

A Logical Framework for Frequent Pattern Discovery in Spatial Data

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Abstract

In recent times, several extensions of data mining methods and techniques have been explored aiming at dealing with advanced databases. Many promising applications of inductive logic programming (ILP) to knowledge discovery in databases have also emerged in order to benefit from semantics and inference rules of first-order logic. In this paper, an ILP framework for frequent pattern discovery in spatial data is presented. The pattern discovery algorithm operates on first-order logic descriptions computed by an initial step of feature extraction from a spatial database. The algorithm benefits of the available background knowledge on the spatial domain and systematically explores the hierarchical structure of task-relevant geographic layers. Preliminary results have been obtained by running the algorithm SPADA on spatial data from an Italian province.

1 Introduction

In recent times, several extensions of data mining methods and techniques have been explored to deal with advanced databases such as spatial databases, temporal databases, object-oriented databases and multimedia databases. Progress in spatial databases, such as spatial data structures (Gütting 1994), spatial reasoning (Egenhofer 1991), and computational geometry (Preparata and Shamos, 1985), etc., paved the way for the study of knowledge discovery in spatial databases which aims at the *extraction of implicit knowledge, spatial relations, or other patterns not explicitly stored in spatial databases* (Koperski, Adhikary and Han 1996). Generally speaking, a *spatial pattern* is a pattern showing the interaction of two or more spatial objects or space-depending attributes according to a particular spacing or set of arrangements (DeMers 2000). For instance, cities across nations are often clustered near lakes, oceans and streams. Actually such an arrangement reveals a *spatial association*, meaning that one spatial pattern is totally or partially related to some other spatial pattern. Furthermore, questions can be raised about the causes not only of single distributions but also of spatially correlated distributions of phenomena. For instance, we may explain that the tendency of cities to cluster near water bodies is driven by the need for sources of drinking water

and recreation and for commerce facilities. Thus, once the language of geography has been acquired, the major tasks among geographers are to observe the relevant spatial features, to identify spatial patterns, to describe and quantify spatial associations and to elicit explanations for pattern interactions. With the advent of geographical information systems (GIS), advanced functionalities of spatial data mining such as frequent pattern discovery are of great interest to GIS users.

The design of algorithms for frequent pattern discovery has turned out to be a popular topic in data mining. This is not surprising given the relevance of *data* and *patterns* in the definition of data mining as a core step in the KDD process (Fayyad, Piatetsky-Shapiro, Smyth 1996). The blueprint for most algorithms proposed in the literature is the *levelwise method* by Mannila and Toivonen (1997), which is based on a breadth-first search in the lattice spanned by a generality order between patterns. The space is searched one level at a time, starting from the most general patterns and iterating between *candidate generation* and *candidate evaluation* phases. Frequent patterns are commonly not considered useful for presentation to the user as such. They can be efficiently post-processed into rules that exceed given threshold values. In the case of *association rules* the threshold values of support and confidence offer a natural way of pruning weak and rare rules (Agrawal and Srikant 1994).

In this paper, we propose a logical framework for frequent pattern discovery in spatial data. The main novelty with respect to previous contributions to spatial data mining (Koperski and Han 1995) is the expressive power of the language chosen for representing both data and patterns. Indeed, the research to date in the field has generally taken the path of merely embedding spatial constructs on the top of well-established statistical techniques in order to accommodate the space dimension (Roddick and Spiliopoulou 1999). We claim the application of Inductive Logic Programming (ILP) methods and techniques (Lavrac and Dzeroski 1994) to knowledge discovery in spatial databases in order to benefit from semantics and inference rules of first-order logic.

The paper is organized as follows. Section 2 will introduce the task of mining spatial association rules viewed as context for frequent pattern discovery in spatial data. In Section 3, representation, problem and algorithmic issues in the ILP approach to the task at hand will be

discussed and illustrated by means of a sample task of frequent pattern discovery in data of an Italian province. Conclusions and future work are given in Section 4.

