When Regions Start to Move

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Abstract

In this paper, we discuss a formalism for modeling regions that are exposed to movement or deformation. The basis of our formalism is the RCC theory, which uses topological relations between regions to reason about them. By using fuzzy set theory and the notion of conceptual neighbors, we are able to deal with regions that can move or change over time. In addition, we introduce an algorithm for reasoning about these fuzzy sets, based on the path-consistency algorithm developed by Allen for reasoning in the interval algebra.

Introduction and Motivation

In the last two decades, the amount of work on qualitative spatial reasoning based on sets of relations among objects has increased steadily. Early approaches mainly used extensions of Allen's interval algebra (Allen 1983) for reasoning about space. In (Guesgen & Hertzberg 1993), for example, we introduce a form of spatial reasoning that extends Allen's relations to the three dimensions of space by applying very simple methods for constructing higher-dimensional models and for reasoning about them. Freksa (1990) uses the same set of relations and shows that for an important class of problems, only a small subset of all possible combinations of spatial relations can occur. By restricting himself to sets of conceptually neighboring relations, he can restrict the complexity of the constraint satisfaction algorithms significantly.

Hernández (1991) introduces an extension of Allen's approach to represent the spatial features occurring in 2D projections of 3D scenes. He suggests to establish spatial relations between objects by splitting them up into two aspects: projection and orientation. Mukerjee and Joe's work (1990) is similar to Hernández's approach. Objects of a two-dimensional world are characterized by the directions in which the objects are moving and by associating with the objects trajectories along which they are moving.

Kettani and Moulin (1999) use the notion of spatial conceptual maps to generate and describe routes in a qualitative way. Their spatial models are based on the notion of object influence areas. These areas determine how people reason about objects, evaluate metric measures, qualify distances between objects, etc. Musto *et al.* (2000) also use a qualitative approach to describe routes (or courses of motion, as they call them). They use qualitative motion vectors to abstract from irrelevant details of a course of motion.

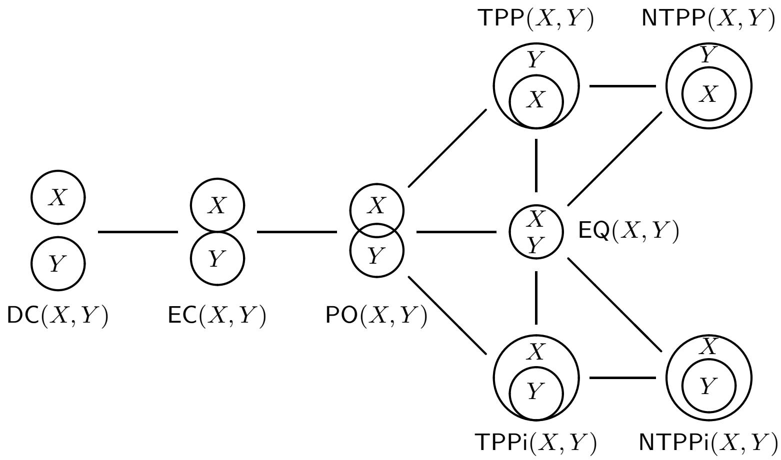
In recent years, the RCC theory (Randell, Cui, & Cohn 1992) has gained a particular interest in the research community. This first-order theory is based on a primitive relation, called connectedness, and uses eight topological relations, defined on the basis of connectedness, to provide a framework to reason about regions. Although the regions are usually considered to be static, there has been some success with applying the RCC theory to a dynamic environment. In (Cui, Cohn, & Randell 1992), for example, the RCC theory is used as basis for qualitative simulations.

The application that we have in mind here is not a qualitative simulation of a changing environment (as in (Cui, Cohn, & Randell 1992)) but a robust technique for reasoning about spatial descriptions that may or may not change from one time instance to the other. Assume, for example, that a fire agent has to decide what action to perform next.[1] The agent has a description of the world to base its decision on. However, by the time the agent has made its decision, the world might have changed: fire, which previously had not spread over a residential area, might now be overlapping that area, or a road adjacent to a collapsing building might now be covered with debris from that building.

