Handling Qualitative Preferences Using Normal Form Functions

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Abstract

Reasoning about preferences is a major issue in many decision making problems. Recently, a new logic for handling preferences, called *Qualitative Choice Logic (QCL)*, was presented. This logic adds to classical propositional logic a new connective, called *ordered disjunction* symbolized by $\vec{\times}$. That new connective is used to express preferences between alternatives.

QCL was not designed to handle conditional preferences, even if it is possible to express an implication with a preference on the left hand side, for instance "(Air France $\vec{\times}$ Virgin) $\Rightarrow$ Hotel Package". However, using *QCL* semantics, there is no difference between such material implication "(Virgin $\vec{\times}$ Air France) $\Rightarrow$ Hotel Package" and the purely propositional formula "(Air France $\vee$ Virgin) $\Rightarrow$ Hotel Package ". Indeed, the negation in *QCL* gets rid of nested ordered disjunctions. Furthermore, the negation in *QCL* misses some desirable properties from propositional calculus.

In this paper, we present a new semantics for the *QCL* language that addresses those problems. We describe a general framework for handling qualitative preferences. That framework is based on normal form functions that transform general *QCL* formulas into basic choice formulas, which are simple formulas (ordered disjunction of propositional formulas). We formulate the properties of our normal form function that overcome current *QCL* limitations.

Introduction

Decision analysis and Artificial Intelligence have been developed almost separately. Decision analysis is concerned with aggregation schemes and has relied mostly on numerical approaches, while Artificial Intelligence deals with reasoning and has an important logically oriented tradition (Minker 2000). Artificial Intelligence methods can contribute to a more implicit and compact representation of "agent's" preferences. This line of research has been recently illustrated in various ways by AI researchers (Lang, van der Torre, & Weydert 2002; Boutilier 1994; Tan & Pearl 1994a; Brewka 2002).

Recently, a new logic for representing choices and preferences has been proposed (Brewka, Benferhat, & Le Berre

2004). This logic, called *Qualitative Choice Logic (QCL)*, is an extension of propositional logic. The non-standard part of *QCL* logic is a new logical connective $\vec{\times}$, called *Ordered disjunction*, which is fully embedded in the logical language. Intuitively, if A and B are propositional formulas then A $\vec{\times}$ B means: "if possible A, but if A is impossible then at least B". *QCL* logic has strong connections with possibilistic logic, and several extensions of *QCL* have been proposed (e.g, (Benferhat, Brewka, & Le Berre 2004), (Brewka, Niemela, & Syrjanen 2004)).

As a consequence, *QCL* logic can be very useful to represent preferences for that framework. However, its inference relation is less satisfactory. Assume that we want to represent the options concerning a travel from Paris to Key West. Assume that a travel agency has the following rules "customers preferring Air France to Virgin also buy a hotel package" and "customers preferring Virgin to Air France do not buy a hotel package". When a travel agency meets a customer that actually prefers Air France to Virgin, the expected behavior of its information system is to propose a hotel package to that customer. Unfortunately, the QCL logic does not allow us to infer such a conclusion. It will infer both that a package should and should not be proposed.

In fact, the way the negation (and the conditional) is handled in *QCL* logic is not fully satisfactory. In *QCL* when a negation is used on a *QCL* formula with ordered disjunctions, that negated *QCL* formula is logically equivalent to a propositional formula obtained by replacing the ordered disjunction ($\vec{\times}$) by the propositional disjunction ($\vee$).

This is really a limitation, since for instance *QCL* does not make a distinction between the three rules: "Air France $\vec{\times}$ Virgin $\Rightarrow$ FirstClass" (people preferring Air France to Virgin travel in first class), "Virgin $\vec{\times}$ Air France $\Rightarrow$ FirstClass" (people preferring Virgin to Air France travel in first class) and "Air France $\vee$ Virgin $\Rightarrow$ FirstClass" (people flying on Air France or Virgin travel in first class).

This paper proposes to equip the *QCL* framework with inference relation that overcome the *QCL* original inference relation. The inference relation is constructed using normal form functions.