2 The mining task

The discovery of spatial association rules is a descriptive mining task aiming at the detection of associations between *reference objects* and some *task-relevant objects*, the former being the main subject of the description while the latter being spatial objects that are relevant for the task at hand and spatially related to the former. The discovery process may be activated by a user query expressed in a database mining query language such as

```
MINE ASSOCIATIONS DESCRIBING "large_towns"
WITH RESPECT TO topology(T.geo, R.geo), R.name,
topology(T.geo, W.geo), W.name, topology(T.geo, B.geo),
B.admin_region2
FROM town T, road R, water W, boundary B
WHERE T.type="large" AND distance(T.geo, R.geo) < "5 km"
      AND distance(T.geo, W.geo) < "5 km"
      AND distance(T.geo, B.geo) < "30 km"
```

where large towns play the role of reference objects while roads, water bodies and boundaries play the role of geographic layers from which task-relevant objects are taken. Query processing involves massive spatial computation to extract spatial relations from the underlying spatial database. Some kind of taxonomic knowledge on task-relevant geographic layers may also be taken into account to get descriptions at different concept levels (*multiple-level association rules*). As usual in the problem setting of association rule mining, we search for associations with large support and high confidence (*strong rules*).

Formally, the problem can be stated as follows:

Given

- a spatial database SDB,
- a set of reference objects S ,
- some task-relevant geographic layers R_k , $1 \leq k \leq m$, together with spatial hierarchies defined on them,
- a couple of thresholds for each level l in the spatial hierarchies, $\text{minsup}[l]$ and $\text{minconf}[l]$

Find strong multiple-level spatial association rules.

The most representative work in the literature for the mining task of interest is the progressive-refinement method by Koperski and Han (1995). It relies on the so-called attribute-value approach (AV) to data mining, namely it can deal only with data represented by attribute-value couples. It is noteworthy that the AV approach to spatial data mining suffers from the following limits:

- Patterns are represented by languages with the expressive power of propositional logic
- Domain knowledge is represented in a poor form
- Mining is restricted to a single relation/data file

The ILP approach promises to overcome these limits. To the best of our knowledge, the only contribution from ILP to spatial data mining is the system GwiM (Popelinski

1998). Anyway, no insight in the algorithmic issues has been provided. A proposal of logical framework inspired to the work on mining association rules from multiple relations by Dehaspe and De Raedt (1997) is sketched in the following Section.

3 The logical framework

The basic idea in our proposal of logical framework is that a spatial database boils down to a deductive relational database (DDB) once the spatial relationships between reference objects and task-relevant objects have been extracted. Indeed, DDBs define relations both extensionally as ground facts (extensional database, EDB) and intensionally as rules (intensional database, IDB). Thus, the expressive power of first-order logic in databases allows to specify background knowledge (BK) such as spatial hierarchies, spatial constraints and rules for spatial qualitative reasoning.

3.1 Representation issues

Let $L = \{a_1, a_2, \dots, a_l\}$ a set of Datalog atoms of the form $p(t_1, \dots, t_n)$, where each term t_i may be either a variable or a constant (Ceri, Gottlob and Tanca 1989). A conjunction of atoms is named *atomset*. In our framework patterns are represented as atomsets. Since the ILP approach operates in the context of a DDB, we denote the DDB at hand $D(S)$ to mean that it is obtained by adding spatial relations extracted from SDB as concerns the set of reference objects S to the previously supplied BK. The tuples in $D(S)$ can be grouped into distinct subsets: Each group, uniquely identified by the corresponding reference object $s \in S$, is called *spatial observation* and denoted $O[s]$. Actually, a spatial observation is multi-key, namely it contains not only spatial relations between the reference object $s \in S$ and some task-relevant object $r_i \in R_k$ but also spatial relations between r_i and some $s' \in S$. Thus, a spatial observation is given by

$$O[s] = O[s|s] \cup \{O[r_i|s] \mid \exists \text{ tuple } \theta \in D(S): \theta(s, r_i)\} \quad 1 \leq i \leq n$$

where $O[r_i|s]$ is the observation with key r_i given s .

