2D orientations: $\mathcal{CYC}_t$ has 24 atomic relations, thus 2^{24} (general) relations, and the unique relation *cyc* of the CYCORD theory (Megiddo 1976; Röhrig 1994) is just one particular relation of $\mathcal{CYC}_t$. The atomic relations of $\mathcal{CYC}_t$ are written as $b_1b_2b_3$, where b_1, b_2, b_3 are atomic relations of $\mathcal{CYC}_b$, and such an atomic relation is interpreted as follows: $(\forall x, y, z \in 2D\mathcal{O})(b_1b_2b_3(x,y,z) \Leftrightarrow b_1(y,x) \wedge b_2(z,y) \wedge b_3(z,x))$. An illustration of the 24 atomic relations can be found in (Isli & Cohn 2000). A (general) relation of $\mathcal{CYC}_t$ is any subset, say R, of the set $\mathcal{CYC}_t$-at of all 24 atomic relations: $(\forall x, y, z \in 2D\mathcal{O})(R(x,y,z) \Leftrightarrow \bigvee_{r \in R} r(x,y,z))$. The $\mathcal{CYC}_t$ converse table, rotation table and composition tables can be found in (Isli & Cohn 2000).

CSPs of c-intervals and CSPs of 2D orientations

We define a $\mathcal{CIA}$ CSP as a CSP of which the constraints are $\mathcal{CIA}$ relations on pairs of the variables; a $\mathcal{CYC}_t$ CSP as a CSP of which the constraints are $\mathcal{CYC}_t$ relations on triples of the variables. For $\mathcal{CIA}$ CSPs, the universe is the set cI of c-intervals; for $\mathcal{CYC}_t$ CSPs, the universe is the set $2D\mathcal{O}$ of 2D orientations. We use the term $\mathcal{CYC}$-CSP to refer to a CSP which is either a $\mathcal{CIA}$ CSP or a $\mathcal{CYC}_t$ CSP.

A $\mathcal{CIA}$-matrix (resp. $\mathcal{CYC}_t$-matrix) of order n is a constraint matrix of order n of which the entries are $\mathcal{CIA}$ (resp. $\mathcal{CYC}_t$) relations. The constraint matrix associated with a $\mathcal{CIA}$ CSP (resp. $\mathcal{CYC}_t$ CSP) is a $\mathcal{CIA}$-matrix (resp. $\mathcal{CYC}_t$-matrix).

A scenario of a $\mathcal{CYC}$-CSP is a refinement P' such that all entries of $M^{P'}$ are atomic relations. A consistent scenario is a scenario which is consistent. An atomic $\mathcal{CYC}$-CSP is a $\mathcal{CYC}$-CSP of which all entries of the constraint matrix are atomic relations.

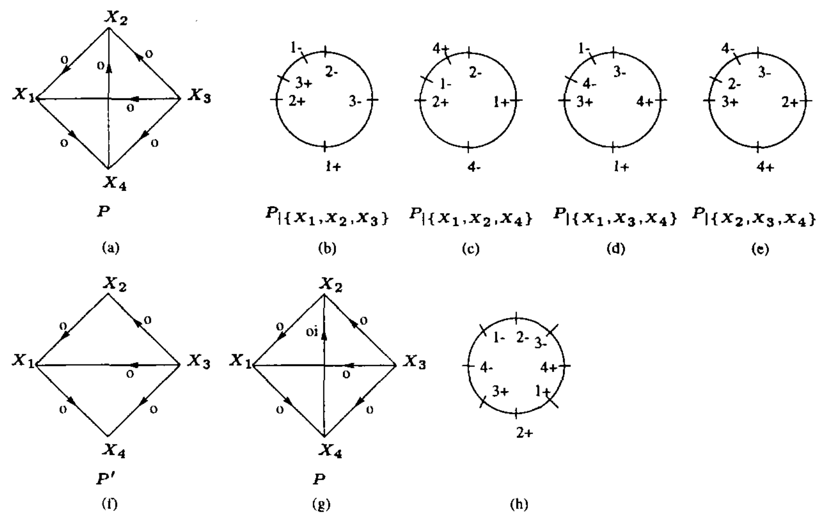

Figure 1: Incompleteness of path consistency for $\mathcal{CIA}$ atomic relations.

