Revising Contextual Theories

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Abstract

The aim of this paper is to present a procedure for revising contextual theories. Contextual theories have been introduced by J. McCarthy and S. Buvač (McCarthy & Buvač 1994) and they are based on the following principle: generally, a formula is not true in an absolute way but should be considered in a relative way. Contextual reasoning aims at explictly stating that a statement ϕ is holding in a context c. Since contexts may be nested, a contextual theory may be considered as a tree where each node represents a context and its associated propositional theory built on its own vocabulary. The model for revising contextual theories is based on an order over the propositional models associated to each node. We show that our revision model satisfies the AGM postulates (previously redefined for handling contextual theories).

Introduction

This paper studies the revision of contextual theories introduced by J. McCarthy and S. Buvač (McCarthy 1989; McCarthy & Buvač 1994). Contextual reasoning is based on the postulate that a formula is true in a specific context. A context is an abstract entity highlighting the relative truth of a formula (e.g. an agent, a time point, a place...). Contexts can be nested and thus, statements can be stated in sequences of contexts, for instance, in *Europe* (a context), in *2002* (a second context nested in the first one), *population_350millions* (the statement). All statements are stated in an initial context (Buvač, Buvač, & Mason 1995; Perrussel 1998a). This point of view on knowledge leads us to represent contextual theories as trees where a node represents a context, its associated vocabulary specifying what is meaningful and meaningless in a context, and a propositional theory built on this vocabulary. In this paper we present a procedure for revising contextual theories. This procedure is based on a preference relationship defined over the propositional models associated to each node. Next, the revision procedure considers the minimal models (w.r.t. to the preferences) in order to build the revised contextual theory. We show that our procedure satisfies the AGM postulates (Alchourrón, Gärdenfors, & Makinson 1985).

The paper is organized as follows: in section 2, we define contextual theories. In section 3, we introduce contextual models with preferences and, next, in section 4, we describe the revision procedure. In section 5, we recall the AGM postulates and present some results. Section 6 concludes the paper by considering some open issues.

Contextual Theories

As mentioned above, we consider that contextual knowledge is organized in a tree like structure. Each node of the tree represents a context and thus a path represents a sequence of contexts. Let $\mathcal{C}$ be a set of symbols of contexts and $\mathcal{A}$ be the set of trees build over $\mathcal{C}$.

Definition 1 (Tree of Contexts) A tree of context is build according to the following rules: (i) $() \in \mathcal{A}$ (the empty tree), (ii) if $c \in \mathcal{C}$ then $(c) \in \mathcal{A}$; (iii) if $c \in \mathcal{C}$ et $a_1...a_n \in \mathcal{A}$ then $(c, (a_1...a_n)) \in \mathcal{A}$. Every subtree $a_1..a_n$ is rooted in a different context.

Notice that we restrict our proposal to finite trees. Now, we define some notations and functions for handling contextual trees. Let $A = (c, (a_1, ...a_n))$ be a tree. $root(A)$ returns the initial context of the tree (the root), i.e. c. $rst(A)$ returns the set of the sub-trees of A, i.e. $\{a_1, ..., a_n\}$. Let Σ_A be the set of paths built over A. A path is a sequence of contexts $\sigma = [c_0, c_1, c_2...c_n]$. The operator $*$ is used for merging sequences of contexts: $\sigma * \sigma' = [c_0...c_n, c'_0...c'_k]$ (whenever it's clear, $\sigma * c$ stands for $\sigma * [c]$ where $c \in \mathcal{C}$).

Contextual knowledge is rooted in the initial context of A. Statements may concern this initial context or some sub-contexts. If we consider, the example mentioned in the introduction, statements may consider the initial context ($Europe$) or some sub-contexts (i.e. *2002*). In a node (a context), knowledge is expressed in propositional logic ($\mathcal{L}_0$). Assume that $\top$ (true) and $\bot$ (false) belong to $\mathcal{L}_0$. Let csq be the closure operation under classical consequence logic. We associate a propositional theory T_n to each node n. Theories are closed under the consequence logic: $T_n = csq(T_n)$.