This paper is another attempt at extending the RCC theory to make it suitable for reasoning about space in dynamic environments (in particular, regions with movement or deformation). Similar to the approach discussed in (Cui, Cohn, & Randell 1992), we utilize the neighborhood structure that is inherent in the RCC theory. In addition to that, we define fuzzy sets for the relations between regions based on the neighborhood structure and the direction of the movement or deformation of the regions.

The paper is organized as follows. We start with a brief review of the RCC theory and its application to dynamic environments. We then show how information about movement and deformation can be encoded in the RCC theory by associating fuzzy sets with the relations. Finally, we will

[1]This example is taken from the RoboCup Rescue Simulation project (Kitano *et al.* 1999). The aim of this project is to simulate a disaster area (e.g., an area after an earthquake) and to implement agents (e.g, fire agents) that act intelligently in this simulation.



Relation	**Interpretation**
$\mathsf{DC}(X,Y)$	X disconnected from Y
$\mathsf{EC}(X,Y)$	X externally connected to Y
$\mathsf{PO}(X,Y)$	X partially overlaps Y
$\mathsf{EQ}(X,Y)$	X identical with Y
$\mathsf{TPP}(X,Y)$	X tangential proper part of Y
$\mathsf{TPPi}(X,Y)$	Y tangential proper part of X
$\mathsf{NTPP}(X,Y)$	X nontangential proper part of Y
$\mathsf{NTPPi}(X,Y)$	Y nontangential proper part of X

Figure 1: The RCC8 relations arranged in a graph showing the conceptual neighbors.

introduce the means to reason about these fuzzy sets.

Brief Review of the RCC Theory

The basis of the RCC theory is the connection relation, which is a reflexive and symmetric relation, satisfying the following axioms:

1. For each region X: $\mathsf{C}(X,X)$
2. For each pair of regions X, Y: $\mathsf{C}(X,Y) \rightarrow \mathsf{C}(Y,X)$

From this relation, additional relations can be derived, which include the eight jointly exhaustive and pairwise disjoint RCC8 relations (see Figure 1):

$$\mathsf{RCC8} = \{\mathsf{DC}, \mathsf{EC}, \mathsf{PO}, \mathsf{EQ}, \mathsf{TPP}, \mathsf{TPPi}, \mathsf{NTPP}, \mathsf{NTPPi}\}$$

There are different ways to reason about RCC8 relations. Since the RCC theory is expressed in first-order predicate logic, theorem provers can be used to infer new relations from a set of given ones. More popular, however, is reasoning based on a composition table similar to the one used in (Allen 1983), which describes how relations depend on each other. In particular, given the relation R_1 between the regions X and Y, and the relation R_2 between the regions Y and Z, the composition table determines the relation R_3 between the regions X and Z, i.e., $\mathsf{R}_3 = \mathsf{R}_1 \circ \mathsf{R}_2$. In the case of a set of regions $\mathcal{X}$ with more than three regions, the composition table can be applied repeatedly to three-element subsets of $\mathcal{X}$ until no more relations can be updated, resulting in a set of relations that is locally consistent.

The RCC theory was originally intended for static descriptions, but it can also be applied to reason about dynamic environments, i.e., environments with movements or deformations of the regions. Assuming that these movements or deformations are continuous, conceptual neighborhoods (Freksa 1992) can be used to describe the way in which relations can change. Two relations on regions X and Y are conceptual neighbors if the shape of X or Y can be continuously deformed such that one relation is transformed into the other relation without passing through a third relation. Figure 1 shows the conceptual neighbors for the RCC8 relations.

Although conceptual neighborhoods provide the basis for reasoning about space with movement and deformation, it is not sufficient for many real-world problems, since it disregards any information about the direction of the movement or deformation. If, for example, we know that two regions are partially overlapping and moving away from each other, then we can conclude that in the next time instance they are either still partially overlapping or externally connected to each other. It is not possible that one region becomes a tangential proper part of the other region (although the conceptual neighborhood graphs suggests this as well).