The rest of this paper is organized as follows. First, we recall the concepts of the Basic Choice Formulas (BCF) and General Choice Formulas (GCF). Then we introduce the concept of Normal Form Functions that transform General Choice Formulas into Basic Choice Formulas. In particular, we present a new Normal Form Function that correctly handles the negation of preferences. The last section provides some related works.

The QCL language

This section presents the QCL language, which is in fact composed of three encapsulated sub-languages: Propositional Logic Language, the set of Basic Choice Formulas (BCF) and the set of General Choice Formulas (GCF). These sub-languages are presented in the following subsections. We also present the inference relation from BCF theories.

Basic Choice Formulas (BCF)

Let PS denotes a set of propositional symbols and $PROP_{PS}$ denotes the set of propositional formulas that can be built using classical logical connectives ($\Leftrightarrow, \Rightarrow, \wedge, \vee, \neg$) over PS.

Basic choice formulas are ordered disjunctions of propositional formulas. They propose a simple way to order available alternatives. Given a set of propositional formulas $a_1, a_2, ..., a_n$, the formula $a_1 \vec{\times} a_2 \vec{\times} \ldots \vec{\times} a_n$[1] is used to express an ordered list of alternatives: some a_i must be true, preferably a_1, but if this is not possible then a_2, if this is not possible a_3, etc.

The language composed of *basic choice formulas* and propositional formulas is denoted by BCF_{PS}, and defined by:

Definition 1 *The set BCF_{PS} of basic choice formulas is the smallest set of words defined inductively as follow:*

1. If $\phi \in PROP_{PS}$ then $\phi \in BCF_{PS}$
2. If $\phi, \psi \in BCF_{PS}$ then $(\phi \vec{\times} \psi) \in BCF_{PS}$
3. Every *basic choice formula* is only obtained by applying the two rules above a finite number of times.

BCF formulas represent simples alternatives between propositional formulas. The language of *basic choice formulas* has strong relationships with possibility theory, (see (Benferhat, Brewka, & Le Berre 2004) for more details). In particular, any basic choice formula can be presented by guarented possibility distribution, and conversly.

General Choice Formulas

General Choice Formulas represent any formula that can be obtained from PS using connectors $\vec{\times}, \wedge, \vee, \neg$. The language composed of *general choice formulas* is denoted by QCL_{PS}.

The language QCL_{PS} is defined inductively as follows:

[1]The operator $\vec{\times}$ is associative. Hence, $a_1 \vec{\times} a_2 \vec{\times} \ldots \vec{\times} a_n$ is used as a shorthand for $(((a_1 \vec{\times} a_2) \vec{\times} a_3) \ldots \vec{\times} a_n \ldots)))$.

1. If $\phi \in BCF_{PS}$ then $\phi \in QCL_{PS}$
2. If $\phi, \psi \in QCL_{PS}$ then $(\phi \wedge \psi), \neg(\psi), (\phi \vee \psi), (\phi \vec{\times} \psi) \in QCL_{PS}$.
3. The language of QCL_{PS} is only obtained by applying the two rules above a finite number of times.

GCF formulas represent the whole set of formulas that can be built using for connectives ($\vec{\times}, \wedge, \vee, \neg$).

Example 1 *The formula "AirFrance $\vec{\times}$ Virgin" is a Basic Choice Formula, while the formula "(AirFrance $\vec{\times}$ (Virgin $\wedge \neg KLM$))$\vee$FirstClass" is a General Choice Formula. (For more details, see (Brewka, Benferhat, & Le Berre 2004)).*

Inference from BCF theories

The semantics of BCF is based on the degree of satisfaction of a formula in a particular model I. As in standard propositional logic, an interpretation I is an assignment of the classical truth values T,F to the atoms in PS. I will be represented by the set of its satisfied atoms.