Contexts have different vocabularies specifying what is defined in their "universe". Thus some logical statements may be meaningful in some contexts and may be meaningless in others. Let $\mathcal{P}$ be the set of propositional symbols and $Voc : \Sigma_A \rightarrow 2^{\mathcal{P}}$ be a function specifying the vocabulary associated to a node. We assume that $\top$ and $\bot$ belong to all vocabularies ($\forall \sigma \in \Sigma_A(\{\top, \bot\} \subseteq Voc(\sigma))$). In order to define if a formula is meaningful or meaningless in a sequence of contexts, we introduce a function Voc: for every propositional formula ϕ, Voc returns the set of propositional symbols appearing in ϕ: $\mathsf{Voc} : \mathcal{L}_0 \rightarrow 2^{\mathcal{P}}$.

Thus, a contextual theory is a set of propositional statements defined on a specific vocabulary and a set of statements about other contexts. This set is in fact a set of "sub"-contextual theories.

Definition 2 (Contextual Theory) A contextual theory K defined over a tree of contexts A and rooted in an initial sequence σ is a couple $\langle \Gamma, \bigcup_{a \in rst(A)} \{K_a\} \rangle$ such that $\Gamma \in 2^{\mathcal{L}_0}$, $\mathsf{Voc}(\Gamma) \subseteq Voc(\sigma * root(A))$ and every K_a is a contextual theory rooted in the sequence $\sigma * root(A)$.

Let A be a tree of contexts. We consider a special contextual theory $\top_A$ where every propositional theory is equivalent to true. let $\top_A$ be this theory such that $\top_A = \langle \top, \cup_{a \in rst(A)} \{\top_a\} \rangle$. In order to merge two theories, we consider a union-like operator. Let $\sqcup$ be the union of two contextual theories associated to a tree A. If $K = \langle \Gamma_0, \cup_{a \in rst(A)} K_a \rangle$ and $K' = \langle \Gamma'_0, \cup_{a \in rst(A)} K'_a \rangle$ then $K \sqcup K' = \langle csq(\Gamma_0 \cup \Gamma'_0), \bigcup_{a \in rst(A)} \{K_a \sqcup K'_a\} \rangle$. We use the symbol $\sqsubseteq$ for describing the inclusion of a first theory in a second one: $K \sqsubseteq K' \iff \exists K''(K'' \sqcup K = K')$.

After describing contextual theories in a syntactic way, we consider their semantics. Considering a contextual approach in the semantics means that only a subset of the propositional symbols set $\mathcal{P}$ may be valued (Buvač, Buvač, & Mason 1995). A contextual model is a tree where we associate to every node a set of propositional truth assignments.

Definition 3 (Contextual Model) Let $M_A = \langle W, \cup_a \in rst(A)\{M_a\} \rangle$ be a contextual model associated to a tree of contexts A such that W is a set of propositional truth assignments $w : \mathcal{P} \rightarrow \{1, 0\}$ where w could be partial and a set of contextual models (a contextual model for each sub-tree). If $w \in W$, $Dom(w)$ returns the range of w: a subset of $\mathcal{P}$.

We say that a contextual model $M = \langle W, \cup\{M_a\} \rangle$ is included in a model $M' = \langle W', \cup\{M'_a\} \rangle$, $M \subseteq M'$, iff the set W is included or is equal to the set W' and every sub-contextual model M_a is included in $M_{a'}$ ($M_a \subseteq M_{a'}$).

Let A be a tree of contexts, a contextual theory $\langle \Gamma, \bigcup_{a \in rst(A)} \{K_a\} \rangle$ is satisfied by a contextual model $\langle W, \cup_a \in rst(A)\{M_a\} \rangle$ if the propositional theory Γ is satisfied and if all the sub-theories K_a are satisfied by their respective contextual models M_a. More formally ($\models_{\mathcal{L}_0}$ stands for the propositional satisfaction relation):

Definition 4 ($\models$) Let A be a tree of contexts, Voc its associated vocabulary, σ an initial sequence of contexts and $M_A = \langle W, \Omega \rangle$ a contextual model (Ω represents the set of nested models). Let $w \in W$ be a propositional truth assignment and K_A be a contextual theory satisfying the vocabulary constraints mentioned in the definition 2. The relationship $M, w \models K$ is defined if

1. $\mathsf{Voc}(\Gamma_{root(A)}) \subseteq Dom(w)$,
2. the relationship $\models$ is defined for all the nested models $M_a \in \Omega$.