Modeling Movement and Deformation

Movement and deformation is closely related to the notion of direction. The idea of incorporating directions into a static spatial theory is not new. Renz (2001), for example, introduces the directed interval algebra, which uses 26 base relations to describe the relationship between two directed intervals. However, this approach cannot directly be applied to the RCC theory, because movement or deformation is not aligned to a particular axis in this theory (see Figure 2). A purely qualitative approach to modeling movements or deformations of regions in the RCC theory, similar to the one used in the directed interval algebra, would lead to descriptions that are too coarse to make meaningful inferences. On the other hand, precise mathematical descriptions of movements or deformations are often too complex. In this paper, we are suggesting a formalism that is more powerful than the analog of the directed interval algebra for regions but more feasible from the computational viewpoint than a precise mathematical one.

As indicated above, our approach is based on the notion of conceptual neighbors. Given a particular relation between two regions X and Y, this relation may change due to movement or deformation of the regions. However, it is likely that the new relation is a conceptual neighbor of the origi-

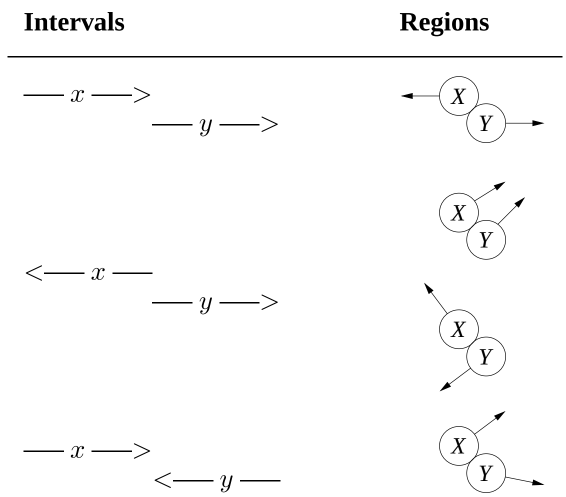


Figure 2: All possible movements/deformations in the directed interval algebra for the meets relation as opposed to some examples of movements/deformations in the RCC theory for the EC relation.

nal relation, or if this it not the case, at least a neighbor of the neighbor of the original relation, and so on. To quantify this fact, we replace the original relation with a fuzzy set on RCC8 relations (i.e., we replace the original crisp relation with an imprecise one).

A fuzzy set $\tilde{R}$ of a domain D is a set of ordered pairs, $(d, \mu_{\tilde{R}}(d))$, where d is an element of the underlying domain D and $\mu_{\tilde{R}} : D \rightarrow [0, 1]$ is the membership function of $\tilde{R}$. In other words, instead of specifying whether an element d belongs to a subset R of D or not, we assign a grade of membership to d. The membership function replaces the characteristic function of a classical subset of D.

Each RCC8 relation can be associated with a characteristic function, which yields a value of 1 if and only if the argument is equal to the RCC8 relation denoted by the characteristic function:

$$\mu_{\mathsf{R}} : \mathsf{RCC8} \longrightarrow \{0, 1\}$$

$$\mu_{\mathsf{R}}(\mathsf{R}') = \begin{cases} 1, & \text{if } \mathsf{R}' = \mathsf{R} \\ 0, & \text{else} \end{cases}$$

This function is converted into a membership function by replacing its range with the unit interval:

$$\mu_{\widetilde{\mathsf{R}}} : \mathsf{RCC8} \longrightarrow [0, 1]$$

The value of the membership function depends on the movement and deformation of the regions and the distance of the relations in the conceptual neighborhood graph.

For example, if two regions X and Y are externally connected (i.e., $\mathsf{EC}(X, Y)$) and moving towards each other, we would assume that neither $\mathsf{DC}(X, Y)$ nor $\mathsf{EC}(X, Y)$ can be observed in the next time instance, but all the other relations are plausible with decreasing membership grades $m_1 \geq m_2 \geq m_3 \cdots \geq 0$. Figure 3 illustrates this observation.