Definition 2 • *Let $\phi = a_1 \vec{\times} a_2 \vec{\times} \ldots \vec{\times} a_n \in BCF_{PS}$,*
$I \models_k \phi$ iff $I \models a_1 \vee a_2 \vee ... \vee a_n$ and $k = min\{j \,|\, I \models a_j\}$.
• *Let $\phi \in PROP_{PS}$. $I \models_1 \phi$ iff $I \models \phi$.*

Namely, a basic choice formula $a_1 \vec{\times} a_2 \vec{\times} \ldots \vec{\times} a_n$ is satisfied to a degree k by an interpretation I if I satisfies a_k but fails to satisfy a_i for all $1 \leq i < k$. When $I \models_k a_1 \vec{\times} a_2 \vec{\times} \ldots \vec{\times} a_n$ does not hold, we simply write $I \not\models_k a_1 \vec{\times} a_2 \vec{\times} \ldots \vec{\times} a_n$. In particular, if there is no k such that $I \models_k a_1 \vec{\times} a_2 \vec{\times} \ldots \vec{\times} a_n$ holds, we write: $I \not\models a_1 \vec{\times} a_2 \vec{\times} \ldots \vec{\times} a_n$. A propositional formula ϕ is satisfied to a degree 1 by I if I is a model of ϕ.

In the rest of the paper, K is a set of propositional formulas which represents knowledge or integrity constraints, and T is a set of general choice formulas. To define the inference relation between $(K \cup T)$ and a propositional formula ϕ, we first need to define the notion of models and preferred models.

Definition 3 *Let K be a set of propositional formulas and T be a set of BCF formulas. An interpretation I is a model of $K \cup T$ iff*

1. *I satisfies K, and*
2. *I satisfies each formula of T to some degree (i.e, $\forall \phi \in T, \exists k$ such that $I \models_k \phi$).*

The satisfaction degree on formulas helps us to determine preferred models. There are different ways of doing this. In (Brewka, Benferhat, & Le Berre 2004) a lexicographic ordering on models, based on the number of formulas satisfied to a particular degree is used. The lexicographic ordering is defined as follows:

Definition 4 *Let $M^k(T)$ denote the subset of formulas of T satisfied by a model M to a degree k. A model M_1 is $K \cup T$-preferred over a model M_2 if there is a k such that $|M_1^k(T)| > |M_2^k(T)|$ and for all $j < k$: $|M_1^j(T)| = |M_2^j(T)|$.*

M is a preferred model of $K \cup T$ iff M is maximally $(K \cup T)$-preferred.

Lastly, the inference relation that allows to infer plausible conclusions from $K \cup T$ is defined as follows:

Definition 5 *Let K be a set of propositional formulas and T be a set of BCF formulas, and ϕ be a propositional formula of $PROP_{PS}$. $K \cup T \mathrel{|\!\sim} \phi$ iff ϕ is satisfied in all preferred models of $K \cup T$.*

Example 2 *As an example consider $K = \emptyset$ and $T = \{Air\text{-}France \stackrel{\rightarrow}{\times} Virgin\}$. We have three models $\{Air\text{-}France\}$, $\{Air\text{-}France, Virgin\}, \{Virgin\}$ with respective satisfaction degree 1, 1, 2.*

This means that $\{Air\text{-}France\}$ and $\{Air\text{-}France, Virgin\}$ are maximally preferred and we have Air-France $\stackrel{\rightarrow}{\times}$ Virgin $\mathrel{|\!\sim}$ Air-France.

A unified framework for handling qualitative preferences

QCL inference relation is fully satisfactory when used on basic choice formulas. However, as we will see later, the inference relation defined in (Brewka, Benferhat, & Le Berre 2004) for General Choice Formulas is not satisfactory. This section first presents a general framework that helps in defining satisfactory inference relation for General Choice Formulas.

Normal Form Function

To define the inference relation on QCL_{PS}, we will take inspiration from the constructive approach that has been used in (Brewka, Benferhat, & Le Berre 2004), namely:

1. First, we transform the set of *general choice formulas* into a set of *basic choice formulas*,

2. Then use Definitions 3, 4, 5 to derive plausible conclusions, since the inference from BCF theories is fully satisfactory.