If $M, w \models K$ is defined, $M_A, w \models K$ holds iff:

1. $w \models_{\mathcal{L}_0} \Gamma_{root(A)}$ and
2. for every model $M_a = \langle W_a, \Omega_a \rangle \in \Omega$, every world $w_a \in W_a$, we get $M_a, w_a \models K_a$.

In other words, if the relationship $\models$ is not defined w.r.t. to the vocabulary constraints, then neither $M_A, w \models K_A$ and $M_A, w \not\models K_A$ are defined.

Revision Contextual Model

As A. Grove (Grove 1988), we adopt a revision function based on a semantical approach. The procedure is based on a set of preferences. Defining an order, when we have a semantical point of view, consists of specifying preferences over valuations. Let $\leq$ be the relationship describing preferences over propositional valuations. The notation $w \leq w'$ means that the plausibility of w is better than or equal to the plausibility of w'. We assume that $\leq$ is reflexive and transitive. The relationship $\leq$ is defined for every sequences of contexts since preferences are specific to each contexts. Let us mention that we can only compare valuations belonging to the same set of valuations.

Definition 5 (Revision Model) A revision model M_r associated to a tree A is a structure $\langle W, \leq, \cup_{a \in rst(A)} \{M^r_a\} \rangle$ where W is a set of valuations, $\leq$ a preorder and $\cup_{a \in rst(A)} \{M^r_a\}$ a set of revision models.

A revision models is a tree of ordered valuations. Our revision function will be based on those pre-orders. Based on a revision model and the preferred valuations associated to each sequence of contexts, we build the tree of minimal valuations. Let us call this resulting contextual model, a *contextual minimal model*.

Definition 6 (Contextual minimal model: $\mathfrak{M}$) Let $M^r = \langle W, \leq, \cup_{a \in rst(A)} \{M^r_a\} \rangle$ be a contextual revision model. The minimal contextual model based on M_r is a contextual model defined as:

$$\langle \{w \in W | \text{ for all } w' \in W, w' \not< w\}, \cup_{a \in rst(A)} \{\mathfrak{M}_a\} \rangle$$

As usual, $w < w'$ stands for $w \leq w'$ and $w' \not\leq w$. The next stage consists of linking a theory and a contextual revision model. Let K be a contextual theory and M_r a revision model. Assume the following constraints. First of all, the minimal contextual model based on M_r satisfies K, i.e. the most plausible valuations are those that are verifying their

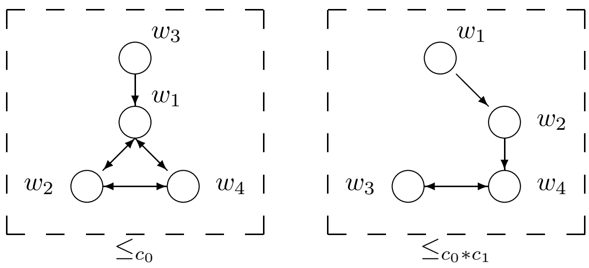

Figure 1: Pre-orders of the revision model M^r described in (2).

associated propositional theory. Next, we constrain the pre-orders such that every relationship $\leq$ is connected, i.e. for all $w, w' \in W$, either $w \leq w'$ or $w' \leq w$ stands. Let $|W|^{\mathfrak{M}}$ be the set of minimal valuations belonging to W. Every possible valuations, w.r.t. the vocabulary specified by Voc has to be defined. Let $\mathcal{W}_\sigma$ be the set of all possible valuations build on the vocabulary $Voc(\sigma)$.