Since there is no algorithm for computing the initial membership grades $m_1, m_2, \ldots$, in general, the grades have to be determined on an intuitive basis. Choosing the right grade for each degree of neighborhood can therefore be a problem. On the other hand, there are experiments showing that fuzzy membership grades are quite robust, which means that it is not necessary to have precise estimations of these grades (Bloch 2000). The explanation given for this observation is twofold: first, fuzzy membership grades are used to describe imprecise information and therefore do not have to be precise, and second, each individual fuzzy membership grade plays only a minor role in the whole reasoning process, as it is usually combined with several other membership grades.

If the membership grades are combined by using the min/max combination scheme, as it is the case in the rest of this paper, we do not need numeric membership grades but can perform reasoning on symbolic values $m_1, m_2, \ldots,$, which solves the problem of determining the initial membership grades. The fact that there is an ordering $m_1 \geq m_2 \geq m_3 \cdots \geq 0$ on the grades suffices to guarantee that we can select the largest/smallest grade from a given set of membership grades, which is essentially what fuzzy reasoning is based upon.

Non-atomic RCC8 relations (i.e., disjunctions of RCC8 relations) can be transformed into fuzzy RCC8 relations by using the same technique as described in the previous section. A non-atomic RCC8 relation is given by a set of atomic RCC8 relations, which is interpreted in a disjunctive way. We therefore transform each atomic relation in the set into a fuzzy RCC8 relation and compute the fuzzy union of the resulting sets.

There are different ways of computing the union, intersection, and compliment of fuzzy sets. Here, we have chosen the min/max combination scheme (Zadeh 1965) to define the membership function of the union, intersection, and complement of fuzzy sets, respectively:

$$\begin{aligned} \mu_{\widetilde{\mathsf{R}}_1 \cup \widetilde{\mathsf{R}}_2}(\mathsf{R}) &= \max\{\mu_{\widetilde{\mathsf{R}}_1}(\mathsf{R}), \mu_{\widetilde{\mathsf{R}}_2}(\mathsf{R})\} \\ \mu_{\widetilde{\mathsf{R}}_1 \cap \widetilde{\mathsf{R}}_2}(\mathsf{R}) &= \min\{\mu_{\widetilde{\mathsf{R}}_1}(\mathsf{R}), \mu_{\widetilde{\mathsf{R}}_2}(\mathsf{R})\} \\ \mu_{\widetilde{\mathsf{R}}_1^C}(\mathsf{R}) &= 1 - \mu_{\widetilde{\mathsf{R}}_1}(\mathsf{R}) \end{aligned}$$

Reasoning about Fuzzy Regions

In order to be able to reason about fuzzy RCC8 relations, we have to define the composition of fuzzy RCC8 relations. In the crisp case, the composition of two relations can be represented as a characteristic function of the following form:

$$\mu_{\mathsf{R}_1 \circ \mathsf{R}_2} : \mathsf{RCC8} \longrightarrow \{0, 1\}$$

The function yields a value of 1 for arguments that are elements of the corresponding entry in the composition table; otherwise, a value of 0:

$$\mu_{\mathsf{R}_1 \circ \mathsf{R}_2}(\mathsf{R}) = \begin{cases} 1, & \text{if } \mathsf{R} \subseteq \mathsf{R}_1 \circ \mathsf{R}_2 \\ 0, & \text{else} \end{cases}$$

For example, if $\mathsf{R}_1 = \mathsf{EC}$ and $\mathsf{R}_2 = \mathsf{TPPi}$, then the characteristic function of the relation $\mathsf{R}_1 \circ \mathsf{R}_2 = \mathsf{EC} \circ \mathsf{TPPi} = \{\mathsf{EC}, \mathsf{DC}\}$ is defined as follows:

$$\mu_{\mathsf{EC} \circ \mathsf{TPPi}}(\mathsf{R}) = \begin{cases} 1, & \text{if } \mathsf{R} \in \{\mathsf{EC}, \mathsf{DC}\} \\ 0, & \text{else} \end{cases}$$