Example 1 Suppose the mining task is to discover associations relating large towns (S) with water bodies (R_1), roads (R_2) and province boundaries (R_3) in the Province of Bari, Italy. We are also given a BK including the spatial hierarchies of interest (see Figure 1 for a graphical representation of the layer of roads).

```
spatial_hierarchy(town, 1, null, [town]).
spatial_hierarchy(town, 2, town, [large_town,
medium_size_town, small_town]).
spatial_hierarchy(town, 3, large_town, [bari, altamura, andria,
barletta, trani, bitonto, molfetta, gravina, monopoli, corato,
gioia_del_colle]).
spatial_hierarchy(town, 3, medium_size_town, [modugno,
palo_del_colle, terlizzi, ruvo, noicattaro, adelfia, grumo,
giovinazzo, mola_di_bari]).
spatial_hierarchy(town, 3, small_town, [palese, bitetto, binetto,
toritto, valenzano, cassano, mariotto, palombaio]).
```

confidence at level l in the spatial hierarchies. An atomset C is *large* (or frequent) at level l if $\sigma(C) \geq \text{minsup}[l]$ and all ancestors of C with respect to the taxonomies are large at their corresponding levels. The confidence of a spatial association rule $A \rightarrow B$ is high at level l if $\phi(B|A) \geq \text{minconf}[l]$. A spatial association rule $A \rightarrow B$ is *strong* at level l if the atomset $A \cup B$ is large and the confidence is high at level l .

3.2 Problem issues

Within the ILP approach, the problem of mining spatial association rules can be decomposed into four sub-problems:

- 1) Extract spatial relationships between reference objects and task-relevant objects
- 2) Represent each extracted relationship as atom
- 3) Find large (or frequent) atomsets
- 4) Generate highly-confident spatial association rules

Both the problem statement and the problem solution are quite complicated since the spatial domain is inherently complex. The preliminary feature extraction step may be performed by the two-step spatial computation proposed by Koperski and Han (1995). Such a pre-processing is necessary for saving computational effort both in time (on-line computation of spatial relations) and in space (materialization of spatial relations). The relations returned by the spatial computation are represented as facts to be inserted into $D(S)$. A solution to the third sub-problem (frequent pattern discovery) is illustrated in the following Section. The sub-problem of generating highly-confident rules from frequent patterns is solved as usual in the problem setting of association rule mining (Agrawal and Srikant 1994).

3.3 Algorithmic issues

The algorithm SPADA (Spatial Pattern Discovery Algorithm) being proposed for frequent pattern discovery in spatial data implements the aforementioned levelwise method (see Figure 2). It can be considered as an extension of WARMR (Dehaspe and De Raedt 1997) to explore systematically the hierarchical structure of task-relevant geographic layers. The pattern space is structured according to θ -subsumption (Plotkin 1970). The *candidate generation* phase consists of a refinement step followed by a pruning step. The former applies a specialization operator under θ -subsumption to patterns previously found frequent by preserving the property of linkedness (Helft 1987). The latter involves verifying that candidate patterns do not θ -subsume any infrequent pattern. The *candidate evaluation* phase is performed by comparing the support of the candidate pattern with the minimum support threshold set for the level being explored. If the pattern turns out not to be a large one, it is rejected. As for the support count, the candidate is transformed into an existential query whose answer set supplies all the substitutions that make the pattern true in $D(S)$. In particular, the number of different bindings for the variable which is the placeholder for

reference objects is assumed as absolute frequency of the pattern in $D(S)$. It is noteworthy that the property of linkedness guarantees the equivalence between the absolute frequency of a pattern and the number of observations covered by the pattern. The support is obtained as relative frequency of the pattern in $D(S)$.

Cycle on the level ($l \geq 1$) of the spatial hierarchies
 Find large 1-atomsets at level l
Cycle on the size ($k > 1$) of the atomsets
 Generate candidate k -atomsets at level l
 from large $(k-1)$ -atomsets

 Generate large k -atomsets at level l
 from candidate k -atomsets
 until no more large atomsets are found.

Figure 2. The algorithm SPADA

A rough preliminary remark on the computational complexity of SPADA leads to the notorious trade-off between expressivity and efficiency in first-order representations. Indeed, it is well known that a simple matching of two expressions with commutative and associative operators (such as the logical OR of atoms in a clause) is NP-complete (Garey and Johnson 1979). Therefore, any known algorithm that checks the coverage of an atomset or equivalently that evaluates a query with respect to a relational database has an exponential complexity. Nevertheless, it has been also proved that queries with up to k atoms, where each atom contains at most j terms, can be evaluated in polynomial time (De Raedt and Dzeroski 1994). Whether these constraints are applicable to the domain of spatial data analysis is still under investigation.