Definition 7 (Complete revision model) Let A be a contextual tree, σ an initial sequence of contexts and $M^r_A = \langle W, \leq, \cup_{a \in rst(A)}\{M^r_a\}\rangle$ a revision model. A revision model M_r is said to be a complete for a contextual theory K_A iff:

- $W = \mathcal{W}_{\sigma * root(A)}$, i.e. all the valuations build on $Voc(\sigma * root(A))$ belongs to W,
- The relation $\leq$ is transitive, reflexive and connected.
- for all $w, w \in |W|^{\mathfrak{M}}$ iff $\mathfrak{M}, w \models K$.
- every revision model M^r_a is a complete revision model rooted in the initial sequence $\sigma * root(A)$.

For instance, let us consider the following contextual tree $A = (c_0, (c_1))$, the empty initial sequence $\sigma = []$ and the contextual theory build on A and rooted in σ ($Voc([c_0]) = Voc([c_0, c_1]) = \{p, q\}$):

$$K = \langle\{p \rightarrow q\}, \{\langle\{p, p \vee q\}, \emptyset\rangle\}\rangle \quad (1)$$

Assume the following revision model:

$$M^r = \langle\{w_1, w_2, w_3, w_4\}, \leq_{c_0}, \{\langle\{w_1, w_2, w_3, w_4\}, \leq_{c_0 * c_1}, \emptyset\rangle\}\rangle \quad (2)$$

Valuations are: $w_1 = \{\bar{p}, \bar{q}\}$, $w_2 = \{\bar{p}, q\}$, $w_3 = \{p, \bar{q}\}$ et $w_4 = \{p, q\}$ where $\bar{p}$ means $w_i(p) = 0$. The pre-orders $\leq_{c_0}$ et $\leq_{c_0 * c_1}$ are described figure 1 where $w_i \rightarrow w_j$ stands for $w_j \leq w_i$. The model is complete and we have the minimal model $=\langle\{w_1, w_2, w_4\}, \{\langle\{w_3, w_4\}, \emptyset\rangle\}\rangle$.

Now, we can specify our revision function for revising contextual theories. Our revision function is based on the works of C. Boutilier (Boutilier 1994; 1995). For revising a theory K by a second theory K', we extract the minimal models satisfying K' from the complete revision model M_r associated to K.

Let A be a contextual tree, σ an initial sequence, $M_r = \langle W, \leq, \bigcup_{a \in rst(A)}\{M^r_a\}\rangle$ a complete revision model for a contextual theory K and $K' = \langle \Gamma'_{root(A)}, \bigcup_{a \in rst(A)} K'_a\rangle$ a contextual theory. Let $min(K')$ be the minimal contextual model based on M_r satisfying K'.
In order to define $min(K')$, we consider all the minimal valuations in every sequence of contexts. Let $|W|^{min(K')}$ be the set of minimal truth assignments belonging to W and satisfying $\Gamma'_{root(A)}$:

$$|W|^{min(K')} = \{w \in W | w \models_{\mathcal{L}_0} \Gamma'_{root(A)} \text{ and for all } w' \in W \text{ if } w' < w \text{ then } w' \not\models_{\mathcal{L}_0} \Gamma'_{root(A)}\}$$

According to this definition, $min(K')$ is a contextual model defined as follows:

$$min(K') = \langle |W|^{min(K')}, \cup_{a \in rst(A)}\{min(K'_a)\}\rangle$$

We get a similar definition, when we consider all the valuations (and thus without considering the preferenceces): let $|W|^{K'}$ be the set of propositional truth assignments satisfying $\Gamma'_{root(A)}$:

$$|W|^{K'} = \left\{w \in W' | w \models_{\mathcal{L}_0} \Gamma'_{root(A)}\right\}$$

As for $min(K')$ we get the contextual model $\|K'\|$ based on M_r and satisfying K':

$$\|K'\| = \langle |W|^{K'}, \cup_{a \in rst(A)}\{\|K'_a\|\}\rangle$$

Let us notice that $min(K')$ and $\|K'\|$ will be defined if and only if vocabulary constraints are not violated (cf. definition 2).

