A Model for Qualitative Spatial Reasoning Combining Topology, Orientation and Distance

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Abstract

Much work has been done in the area of qualitative spatial reasoning over the past years, with application in various domains. However, existing models only capture particular aspects of the spatial relations between objects and therefore are unable to represent these relations accurately. In this paper, we draw together two existing approaches to provide a calculus taking into account topology, orientation and distances, while keeping in mind cognitive considerations. Provided that some constraints are imposed on the spatial objects and the frame of references, our model has been successfully tested to infer implicit constraints from a knowledge base. With further improvements, it has applications to GISs and robot navigation.

Introduction

Representing and dealing with spatial knowledge is undeniably very important in many fields, especially in Artificial Intelligence, as it has a diverse range of applications. A common way to model spatial knowledge is to use a quantitative approach, through coordinates and numerical values. Since a quantitative approach does not lend itself to handling inexact knowledge easily, it is often assumed that the exact position of each object is known if this representation is used. It is therefore not surprising that another approach has gained more and more attention during the last 20 years: qualitative reasoning. Although a considerable number of calculi address the problem of representing space qualitatively, in certain cases they are still insufficient.

Qualitative Spatial Knowledge

Qualitative reasoning, unlike quantitative reasoning, deals with commonsense knowledge without using numerical values, like exact coordinates or metrics. The qualitative approach is closer to the way humans represent and reason with spatial knowledge in everyday life (Hernández, 1994) and thus is more meaningful. For example, "object A is twice as big as object B" makes more sense for people than

"object A is 1.825 times bigger than object B". The traditional quantitative approach, used in various areas such as robotics or computational geometry, has several drawbacks when no exact or precise data is available, as highlighted by (Hernández, 1994). This includes complexity (the positions of all the objects have to be known, even if we do not need them), partial and uncertain information (when a value is not known exactly, it has to be either ignored or assigned) and missing adequacy: "quantities approaches … force us to use quantities to express even qualitative facts" (Hernández, 1994, p.3). An interesting property of the qualitative approach is that only as many distinctions as necessary are made: for example in some applications just knowing if an obstacle is to the back, front, left or right of a robot may suffice. Also, a coarse level of granularity for reasoning can be used to handle vague knowledge. For example, if the direction of the goal for a robot is not clearly defined and can be either North, North-West or North-East, then at the coarser level of granularity, only the fact that the goal is somewhere North (as opposed to South) will be retained.

Current Limitations

Due to the complexity of space, none of the approaches completely solves the problem of providing a calculus to represent and reason with all the aspects of space. In fact, research has rather concentrated on single aspects of space such as topology, orientation, distances and sizes, or combinations of those. However, in certain cases, the existing models are insufficient to describe accurately the spatial relations between objects. As an example, let us consider the following situations, which can arise in robot navigation: *Obstacle B is in front of obstacle A, but far away* and Obstacle *B is in front of and very close to Obstacle A, but they are disjoint*. Using models combining topology and orientation, as in (Hernandez, 1994), would result in the following relation for both cases:

B (disjoint, front) A

The notion of distance is lost and thus the robot does not know whether it could pass between A and B. On the other hand, a model like the one in (Clementini, Di Felice and Hernández, 1997), which combines orientation and

distances, would not represent the topological relation between the obstacles.

Overview of the Proposed Approach

In this paper, we introduce a new model which draws together two existing approaches, namely the frameworks developed by Hernández (1994) and Clementini, Di Felice and Hernández (1997), in order to provide a calculus that takes into account topology, orientation and distance in a cognitively adequate way. Such an approach can have diverse applications (e.g., in GISs), which allow the user to make qualitative statements that meet their needs. As an example, a user could ask *"What are the regions C that have a boundary with country A and are close to country B?"* Another possible application is in the field of robot navigation. Our model could for instance help a robot avoid obstacles given their relative position to one another and also help it manage its energy consumption.

The rest of this paper is organised as follows: Section 2 gives an overview of relevant work in the field of qualitative spatial knowledge. Section 3 introduces our proposed model for representing space. In Section 4 it is shown how reasoning is done. Results are discussed in Section 5 and finally Section 6 summarises this paper and introduces further improvements our model could benefit from.

Related Work

Research has mostly focused on topology, orientation and distances, which are the most important aspects of space. We describe here some of the work that has been done in this area.

Topology

Topological relations between objects are the aspect of space that has been addressed the most by research in Artificial Intelligence. Topology deals with inherent properties between objects, i.e. two objects are disjoint from each other, they touch each other or one is included in the other. One of the most famous calculi in this area is the 4-intersection model introduced by Egenhofer (1991), which considers the four intersections of the interiors (°) and the boundaries (∂) of two sets. These four intersections lead to a total of 16 topological relations, but only eight are possible due to the physical properties of the 2-D space.