Example 3 The algorithm SPADA has been run on the mining task in Example 1 with support thresholds $\text{minsup}[1]=70\%$, $\text{minsup}[2]=68\%$, and $\text{minsup}[3]=50\%$. Some interesting patterns have been discovered. For instance, at level $l=2$ in the spatial hierarchies, the following candidate C :

$\text{is_a}(X, \text{large_town}), \text{intersects}(X, R), \text{is_a}(R, \text{main_trunk_road}),$
 $\text{intersects}(Y, R), \text{diff}(Y, X), \text{is_a}(Y, \text{large_town})$

has been generated after $k=5$ refinement steps and evaluated with respect to $D(S)$ by means of the query:

?- $\text{is_a}(X, \text{large_town}), \text{intersects}(X, R), \text{is_a}(R, \text{main_trunk_road}),$
 $\text{intersects}(Y, R), \text{diff}(Y, X), \text{is_a}(Y, \text{large_town})$

The answer set includes two substitutions, $\theta_1 = \{X \backslash \text{barletta}, R \backslash \text{ss16}, Y \backslash \text{bari}\}$ and $\theta_2 = \{X \backslash \text{barletta}, R \backslash \text{ss16bis}, Y \backslash \text{bari}\}$. Therefore, the spatial observation $O[\text{barletta}]$, shown in Example 2, is covered. However, while computing the support, the two substitutions count as only one because both refer to the same large town. Since ten of eleven spatial observations are covered and all the ancestor patterns are large at their level ($l \leq 2$), the pattern is a large one at level $l=2$ with support 91%. For the sake of clarity, the following pattern discovered after $k=5$ refinement steps at level $l=1$

$\text{is_a}(X, \text{large_town}), \text{intersects}(X, R), \text{is_a}(R, \text{road}),$
 $\text{intersects}(Y, R), \text{diff}(Y, X), \text{is_a}(Y, \text{large_town})$

```

spatial_hierarchy(road, 1, null, [road]).
spatial_hierarchy(road, 2, road, [motorway, main_trunk_road,
regional_road]).
spatial_hierarchy(road, 3, motorway, [a14]).
spatial_hierarchy(road, 3, main_trunk_road, [ss16, ss16bis, ss96,
ss98, ss99, ss100]).
spatial_hierarchy(road, 3, regional_road, [r16, r93, r97, r170,
r171, r172, r271, r378]).
spatial_hierarchy(water, 1, null, [water]).
spatial_hierarchy(water, 2, water, [sea, river]).
spatial_hierarchy(water, 3, sea, [adriatico]).
spatial_hierarchy(water, 3, river, [ofanto, lacone]).
spatial_hierarchy(boundary, 1, null, [boundary]).
spatial_hierarchy(boundary, 2, boundary, [fg_boundary,
ta_boundary, br_boundary, mt_boundary, pz_boundary]).
is_a(X, Y) :- spatial_hierarchy(_, _, Y, Nodes),
member(X, Nodes).
is_a(X, Y) :- spatial_hierarchy(Root, _, Father, Nodes),
member(X, Nodes), is_a(Father, Y).

```

Here, the *is-a* relationship is overloaded, namely it may stand for *kind-of* as well as for *instance-of* depending on the context. Spatial relations between objects in S and objects in any of R_1 , R_2 and R_3 , are extracted by means of spatial computation and transformed into facts of kind $\langle \text{spatial relation} \rangle (\text{RefObj}, \text{TaskRelevantObj})$ to be added to $D(S)$. Spatial observations are portions of $D(S)$, each concerning a reference object. In our case, there are eleven distinct spatial observations, one for each large town. For instance, $O[\text{barletta}]$ is given by the union of the following sets of ground facts