Indeed, having a set of preferences in a normal form, namely having a set of basic choice formula allows us to reuse various non-monotonic approaches such as possibilistic logic (Dubois, Lang, & Prade 1994) or compilation of stratified knowledge bases (Coste-Marquis & Marquis 2004).

Step 1 is achieved by what we call "Normal Form Function" which is defined as follows:

Definition 6 *A normal form function, denoted by $\mathcal{N}$: $QCL_{PS} \rightarrow BCF_{PS}$, is a function that assigns to each general choice formula, a basic choice formula.*

Step 2 is formally defined as follows:

Definition 7 *Let $\mathcal{N}$ be a normal form function. Let I be an interpretation and ϕ be a general choice formula in QCL_{PS}. We say that I satisfies ϕ to a degree k, denoted by $\models_k^{\mathcal{N}}$, iff I satisfies the normal form of ϕ to a degree k.*

$$I \models_k^{\mathcal{N}} \phi \text{ iff } I \models_k \mathcal{N}(\phi),$$

where $\models_k$ is given by Definition 2.

Given K, T, the definition of consequence relation, denoted by $\models^{\mathcal{N}}$ is parameterized with the normal form function $\mathcal{N}$:

Definition 8 *Let $\mathcal{N}$ be a normal form function. Let K be a set of propositional formulas and T be a set of general choice formulas. Let $T' \subseteq BCF_{PS}$ be a set of basic choice formulas obtained by replacing each $\phi \in T$ by $\mathcal{N}(\phi)$. Then $\phi \in PROP_{PS}$ is a consequence of $K \cup T$ iff ϕ is a consequence of $K \cup T'$, i.e.*

$$K \cup T \mathrel{|\!\sim^{\mathcal{N}}} \phi \text{ iff } K \cup T' \mathrel{|\!\sim} \phi$$

where $\mathrel{|\!\sim}$ is given by definition 5.

The next step is to find some natural properties for any normal form function. This is the aim of the following section which provides some properties (that any normal form function should satisfy). But first, we need to introduce the notion of equivalence between BCF formulas, which is given by the following definition:

Definition 9 *Two BCF formulas ϕ and ψ are said to be equivalent, denoted simply by $\phi \equiv \psi$, if:*

- *For all interpretation I, and integer k we have $I \models_k \phi$ iff $I \models_k \psi$.*

Properties of the Normal Form Functions $\mathcal{N}$

The following properties should be satisfied, namely, they are very desirable when we use any normal form function.

Property 1 (Preserving normality)

$$\forall \phi \in BCF_{PS}, \mathcal{N}(\phi) \equiv \phi.$$

This property simply states that basic choice formulas are already in a normal form.

The next one is needed to normalize general choice formulas. It simply states that normal form function should be decomposable.

Property 2 (Decomposition of $\mathcal{N}$ w.r.t.) *Let $\phi, \psi \in QCL_{PS}$.*

disjunction $\mathcal{N}(\phi \vee \psi) \equiv \mathcal{N}(\mathcal{N}(\phi) \vee \mathcal{N}(\psi))$.
conjunction $\mathcal{N}(\phi \wedge \psi) \equiv \mathcal{N}(\mathcal{N}(\phi) \wedge \mathcal{N}(\psi))$.

ordered disjunction $\mathcal{N}(\phi \vec{\times} \psi) \equiv \mathcal{N}(\phi) \vec{\times} \mathcal{N}(\psi)$.
negation $\mathcal{N}(\neg\phi) \equiv \mathcal{N}(\neg\mathcal{N}(\phi))$.

Property 3 is a De-Morgan like property, while property 4 states that the normal form of a double negation on some formula should be equivalent to the normal form of that formula.

Property 3 (De Morgan) *Let $\phi, \psi \in QCL_{PS}$.*

- $\mathcal{N}(\neg(\phi \wedge \psi)) \equiv \mathcal{N}(\mathcal{N}(\neg\phi) \vee \mathcal{N}(\neg\psi))$,
- $\mathcal{N}(\neg(\phi \vee \psi)) \equiv \mathcal{N}(\mathcal{N}(\neg\phi) \wedge \mathcal{N}(\neg\psi))$.