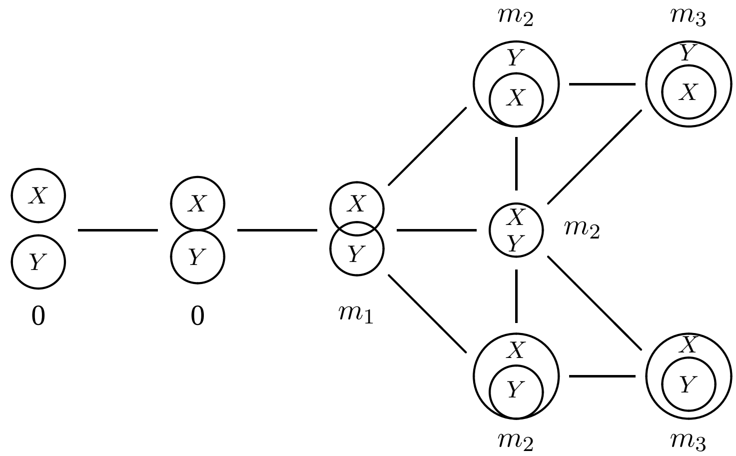


Figure 3: The assignment of membership grades to the RCC8 relations with $\mathsf{EC}(X, Y)$ as reference relation.

Adopting the min/max combination scheme from fuzzy set theory, we can now define the fuzzy composition $\widetilde{\mathsf{R}}_1 \circ \widetilde{\mathsf{R}}_2$ of two fuzzy RCC8 relations $\widetilde{\mathsf{R}}_1$ and $\widetilde{\mathsf{R}}_2$ as the following fuzzy RCC8 relation:

$$\{(\mathsf{R}, \mu_{\widetilde{\mathsf{R}}_1 \circ \widetilde{\mathsf{R}}_2}(\mathsf{R})) \mid \mathsf{R} \in \mathsf{RCC8}\}$$

where $\mu_{\widetilde{\mathsf{R}}_1 \circ \widetilde{\mathsf{R}}_2}$ is given by the following:

$$\mu_{\widetilde{\mathsf{R}}_1 \circ \widetilde{\mathsf{R}}_2}(\mathsf{R}) = \max_{\substack{\mathsf{R'}_1, \mathsf{R'}_2 \in \mathsf{RCC8} \\ \mu_{\mathsf{R'}_1 \circ \mathsf{R'}_2}(\mathsf{R})=1}} \{\min\{\mu_{\widetilde{\mathsf{R}}_1}(\mathsf{R'}_1), \mu_{\widetilde{\mathsf{R}}_2}(\mathsf{R'}_2)\}\}$$

The fuzzy composition of relations plays a central role in a number of algorithms for reasoning about fuzzy RCC8 relations. One of these algorithms is an Allen-type algorithm for computing local consistency in networks of fuzzy RCC8 relations. Input to this algorithm is a set of regions and a set of (not necessarily atomic) fuzzy RCC8 relations. The aim of the algorithm is to transform the given relations into a set of relations that are consistent with each other.[2] This is achieved through an iterative process that repeatedly looks at three regions X, Y, and Z, and their fuzzy relations $\widetilde{\mathsf{R}}_1(X, Y)$, $\widetilde{\mathsf{R}}_2(Y, Z)$, and $\widetilde{\mathsf{R}}_3(X, Z)$, computes the composition of two of the relations, and compares the result with the third relation:

$$\widetilde{\mathsf{R}}_3(X, Z) \;\leftarrow\; \widetilde{\mathsf{R}}_3(X, Z) \cap [\widetilde{\mathsf{R}}_1(X, Y) \circ \widetilde{\mathsf{R}}_2(Y, Z)]$$

Figure 4 shows pseudocode for the extended algorithm; a more elaborate discussion of the algorithm can be found elsewhere (Guesgen, Hertzberg, & Philpott 1994).

Unlike Allen's original algorithm, the fuzzy version of the algorithm does not make a yes/no decision about whether a relation is admissible or not, but computes a new membership grade for that relation. The new membership grade is compared with the initial membership grade of the relation. If the new grade is smaller than the initial grade, the membership grade of the relation is updated with the new grade.