Incompleteness of path consistency

Balbiani and Osmani (2000) have built a composition table for the algebra $\mathcal{CIA}$. The table can be used by a constraint propagation algorithm, such as Allen's (1983), to propagate knowledge expressed in $\mathcal{CIA}$ as a $\mathcal{CIA}$ CSP. Once such an algorithm applied to such a CSP, say P, has completed, the CSP P verifies the following: (1) $(\forall i,j)(((M^P)_{ij})^{\smile} = (M^P)_{ji})$; and (2) $(\forall i,j,k)((M^P)_{ij} \subseteq (M^P)_{ik} \circ (M^P)_{kj})$. In other words, the CSP P is made path consistent. Balbiani and Osmani (2000) have shown that when the input CSP is "nice", path consistency decides its consistency. Since it is not the case that all $\mathcal{CIA}$ atomic CSPs are nice, one question not answered by previous work (Balbiani & Osmani 2000; Hornsby, Egenhofer, & Hayes 1999) is whether path consistency decides consistency for $\mathcal{CIA}$ atomic CSPs. For an Allen-style algebra, a positive answer to such a question is important for at least two reasons: (1) complete knowledge can be checked for consistency in polynomial time (in the case of $\mathcal{CIA}$, complete knowledge describes a scene of c-intervals where we know precisely the $\mathcal{CIA}$ atomic relation holding on each pair of the involved c-intervals; complete knowledge coincides thus with a $\mathcal{CIA}$ atomic CSP); and (2) a general $\mathcal{CIA}$ CSP can be solved using a backtracking search algorithm such as Ladkin and Reinefeld's (1992). We show that the answer to the question of whether path consistency decides consistency for $\mathcal{CIA}$ atomic CSPs is unfortunately negative; this is done in the following example which provides a $\mathcal{CIA}$ atomic CSP that is path consistent but inconsistent.

Example 1 *Consider the $\mathcal{CIA}$ CSP P given as a constraint graph in Figure 1(a). Figures 1(b–e) provide for each of the four 3-variable sub-CSPs of P, $P_{|\{X_i,X_j,X_k\}}, i,j,k \in \{1,2,3,4\}$, a consistent instantiation (the instantiation of variable $X_i, i = 1\ldots4$, is given as a c-interval $(i-,i+)$). Thus all 3-variable sub-CSPs of P are consistent, which derives straightforwardly from path consistency of P: $(\forall i,j,k)((M^P)_{ij} \subseteq (M^P)_{ik} \circ (M^P)_{kj})$. Path consistency of P, on the other hand, can be checked using Balbiani and Osmani's (2000) composition table: for the purpose of the example, the only thing that needs to be known is that $\{o, oi\} \subseteq o \circ oi$. The point now is that none of the consistent instantiations provided in Figures 1(b–e) can be consistently extended to the missing fourth variable of P. Indeed, if we consider the CSP P' of Figure 1(f), which differs from P in that the relation on the pair (X_4, X_2) is not o but the (unshown) universal relation (P is thus a scenario of P'), P' is consistent and its unique consistent scenario is the one illustrated in Figure 1(g), for which a consistent instantiation is given in Figure 1(h): all edges other than (X_4, X_2) are labelled as in the CSP P (Figure 1(a)); the edge (X_4, X_2) is labelled with oi.*

From Example 1, we get:

Theorem 1 *Path consistency does not decide consistency for $\mathcal{CIA}$ atomic CSPs.*

As a consequence, we cannot use path consistency to check complete knowledge for consistency, and we cannot use Ladkin and Reinefeld's (1992) search algorithm (which uses path consistency as the filtering method during the search) to solve a general $\mathcal{CIA}$ CSP.

The algebra of cyclic time points

In this section, we present a cyclic time point algebra ($\mathcal{CPA}$), which somehow represents for the cyclic time interval algebra ($\mathcal{CIA}$) (Balbiani & Osmani 2000; Hornsby, Egenhofer, & Hayes 1999) what Vilain and Kautz's (1986) linear time point algebra ($\mathcal{LPA}$) represents for Allen's (1983) linear time interval algebra ($\mathcal{LIA}$). The relations of $\mathcal{CPA}$ are ternary relations on (triples of) points of cyclic time: we keep supposing that cyclic time is modelled by $\mathcal{C}_{O,1}$.

The $\mathcal{CPA}$ relations. Consider a configuration of two

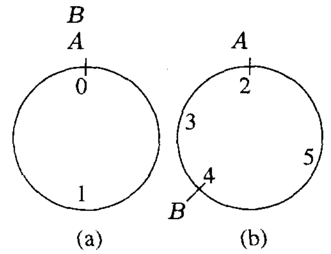

Figure 2: Illustration of the 6 $\mathcal{CPA}$ atomic relations.

points, A and B, of cyclic time: A and B are either equal or distinct from each other (see Figure 2).