Property 4 (Double Negation) *let $\phi, \psi \in QCL_{PS}$*

- $\mathcal{N}(\neg\neg\phi) \equiv \mathcal{N}(\phi)$.

Properties 3 and 4 constrain the framework to be close to the one of propositional logic. As we will see later, property 4 is not satisfied by the *QCL* inference relation defined in (Brewka, Benferhat, & Le Berre 2004).

Instances of Normalization Form Functions

Standard *QCL* and its limitations

Let us first reformulate in this section the inference relation defined in (Brewka, Benferhat, & Le Berre 2004). The reformulation is done by means of normal form function that we denote $\mathcal{N}_{QCL}$ (that approach was already proposed in (Brewka, Benferhat, & Le Berre 2004)).
The objective is to show the limitations of the original transformation.

Definition 10 *We call QCL normal function, denoted by $\mathcal{N}_{QCL}$, a normal function that satisfies the properties 1, 2 and 3, where $\mathcal{N}_{QCL}(\phi \wedge \psi)$, $\mathcal{N}_{QCL}(\phi \vee \psi)$ and $\mathcal{N}_{QCL}(\neg\phi)$ are defined as follows:*

- $\mathcal{N}_{QCL}((a_1 \vec{\times} a_2 \vec{\times} \ldots \vec{\times} a_n) \vee (b_1 \vec{\times} b_2 \vec{\times} \ldots \vec{\times} b_m)) \equiv c_1 \vec{\times} c_2 \vec{\times} \ldots \vec{\times} c_k$, *where $k = max(m, n)$, and*

$$c_i = \begin{cases} (a_i \vee b_i) & if \quad i \le min(m, n) \\ a_i & if \quad m \le i \le n \\ b_i & if \quad n \le i \le m \end{cases}$$

- $\mathcal{N}_{QCL}((a_1 \vec{\times} a_2 \vec{\times} \ldots \vec{\times} a_n) \wedge (b_1 \vec{\times} b_2 \vec{\times} \ldots \vec{\times} b_m)) \equiv c_1 \vec{\times} c_2 \vec{\times} \ldots \vec{\times} c_k$, *where $k = max(m, n)$, and*

$$c_i = \begin{cases} [(a_1 \vee \ldots \vee a_i) \wedge b_i] \vee [a_i \wedge (b_1 \vee \ldots \vee b_i)] \\ \quad if \quad i \le min(m, n) \\ ((a_1 \vee \ldots \vee a_n) \wedge b_i) & if \quad n < i \le m \\ (a_i \wedge (b_1 \vee \ldots \vee b_m)) & if \quad m < i \le n \end{cases}$$

- $\mathcal{N}_{QCL}(\neg(a_1 \vec{\times} a_2 \vec{\times} \ldots \vec{\times} a_n)) \equiv \neg a_1 \wedge \neg a_2 \wedge \ldots \wedge \neg a_n$.

where $a_i, b_j, c_k, \ldots$ are propositional formulas.

As advocated in the previous section, the QCL inference relation does not satisfy Property 4 as it is illustrated by the following example.

Example 3 *Let $\phi = (a \vec{\times} b) \vee \neg c$, where a, b, c are three independent propositional symbols. Using property 3, we have*

$$\begin{aligned} \mathcal{N}_{QCL}(\neg\neg\phi) &\equiv \mathcal{N}_{QCL}(\neg\neg((a \vec{\times} b) \vee \neg c)), \\ &\equiv \mathcal{N}_{QCL}(\neg((\neg a \wedge \neg b) \wedge c)), \quad Let \quad I = \\ &\equiv a \vee b \vee \neg c. \end{aligned}$$

$\{b, c\}$ the interpretation, using definition 3, we have $I \models_2^{\mathcal{N}_{QCL}} \phi$, $I \models_1^{\mathcal{N}_{QCL}} \neg\neg\phi$.
These two formulas have different satisfaction degrees, hence $\mathcal{N}_{QCL}(\neg\neg\phi) \not\equiv \mathcal{N}_{QCL}(\phi)$.