| | |
|--|---|
| $O[\text{barletta} \mid \text{barletta}]$ | $O[\text{ss16} \mid \text{barletta}]$ |
| $\text{is_a}(\text{barletta}, \text{large_town}).$ | $\text{is_a}(\text{ss16}, \text{road}).$ |
| $\text{adjacent_to}(\text{barletta}, \text{adriatico}).$ | $\text{intersects}(\text{bari}, \text{ss16}).$ |
| $\text{intersects}(\text{barletta}, \text{a14}).$ | $\text{intersects}(\text{trani}, \text{ss16}).$ |
| $\text{intersects}(\text{barletta}, \text{ss16}).$ | $\text{intersects}(\text{monopoli}, \text{ss16}).$ |
| $\text{intersects}(\text{barletta}, \text{ss16bis}).$ | $\text{intersects}(\text{molfetta}, \text{ss16}).$ |
| $\text{intersects}(\text{barletta}, \text{r170}).$ | ... |
| $\text{intersects}(\text{barletta}, \text{r193}).$ | $O[\text{ss16bis} \mid \text{barletta}]$ |
| $\text{close_to}(\text{barletta}, \text{fg_boundary}).$ | $\text{is_a}(\text{ss16bis}, \text{road}).$ |
| ... | $\text{intersects}(\text{bari}, \text{ss16bis}).$ |
| $O[\text{adriatico} \mid \text{barletta}]$ | $\text{intersects}(\text{trani}, \text{ss16bis}).$ |
| $\text{is_a}(\text{adriatico}, \text{water}).$ | $\text{intersects}(\text{molfetta}, \text{ss16bis}).$ |
| $\text{adjacent_to}(\text{bari}, \text{adriatico}).$ | ... |
| $\text{adjacent_to}(\text{trani}, \text{adriatico}).$ | $O[\text{r170} \mid \text{barletta}]$ |
| $\text{adjacent_to}(\text{molfetta}, \text{adriatico}).$ | $\text{is_a}(\text{r170}, \text{road}).$ |
| $\text{adjacent_to}(\text{monopoli}, \text{adriatico}).$ | $\text{intersects}(\text{andria}, \text{r170}).$ |
| ... | ... |
| $O[\text{a14} \mid \text{barletta}]$ | $O[\text{r193} \mid \text{barletta}]$ |
| $\text{is_a}(\text{a14}, \text{road}).$ | $\text{is_a}(\text{r193}, \text{road}).$ |
| $\text{intersects}(\text{bari}, \text{a14}).$ | ... |
| $\text{intersects}(\text{trani}, \text{a14}).$ | $O[\text{fg_boundary} \mid \text{barletta}]$ |
| $\text{intersects}(\text{bitonto}, \text{a14}).$ | $\text{is_a}(\text{fg_boundary}, \text{boundary}).$ |
| $\text{intersects}(\text{gioia_del_colle}, \text{a14}).$ | $\text{adjacent_to}(\text{trani}, \text{fg_boundary}).$ |
| $\text{intersects}(\text{molfetta}, \text{a14}).$ | ... |
| ... | |

By definition, the observation encompasses not only spatial relations between the reference object $\text{barletta} \in S$ and task-relevant objects in R_1 (adriatico, etc.), R_2 (a14, etc.), R_3 (fg_boundary, etc.), but also spatial relations between each of these task-relevant objects and some other $s' \in S$ such as $\text{adjacent_to}(\text{bari}, \text{adriatico})$, where $\text{bari} \in S$. \square

To the atomset C we assign an existentially quantified conjunctive formula $\text{eqc}(C)$.

Definition (coverage) An atomset C covers an observation $O[s]$ if $\text{eqc}(C)$ is true in $O[s] \cup \text{BK}$.

Example 2 Let us suppose that BK includes the rule
 $\text{diff}(X, Y) :- X \neq Y$

where \neq is the ISO Prolog Standard built-in predicate for non-unifiability of two variables. The pattern
 $C \equiv \text{is_a}(X, \text{large_town}), \text{intersects}(X, Y), \text{intersects}(Z, Y),$
 $\text{diff}(X, Z), \text{is_a}(Y, \text{road})$

covers the spatial observation $O[\text{barletta}]$ shown in Example 1 because the corresponding existentially quantified conjunctive formula