Research in the area of spatio-temporal reasoning has shown that Allen's algorithm in general only computes local consistency. To obtain a globally consistent network of relations, additional methods have to be used, which usually involves some form of backtracking in the non-fuzzy case. In networks with fuzzy relations, we are seeking some level of optimality, which means that a plain backtracking algorithm is insufficient. Instead, the algorithm must continue after a consistent instantiation is found, if this instantiation is not 'good enough' (in terms of the membership grades of the instantiation). One way to achieve this goal is by applying an optimization technique like branch and bound (Freuder & Wallace 1992), which operates in the same way as backtracking search with some variations:

1. The best instantiation so far is recorded.
2. A search path is abandoned when it is clear that it cannot lead to a better solution.
3. Search stops when all search paths have been either explored or abandoned, or when a perfect instantiation has been found.

Conclusion

In many real-world situations, spatial relations among objects are subject to change over time, due to the fact that regions may alter their position or shape. The purpose of this paper is to introduce a formalism for reasoning about spatial relations that is robust under movement and deformation of regions. This is achieved by converting the RCC8 relations into fuzzy sets and applying a fuzzy RCC8 algorithm to the resulting sets.

Unlike (Cui, Cohn, & Randell 1992), the intention is not to provide a formalism for qualitative simulation, but to provide the basis for reasoning in environments that may (or may not) change from one time instance to the other. As a result, our formalism does not keep track about the changes in

[2] In this context, consistency means that the membership grades are consistent with each other.

Fuzzy RCC8 **Algorithm**

- Let $\widetilde{\mathcal{R}}$ be a set of fuzzy RCC8 relations between regions $\{X_1, X_2, \ldots, X_n\}$.
- While $\widetilde{\mathcal{R}}$ is not empty:
  1. Select a relation $\widetilde{\mathsf{R}}(X_i, X_j) \in \widetilde{\mathcal{R}}$
  2. $\widetilde{\mathcal{R}} \leftarrow \widetilde{\mathcal{R}} - \{\widetilde{\mathsf{R}}(X_i, X_j)\}$
  3. For $k \in \{1, \ldots, n\}$ with $k \neq i, j$:
     $\widetilde{\mathsf{R}}(X_k, X_j) \leftarrow \widetilde{\mathsf{R}}(X_k, X_j) \cap [\widetilde{\mathsf{R}}(X_k, X_i) \circ \widetilde{\mathsf{R}}(X_i, X_j)]$
     If $\widetilde{\mathsf{R}}(X_k, X_j)$ changed, then $\widetilde{\mathcal{R}} \leftarrow \widetilde{\mathcal{R}} \cup \{\widetilde{\mathsf{R}}(X_k, X_j)\}$
     $\widetilde{\mathsf{R}}(X_i, X_k) \leftarrow \widetilde{\mathsf{R}}(X_i, X_k) \cap [\widetilde{\mathsf{R}}(X_i, X_j) \circ \widetilde{\mathsf{R}}(X_j, X_k)]$
     If $\widetilde{\mathsf{R}}(X_i, X_k)$ changed, then $\widetilde{\mathcal{R}} \leftarrow \widetilde{\mathcal{R}} \cup \{\widetilde{\mathsf{R}}(X_i, X_k)\}$

Figure 4: Fuzzy version of Allen's algorithm for the RCC8 relations. Without loss of generality, we assume that $\widetilde{\mathsf{R}}(X_i, X_j)$ is defined for every $i, j \in \{1, 2, \ldots, n\}$ with $i \neq j$, possibly as universal relation $\{(\mathsf{DC}, 1), (\mathsf{EC}, 1), (\mathsf{PO}, 1), \ldots\}$.

the environment, nor does it allow to reason about sequences of changes. Future work might address these problems.

The paper focuses on two reasoning techniques: one based on Allen's algorithm, the other on branch and bound techniques. In general, however, reasoning over fuzzy RCC8 relations does not have to be restricted to these techniques. A network of fuzzy RCC8 relations can be viewed as a constraint network, and the problem of finding a consistent instantiation for such a network as a constraint satisfaction problem. This means that in principle any fuzzy constraint satisfaction algorithm (Guesgen & Philpott 1995) can be used to reason about fuzzy RCC8 relations.

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