Another well-known approach is RCC8, by Randell, Cui and Cohn (1992), which also considers a set of eight jointly exhaustive and pairwise disjoint relations, similar to the ones of the 4-intersection model.

Orientation

Orientation is another important aspect, which has been addressed through various approaches. An example is Freska's Double Cross Calculus (DCC) (1992). This

model uses points as basic entities and allows one to specify the position c of any point with respect to a reference segment ab. Fifteen basic relations are derived from:

- A right-left dichotomy applied from a parallel to ab
- A front-back dichotomy perpendicular to ab in a
- A front-back dichotomy perpendicular to ab in b

Another approach is the Dipole Relation Algebra ($\mathcal{DRA}$), by Moratz, Renz and Wolter (2000), which uses dipoles (i.e., oriented segments) as basic spatial entities.

Distance

Distance is a very complex aspect of space, which might explain why there are not many AI formalisms that deal with this aspect. In fact, the notion of distance depends on many factors, as pointed out by Gahegan (1995). For example, the effect of scale has an influence on our perception of distances and therefore the following statements are not contradictory:

> *Paris is close to Berlin but far from Auckland*

and

> *Paris is close to Auckland but far from the Moon*

The "attractiveness" of objects also plays a role in our perception of distances: being close to a shopping centre and close to a waste dump probably do not mean the same thing for most people.

In addition to that, distances can be measured in different ways, considering for example the cost (e.g., fuel) or the time to get to a particular place. These are not necessarily symmetric (Egenhofer and Mark, 1995).

Moreover, distances cannot be represented without orientation knowledge: for instance, if we just know that *A is far from B* and *B is far from C*, then we do not know how far A is from C. In fact, if the orientation of B w.r.t. A is the same as the orientation of C w.r.t. B, then we can conclude that C is far from A. However, if those orientations are opposite, C may be close to A.

Combinations

Orientation is often combined with distance to get positional information, as in Frank (1991) or Clementini, Di Felice and Hernández (1997). Other combinations are possible, as in Hernández (1994), which combines topology and orientation.

Proposed Representation of Space

We introduce here a framework combining two existing models: (Hernández, 1994), which uses topology and orientation; and (Clementini, Di Felice and Hernández, 1997), which uses orientation and distances. We chose these models because they are efficient in their respective domains, while keeping in mind cognitive considerations.

Representation of Topology

Note that we impose restrictions on the objects, in that they have to be embedded in the 2-D space and have to be convex without holes. A common practice is to consider only the minimum bounding rectangle enclosing the object. For this purpose, we use Hernández's set of eight jointly exhaustive and pairwise disjoint (JEPD) set of relations, based on the 4-intersection model: *disjoint* (d), *tangential* (t), *overlaps* (o), *included* (i), *contains* (c), *included at border* (i@b) and *contains at border* (c@b) and *equal* (=). All those relations give different information concerning the objects. For instance, all relations that imply a contact also give a notion of distance (two objects cannot be far away and overlapping). Some relations provide knowledge about relative size and shape (if A is included in B, then B is necessarily larger than A).

Even though a set of eight JEPD relations seems to be cognitively adequate, as shown for the RCC8 in (Renz, 2002), we sometimes prefer to consider a coarser level of granularities, i.e. using only five relations instead of eight (e.g. if we do not want to make the distinction between i@b and i).

Representation of Orientation

Orientation indicates where objects are placed with respect to one another. It is a ternary relation between the primary object, the reference object and a frame of reference. Let us first discuss the different possible relations between spatial objects. Once again, we can assume different levels of granularities, according to our needs. For example, a very coarse level of granularity would be to only consider a dichotomy-front-back or left-right. A finer granularity would include more distinctions, as left, front, right, back. Note that the names of the different orientations are just labels and can thus be changed according to the application (i.e. using North, South...).

The role of the frame of reference (FofR) is to define the front side of an object. Without it, a given orientation relation between two objects can have several interpretations. Before giving an example, let us first consider the different kinds of FofR. Hernández (1994, p 45) distinguishes three kinds of FofR, namely:

Intrinsic: when the orientation is given by some inherent properties of the reference object

Extrinsic: when external factors impose an orientation on the reference object

Deictic: when the orientation imposed by the point of view from which the reference object is seen

Thus, a statement such as *The ball is in front of the car*, can have different meanings, depending on the type of FofR. For example, if the FofR is intrinsic, then the ball is in front of the physical front of the car; but given a deictic FofR (e.g. the speaker's point of view), the ball can well be on the physical side of the car.