$$\text{eqc}(C) \equiv \exists \text{is_a}(X, \text{large_town}) \wedge \text{intersects}(X, Y) \wedge \text{intersects}(Z, Y) \wedge \text{diff}(X, Z) \wedge \text{is_a}(Y, \text{road})$$

is satisfied by $O[\text{barletta}] \cup \text{BK}$. \square

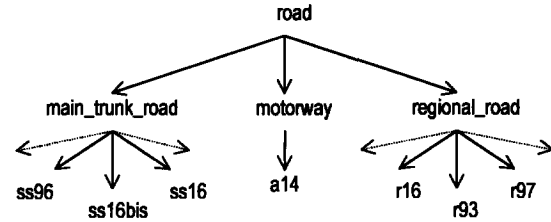


Figure 1. A spatial hierarchy for the layer of roads

Definition Let O be the set of spatial observations in $D(S)$ and O_C denote the subset of O containing the spatial observations covered by the atomset C . The support of C is defined as

$$\sigma(C) = |O_C| / |O|$$

Definition A spatial association rule in $D(S)$ is an implication of the form

$$A \rightarrow B \ (s\%, c\%),$$

where $A \subseteq L$, $B \subseteq L$, $A \cap B = \emptyset$, and at least one atom in $A \cup B$ represents a spatial relationship. The percentages $s\%$ and $c\%$ are respectively called the support and the confidence of the rule, meaning that $s\%$ of spatial observations in $D(S)$ are covered by $A \cup B$ and $c\%$ of spatial observations in $D(S)$ that are covered by A are also covered by $A \cup B$.

Definition The support and the confidence of a spatial association rule $A \rightarrow B$ are given by $s = \sigma(A \cup B)$ and $c = \varphi(B|A) = \sigma(A \cup B) / \sigma(A)$.

The frequency of a pattern depends on the level currently explored in the hierarchical structure of task-relevant geographic layers.

Definition Let $\text{minsup}[I]$ and $\text{minconf}[I]$ be two thresholds setting respectively the minimum support and the minimum

is one of the large ancestors for the pattern *C*.

Such a way of taking the taxonomies into account during the pattern discovery process implements what we referred to as the systematic exploration of the hierarchical structure of task-relevant geographic layers. Furthermore, it is noteworthy that the use of variables and the addition of the atom $\text{diff}(Y,Z)$ derived from the BK allow the algorithm to distinguish between multiple instances of the same class of spatial objects (e.g. the class *large_town*).

During the transformation of frequent patterns into rules, the following strong rule (91% support, 91% confidence)

$\text{is_a}(X, \text{large_town}) , \text{is_a}(Y, \text{large_town}) , \text{diff}(Y,X)$

$\rightarrow \text{intersects}(X,R) ; \text{is_a}(R, \text{main_trunk_road}) , \text{intersects}(Y,R)$

has been derived from the pattern *C*. It states that "Given that 91% of large towns intersect a main trunk road which in turn is intersected by another large town distinct from the previous one, 91% of pairs of distinct large towns are crossed by the same main trunk road". □

4 Conclusions and future work

A logical framework for pattern discovery in spatial data has been sketched. The sample task shows that the expressive power of first-order logic enables us to tackle applications that cannot be handled by the AV approach. The work being presented in this paper is in partial fulfillment of the research objectives set by the project SPIN! (Spatial Mining for Data of Public Interest) funded by the European Union.

For the future, we plan to optimize and test the algorithm SPADA on real-world data sets. Besides the issues of efficiency and scalability that are of great interest to data mining community, the issue of robustness (noise handling, for instance) will be faced. It is noteworthy that very few works tackled this problem in data mining, generally because huge amounts of data to be mined are available. In this case, the presence of low levels of noise can be easily kept under control by tuning the two main parameters of the association rule mining algorithms, namely support and confidence. In spatial data mining, robustness has another facet. Indeed, while the discovery of association rules in transactions requires little transformation of stored data, the task of mining spatial association rules relies on a more complex data pre-processing which is error-prone. For instance, the generation of the predicates *close_to* or *adjacent_to* is based on the user-defined semantics of the closeness and adjacency relations, which should necessarily be approximated. Further work on the automated extraction of symbolic descriptions from vectorised maps is expected to give some hints on this issue.

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