The Revision Procedure

Revising a contextual theory K built on a contextual tree A consists of revising the propositional theories. The smallest revision step is to change only one propositional theory belonging to K. In order words, the input of a revision is a propositional formula ϕ and a sequence of contexts $\sigma \in \Sigma_A$ describing which propositional theory should be revised. The first stage of the revision procedure is to map the structure $\langle\sigma, \phi\rangle$ to a contextual theory k ($\sigma = [c_0, c_1, c_2 ... c_n]$):

$$k = \langle\top, \{...\langle\top, \{...\langle\{\phi\}, \{...\}\rangle ...\}\rangle ...\}\rangle$$

i.e. k is a contextual theory where every propositional theory is defined as equal to $\top$ except the theory associated to the sequence σ. If k is a contextual theory for A, i.e., k do not violate the vocabulary constraints described by Voc, then the revision of K by k and is denoted by K^*_k.

Definition 8 (*) Let K be a contextual theory and $M = \langle W, \leq, \bigcup_{a \in rst(A)}\{M^r_a\}\rangle$ a complete revision model for K. Let $k = \langle\top, \{...\langle\top, \{...\langle\{\phi\}, \{...\}\rangle ...\}\rangle ...\}\rangle$ be the contextual theory associated to a couple $\langle\sigma, \phi\rangle$. The revision of K by k is defined as:

$$K^*_k = \bigsqcup_{min(k) \subseteq \|k'\|} k'$$

where contextual theories k' are defined as k: for every sequence $\sigma = [c_0, c_1, c_2 ... c_m] \in \Sigma_A$ and every propositional formula ψ, k' is defined as the contextual theory $\langle \top, \{...\langle \top, \{...\langle \{\psi\}, \{...\}\rangle ...\}\rangle ...\}\rangle$.

Let us consider again the contextual theory described in 1 and its associated revision model (cf. figure 1). Suppose we revise the propositional theory of the sequence of contexts $[c_0, c_1]$: the statement $\neg p$ have to be incorporated in that theory. According to our definitions, K have to be revised by the theory:

$$k = \langle \{\top\}, \langle \{\neg p\}, \emptyset \rangle \rangle$$

And we get

$$K_k^* = \bigsqcup_{min(k) \subseteq \|k'\|} k' = \langle \{p \rightarrow q\}, \langle \{q\}, \emptyset \rangle \rangle$$

Notice that in the resulting theory K_k^*, only node have been changed: the node associated to the sequence $[c_0, c_1]$.

Based on this revision function for basic change we can easily extend our definition of $*$ for any kind of k. Since every contextual theory may be redefined as a set of couples $\langle$*sequence*, *propositional statement*$\rangle$, we just have to apply the revision function for all the basic couples. The following stage consists of stating if the revision function satisfy the AGM postulates.

Properties of the revision model

In order to prove the main properties of the function $*$, we redefine the AGM postulates for contextual theories (Alchourrón, Gärdenfors, & Makinson 1985). Let A be a contextual tree with a vocabulary specified by Voc. Let K and k be two contextual theories built on A (and thus satisfying the vocabulary constraints). The AGM postulates are redefined as follows:

R1 K_k^* is a contextual theory;

R2 $k \sqsubseteq K_k^*$;

R3 $K_k^* \sqsubseteq K_k^+$ (s.t. $K_k^+ = K \sqcup k$);

R4 If $\neg k \not\sqsubseteq K$ then $K_k^+ \sqsubseteq K_k^*$ (if k is a contextual theory associated to $\langle \sigma, \phi \rangle$, then $\neg k$ is the contextual theory associated to $\langle \sigma, \neg \phi \rangle$);

R5 $K_k^* = Csq(\top_A)$ iff $\models \neg k$ ($Csq(\top_A) = \langle csq(\top), \bigcup_{a \in rst(A)} \top_a \rangle$);

R6 If $\models k \leftrightarrow k'$ then $K_k^* = K_{k'}^*$ ($k \leftrightarrow k'$ iff $(\forall M, w) M, w \models k$ iff $M, w \models k'$);

R7 $K_{k \sqcup k'}^* \sqsubseteq (K_k^*)_{k'}^+$;

R8 If $\neg k' \not\sqsubseteq K$ then $(K_k^*)_{k'}^+ \sqsubseteq K_{k \sqcup k'}^*$;

We have the followings theorems. The first one states that $*$ satisfies the redefined AGM postulates and the second one states that for every revision of a contextual theory, we can defined a revision model. Theorems are stating for a contextual tree and a vocabulary Voc; contextual theories K and k are build on a tree A.