Next example points out another limitation of QCL inference, which concerns the representation of conditional preferences.

Example 4 *This example shows that the QCL framework does not make distinction between the following three rules given in the introduction:*

1. AirFrance $\vec{\times}$ Virgin $\Rightarrow$ FirstClass,

2. Virgin $\vec{\times}$ AirFrance $\Rightarrow$ FirstClass,

3. AirFrance $\vee$ Virgin $\Rightarrow$ FirstClass.

Indeed, we have,

$\mathcal{N}_{QCL}(AirFrance \vec{\times} Virgin \Rightarrow FirstClass)$
$\equiv \mathcal{N}_{QCL}(\mathcal{N}_{QCL} \neg(AirFrance \vec{\times} Virgin) \vee \mathcal{N}_{QCL}(FirstClass))$.
$\equiv \mathcal{N}_{QCL}((\neg AirFrance \wedge \neg Virgin) \vee (FirstClass))$
$\equiv (\neg AirFrance \wedge \neg Virgin) \vee FirstClass$

and,

$\mathcal{N}_{QCL}(Virgin \vec{\times} AirFrance \Rightarrow FirstClass)$
$\equiv \mathcal{N}_{QCL}(\mathcal{N}_{QCL}(\neg(Virgin \vec{\times} AirFrance)) \vee \mathcal{N}_{QCL}(FirstClass))$.
$\equiv \mathcal{N}_{QCL}(\mathcal{N}_{QCL}(\neg Virgin \wedge \neg AirFrance) \vee \mathcal{N}_{QCL}(FirstClass))$
$\equiv \mathcal{N}_{QCL}((\neg Virgin \wedge \neg AirFrance) \vee (FirstClass))$
$\equiv (\neg AirFrance \wedge \neg Virgin) \vee FirstClass$

This last example emphasizes another consequence of the same limitation:

Example 5 *Let us consider the example given in the introduction where our knowledge base K contains*
$\neg Virgin \vee \neg AirFrance$ (1)
and T contains the following preferences:

$$\begin{cases} AirFrance \vec{\times} Virgin \Rightarrow HotelPackage \text{(2)} \\ Virgin \vec{\times} AirFrance \Rightarrow \neg HotelPackage \text{(3)} \\ AirFrance \vec{\times} Virgin \text{(4)} \end{cases}$$

The following truth table summarizes for each formula (1), (2), (3), (4) *from K and T we are interested in, whether it is satisfied (T) (to some degree) or not (-) by a given interpretation.*

AirFrance	Virgin	HotelPackage	(1)	(2)	(3)	(4)
F	F	F	T	T	T	-
F	F	T	T	T	T	-
F	T	F	T	-	T	T
F	T	T	T	T	-	T
T	F	F	T	-	T	T
T	F	T	T	T	-	T
T	T	F	-	-	T	T
T	T	T	-	T	-	T

Table1: The models of $K \cup T$ by using $\mathcal{N}_{\mathcal{QCL}}$.

One can see that $K \cup T$ has no model that satisfies each formula of K and T to some degree. Hence, we get both $K \cup T \hspace{-1mm}\mid\hspace{-1.5mm}\sim^{\mathcal{N}^{QCL}} Hotel$ and $K \cup T \hspace{-1mm}\mid\hspace{-1.5mm}\sim^{\mathcal{N}^{QCL}} \neg Hotel$, while only $K \cup T \hspace{-1mm}\mid\hspace{-1.5mm}\sim^{\mathcal{N}^{QCL}} Hotel$ is desirable.

The following subsection provides an alternative for standard QCL inference relation.

Weak Qualitative Choice Logic (*WQCL*)

The main limitation we identified for *QCL* is the consequence of using the negation on formulas containing ordered disjunctions. This subsection proposes a new normal form function, denoted by $\mathcal{N}_{\mathcal{WQCL}}$, which satisfies all properties given in the previous section, namely properties 1, 2, 3, 4.
In particular, we require that $\mathcal{N}_{\mathcal{WQCL}}(\neg\neg\phi) \equiv \mathcal{N}_{\mathcal{WQCL}}(\phi)$.