1. Case 1 (A and B equal). This gives rise to two atomic relations, $=^{=}$ and $=^{\neq}$, which correspond, respectively, to the regions marked 0 and 1 in Figure 2(a) (region 0 is the location common to A and B, and region 1 is the rest of circle $\mathcal{C}_{O,1}$). Given a third point C of $\mathcal{C}_{O,1}$, we have $=^{=}$ (A, B, C) if C belongs to region 0, and $=^{\neq}$ (A, B, C) if C belongs to region 1.
2. Case 2 (A and B distinct from each other). This gives rise to four atomic relations, $\neq^{=_1}$, $\neq^{bac}$, $\neq^{=_2}$ and $\neq^{bc}$, which correspond, respectively, to the regions marked 2, 3, 4 and 5 in Figure 2(b) (region 2 is the location of A; region 3 is the open arc of $\mathcal{C}_{O,1}$ corresponding to the move in an anti-clockwise direction from A to B; region 4 is the location of B; and region 5 is the open arc of $\mathcal{C}_{O,1}$ corresponding to the move in a clockwise direction from A to B). Given a third point C of $\mathcal{C}_{O,1}$, we have $\neq^{=_1}$ (A, B, C), $\neq^{bac}$ (A, B, C), $\neq^{=_2}$ (A, B, C) or $\neq^{bc}$ (A, B, C) depending on whether C belongs to region 2, to region 3, to region 4 or to region 5.

A (general) relation of $\mathcal{CPA}$, say R, is any subset of the set $\mathcal{CPA}$-at $= \{=^{=}, =^{\neq}, \neq^{=_1}, \neq^{bac}, \neq^{=_2}, \neq^{bc}\}$ of all $\mathcal{CPA}$ atomic relations: $(\forall x, y, z)(R(x, y, z) \Leftrightarrow \bigvee_{r \in R} r(x, y, z))$. Since $\mathcal{CPA}$ is a subalgebra of Isli and Cohn's (2000) $\mathcal{CYC}_t$, the $\mathcal{CPA}$ converse, rotation and composition tables can be obtained in a straightforward manner from the $\mathcal{CYC}_t$ converse, rotation and composition tables (Isli & Cohn 2000).

We define the pointisable part, $p\mathcal{CIA}$, of $\mathcal{CIA}$ as the set of all $\mathcal{CIA}$ relations one can translate into a conjunction of $\mathcal{CPA}$ relations on endpoints of the involved intervals. The enumeration of $p\mathcal{CIA}$ can be found in the longer version of this work (Isli 2000).

The solution search procedure

We are now in a position to present a complete solution search procedure for $\mathcal{CIA}$ CSPs: (1) use Ladkin and Reinefeld's (1992) procedure to search for a path consistent scenario: use of path consistency in the preprocessing step and as the filtering method during the search, and the policy of instantiating an edge with an atomic relation at each node of the search tree; (2) when the search reaches a leaf without detecting any inconsistency (we are then in the presence of a path consistent scenario), translate the scenario of the original CSP into $\mathcal{CPA}$ (thus into $\mathcal{CYC}_t$) and use Isli and Cohn's (2000) complete search procedure to solve the resulting $\mathcal{CYC}_t$ CSP.

Summary

We have considered reasoning in an algebra of cyclic time intervals, $\mathcal{CIA}$, recently known in the literature (Balbiani & Osmani 2000; Hornsby, Egenhofer, & Hayes 1999). We have shown that path consistency does not decide consistency for the $\mathcal{CIA}$ subset consisting only of the atomic relations. This means that we cannot use path consistency to check consistency of complete knowledge expressed in $\mathcal{CIA}$ as a CSP, and we cannot use the well-known Ladkin and Reinefeld's (1992) search procedure to solve a general $\mathcal{CIA}$ CSP: completeness of Ladkin and Reinefeld's procedure is lost because of the incompleteness of path consistency for $\mathcal{CIA}$ atomic relations (Theorem 1). We have then shown how to get a complete search procedure for $\mathcal{CIA}$ CSPs by combining Ladkin and Reinefeld's (1992) search procedure with Isli and Cohn's (2000) complete search procedure for ternary CSPs expressed in another algebra, $\mathcal{CYC}_t$, also recently known in the literature: (1) use Ladkin and Reinefeld's procedure (Ladkin & Reinefeld 1992) with its policy of using path consistency in the preprocessing step and as the filtering method during the search, and of instantiating an edge with an atomic relation at each node of the search tree; and (2) when the search reaches a leaf without detecting any inconsistency (we are then in the presence of a path consistent scenario), translate the scenario of the original CSP into the cyclic time point algebra $\mathcal{CPA}$, also defined in this work, and is a subalgebra of the algebra $\mathcal{CYC}_t$ of 2D orientations (Isli & Cohn 2000), and use Isli and Cohn's complete search procedure (Isli & Cohn 2000) to solve the resulting $\mathcal{CPA}$ CSP.

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