Theorem 1 Let k be a contextual theory associated to a couple $\langle \sigma, \phi \rangle$. Every complete revision model M_r for a contextual theory K defines the revision of K by k so that the postulates R1-R8 are satisfied.

Theorem 2 Let Rev be a revision function satisfying the postulates R1-R8. If $\mathsf{Rev}(K, k)$ is the contextual theory resulting of the revision of K by k then for every K and k there exists a complete revision model M_r such that $\mathsf{Rev}(K, k) = K_k^*$.

Proofs of the theorems are based on (Perrussel 1998b).

Linking contextual and propositional theory revision

We have defined contextual theories in a tree-like structure. Revision action are considered in a basic way: changing a propositional theory in a node. Our revision function $*$ ensures us that just one node has been modified. Since $*$ revises only one node, our change function is close to the functions for revising a set of propositional statements. In this section, we revisit our revision function by considering the revision functions for handling propositional theories.

Let σ be the sequence of contexts representing a node. Let Γ_σ be the associated propositional theory. Assume a model $M_\sigma = \langle W_\sigma, \leq_\sigma \rangle$ defined for every σ. The model M_σ is similar to the models of revision introduced by A. Grove (Grove 1988) and C. Boutilier (Boutilier 1994; 1995). If we consider the models of revision presented by C. Boutilier, a model is a structure $M_B = \langle W, \leq, \varphi \rangle$ s.t.

- $W = W_\sigma$,
- $\leq = \leq_\sigma$ and
- for every symbol p, $\varphi(w, p) = w_\sigma(p)$.

M_B satifies the following constraints (Boutilier 1994):

- all the possible worlds are defined,
- $\leq$ is reflexive, transitive and connected and
- the minimal worlds, w.r.t. $\leq$, satisfy Γ_σ and are the only ones which satisfy Γ_σ.

M_B is a model for revising propositional theories. Let Rev_{M_B} be a revision function for revising a propositional theory based on M_B and satisfying the AGM postulates:

$$\mathsf{Rev}_{M_B}(\Gamma, \phi) = \{\psi | min_{M_B}(\phi) \subseteq \|\psi\|_{M_B}\}$$

such that $min_{M_B}(\phi)$ is the set of minimal worlds satisfying ϕ (w.r.t. $\leq$) and $\|\psi\|_{M_B}$ is the set of worlds satisfying ψ. Based on this function, we can reformulate our revision function[1] :

$$K_{\langle \sigma, \phi \rangle}^* = \begin{cases} \left\langle \mathsf{Rev}_{M_B}(\Gamma_{root(A)}, \phi), \bigcup_{a \in rst(A)} \{K_a\} \right\rangle \\ \qquad \text{if } \sigma = \langle root(A) \rangle \\ \left\langle \Gamma_{root(A)}, \bigcup_{a \in rst(A)} \{K_{a, \langle \sigma - root(A), \phi \rangle}^*\} \right\rangle \\ \qquad \text{otherwise} \end{cases}$$

[1]If $\sigma = [c_0, c_1, ..., c_n] \in \Sigma_A$ then $\sigma - root(A) = [c_1, ..., c_n]$.

Conclusion

In this article, we have proposed a function procedure for revising contextual theories organized in a tree-like structure (McCarthy & Buvač 1994; Buvač, Buvač, & Mason 1995). We have defined contextual theories as local propositional theories build on specific vocabularies. In a first stage we have introduced preferences in contextual models. In a second stage, we have proposed a revision function which considers the best elements for revising a contextual theory. We have shown that the revision function satisfies the AGM postulates. This work is a first step for handling non-monotonicity in contextual reasoning.

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