We propose *WQCL* which is characterized by new negation and conjunction. The conjunction in *WQCL* can be considered as a weaker version of the conjunction available in *QCL*. Indeed, the number of interpretations satisfying $\mathcal{N}_{\mathcal{WQCL}}(\phi \wedge \psi)$ is smaller than the number of interpretations satisfying $\mathcal{N}_{\mathcal{QCL}}(\phi \wedge \psi)$. The $WQCL$ logic is particularly adapted for handling positive preferences, namely preferences that only help to rank-order solution and never excludes solutions. Positive preferences are preferences where the last option is always a tautology, hence preferences will always be satisfied.

In order to fully characterize $\mathcal{N}_{\mathcal{WQCL}}$, we only need to define $\mathcal{N}_{\mathcal{WQCL}}(\phi \wedge \psi)$ and $\mathcal{N}_{\mathcal{WQCL}}(\neg\phi)$, where ϕ and ψ are basic choice formulas.

Definition 11 *We call $WQCL$ normal function, denoted by $\mathcal{N}_{\mathcal{WQCL}}$, a normal function that satisfies the properties 1, 2, 3 and 4, where $\mathcal{N}_{\mathcal{WQCL}}(\phi \wedge \psi)$ and $\mathcal{N}_{\mathcal{WQCL}}(\neg\phi)$ are defined as follows:*

Let $\phi = a_1 \vec{\times} a_2 \vec{\times} \ldots \vec{\times} a_n$, and $\psi = b_1 \vec{\times} b_2 \vec{\times} \ldots \vec{\times} b_m$ *such that a_i's and b_i's are propositional formulas.*

- $\mathcal{N}_{\mathcal{WQCL}}((a_1 \vec{\times} a_2 \vec{\times} \ldots \vec{\times} a_n) \wedge (b_1 \vec{\times} b_2 \vec{\times} \ldots \vec{\times} b_m)) \equiv c_1 \vec{\times} c_2 \vec{\times} \ldots \vec{\times} c_k$ *where k = max(m, n), and*

$$c_i = \begin{cases} (a_i \wedge b_i) & if \quad i \leq min(m,n) \\ a_i & if \quad m \leq i \leq n \\ b_i & if \quad n \leq i \leq m \end{cases}$$

- $\mathcal{N}_{\mathcal{WQCL}} \neg(a_1 \vec{\times} a_2 \vec{\times} \ldots \vec{\times} a_n) \equiv \neg a_1 \vec{\times} \neg a_2 \vec{\times} \ldots \vec{\times} \neg a_n$.

Example 6 *Let us continue examples 3 and 4, we have*

$$\begin{aligned} \mathcal{N}_{\mathcal{WQCL}}(\neg\neg\phi) &\equiv \mathcal{N}_{\mathcal{WQCL}}(\neg\neg((a \vec{\times} b) \vee \neg c)) \\ &\equiv \mathcal{N}_{\mathcal{WQCL}}(\neg((\neg a \vec{\times} \neg b) \wedge c)) \\ &\equiv \mathcal{N}_{\mathcal{WQCL}}((a \vec{\times} b) \vee \neg c). \\ &\equiv (a \vee \neg c) \vec{\times} b. \end{aligned}$$

Hence, in contrast with $\mathcal{N}_{\mathcal{QCL}}$, we do not loose the ordered disjunction in the scope of a negation.

Let $I = \{b, c\}$ the interpretation, we have $I \models_2^{\mathcal{N}_{\mathcal{WQCL}}} \phi$, $I \models_2^{\mathcal{N}_{\mathcal{WQCL}}} \neg\neg\phi$.
These two formulas have the same satisfaction degrees, hence $\mathcal{N}_{\mathcal{WQCL}}(\neg\neg\phi) \equiv \mathcal{N}_{\mathcal{WQCL}}(\phi)$.

A more interesting result arises when considering implications:
$\mathcal{N}_{\mathcal{WQCL}}((AirFrance \vec{\times} Virgin) \Rightarrow (FirstClass))$

$\equiv (\neg AirFrance \vee FirstClass) \vec{\times} (\neg Virgin)$ *, While*

$\mathcal{N}_{\mathcal{WQCL}}((Virgin \vec{\times} AirFrance) \Rightarrow FirstClass)$
$\equiv (\neg Virgin \vee FirstClass) \vec{\times} (\neg AirFrance)$

Therefore contrary to standard QCL, the order of preferences which is preserved is very important.

Definition 12 *Let K be a set of propositional formulas and T be a set of GCF formulas, and ϕ be a propositional formula of $PROP_{PS}$. Let T' be a set of preferences obtained from T by replacing each ϕ in T by $\mathcal{N}_{\mathcal{WQCL}}(\phi)$.*
$K \cup T \hspace{-1mm}\mid\hspace{-1.5mm}\sim^{WQCL} \phi$ *iff $K \cup T' \hspace{-1mm}\mid\hspace{-1.5mm}\sim \mathcal{N}_{\mathcal{WQCL}}(\phi)$.*

Example 7 *Let us consider again Example 5. The following truth table summarizes at which degree of satisfaction is satisfied each formula with the new inference relation WQCL.*

AirFrance	Virgin	HotelPackage	(1)	(2)	(3)	(4)
F	F	F	1	1	1	-
F	F	T	1	1	1	-
F	T	F	1	1	2	2
F	T	T	1	1	1	2
T	F	F	1	2	1	1
T	**F**	**T**	**1**	**1**	**1**	**1**
T	T	F	-	-	-	1
T	T	T	-	1	1	1

Table2: The models of $K \cup T$ by using $\mathcal{N}_{\mathcal{WQCL}}$.

$K \cup T$ has one preferred model (bold line), $I = \{AirFrance, Hotel\}$, from which we obtain the

expected conclusion $K \cup T \mathrel{\vert\!\!\!\sim}^{\mathcal{N}^{WQCL}} Hotel$.

Related Work

The problem of representing preferences has recently drawn attention from Artificial Intelligence researchers. The paper on preference logic (von Wright 1963) addresses the issue of capturing the common-sense meaning of preference through appropriate axiomatizations. The papers on preference reasoning (Wellman & Doyle 1991; Tan & Pearl. 1994b; Boutilier *et al.* 1999) attempt to develop practical mechanisms for making inference about preferences and making decisions. A principal concept there is Ceteris Paribus preference: preferring one outcome to another, everything else being equal. The work on prioritized logic programming and non-monotonic reasoning (Brewka & Eiter 1999; Delgrande, Schaub, & Tompits 2000; Sakama & Inoue 2000) has potential applications to databases.

All theses works bear some relationship to the qualitative Choice Logic which is presented by Brewka et al (Brewka, Benferhat, & Le Berre 2004). However, in the majority of theses works, the negation in the context of representing preferences is not discussed because of the difficulty to interpret what is the meaning of negated preferences.

CP-nets (Boutilier *et al.* 1999) use Bayesian-like structure to represent preferences under again Ceteris Paribus principle. However, CP-nets only represent simple form of conditional preferences. For instance, it can not represent the preference "people preferring Air France to Virgin travel in first class". In this paper, we were not guided by the semantics of negated preferences, but by some desirable logical properties.

Lastly, in (Benferhat, Le Berre, & Sedki 2006) the limitations of *QCL* have been informally advocated in their two page short paper. However it is informal, and no rigorous solution is proposed.

Conclusion

We proposed in this paper a non-monotonic framework for representing preferences. That framework is based on the same language as the QCL logic proposed by (Brewka, Benferhat, & Le Berre 2004). However, it generalizes the way the inference is done by presenting an inference framework based on normal form functions. We also presented some limitations of the original *QCL* inference, especially the side effect resulting from the way negations are handled: negated *QCL* formulas are equivalent to plain propositional formulas. We thus proposed an inference framework for the *QCL* language that overcomes those limitations: the *WQCL* inference which preserves preferential information in negated *QCL* formulas.

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