In Hernández's framework (1994), spatial relations between objects are described with a topological-orientation pair and a FofR. If no FofR is given, it is assumed that the parent object's intrinsic orientation is used as a reference. For example, without any details, we assume that the orientation of objects in a room is given by the intrinsic orientation of a room itself (determined by its main door for instance).

Representation of Distances

As pointed out previously, distance is probably one of the hardest aspects of space to deal with. We use the framework developed by Clementini, Di Felice and Hernández (1997) for dimensionless points, where distance relations are represented between a primary object, a reference object and a FofR. Note that the FofR for distances is different from the one for orientation. A FofR for distances is not only defined by its type (intrinsic, extrinsic, deictic), but also by a scale and a distance system. The purpose of a distance system is to name distances and compare them, and is defined as:

- A set of totally ordered distance relations $Q=\{q_0,q_1,\ldots q_n\}$ (*n* varying with the level of granularity), where q_0 represents the closest distance and q_n the furthest

- An acceptance function mapping the distances q_i to 1-D geometrical interval δ_i representing distance ranges

- An algebraic structure I which defines operations (e.g. $+, <, \leq, \approx, <<$) on the intervals δ_i

The role of the 1-D intervals δ_i is described in the next section. Here, we just consider naming the distances of Q. Depending on the application, the level granularity and thus the number of elements in Q can vary. For example, we could imagine using a very coarse level of granularity only making the distinctions *close* (*cl*) and *far* (*fr*), represented respectively by q_0 and q_1. Other levels could differentiate *close* (*cl*), *medium* (*md*) and far (*fr*) or make five distinctions of distances: *very close* (*vc*), *close* (*cl*), *commensurate* (*cm*), *far* (*fr*) and *very far* (*vf*).

We only consider the case of isotropic distances (i.e., the distance ranges are equal in all the directions around the reference object) and thus the plane is separated into as many regions centred around the reference object as there are different distances, with the outermost region going to infinity.

Combining Knowledge

In our model, the relation between a primary object PO and a reference object RO is given with respect to a FofR for orientation (FofR_O) and a FofR for distances (FofR_D):

<PO [topological,orientation,distance] RO, FofR_O, FofR_D>

However, we restrict our model here in that all the orientation relations are given using the intrinsic orientation of the parent object as a reference. Also, we

only consider the case where all the distances are given using the same FofR for distance with an extrinsic type (given, for example, by the cartographic projection commonly used). Let us consider the example of the position of objects in a room. The statement *The TV is far from and on the left of the window W* is represented by:

TV [d,l,fr] W

Here, we assume that the relation *left of* uses the intrinsic orientation of the room as a reference, and that the frame of reference for distance is given by the dimension of the room, and a scale whose order of magnitude is the meter. In the case where the relation is not clearly defined, it can be given by a set of relations, e.g.:

TV {[d,l,fr];[d,lb,fr]} W

TV {[d,l-lb,fr]} W (condensed form)

Note that topology and distance are not independent: it is not conceivable to consider two objects in contact with each other while the distance separating them is not the smallest possible at the given level of granularity. In other words, if we know that object A is tangential to object B, then the distance between them should be q_0.

Qualitative Reasoning

Reasoning about qualitative spatial knowledge refers to infering implicit knowledge from explicit knowledge, answering spatial queries, maintaining consistency in the knowledge base and more. In this paper, we only consider the inference of implicit knowledge from new knowledge added to the knowledge base. For that, we apply a constraint propagation algorithm similar to the one introduced by Allen (1983), which makes use of a composition table. Given the relation between A and B and the relation between B and C, the composition table specifies all possible relations between A and C.

In our approach, we are not using a composition table for topological/orientation/distance triplets directly. Instead, we use the tables given in the original papers and combine them within the algorithm, as explained later.

Composition Tables for Topology/Orientation

Hernández (1994) showed that the composition tables for topological/orientation pairs contain fewer possible relations than the tables for topology and orientation taken separately. This is logical, since the combination of the two often narrows down the number of possibilities. For example, knowing that *A d B* and *B d C* does not allow us to infer new knowledge about the relation between A and C: it can be any of the eight base relations. However, if we know *A d,l B* and *B d,l C*, then necessarily *A d,l C*.

Composition Tables for Distances

Deriving the composition tables for distances cannot be done without having knowledge about the orientation of

objects. Let us first define Θ_{AB} as the orientation of B w.r.t. A; and d_{AB} as the qualitative distance between A and B. Clementini, Di Felice and Hernández (1997) propose several algorithms to derive the composition tables for distances for three cases: when $\Theta_{BC} = \Theta_{AB}$ (Algorithm 1); when Θ_{BC} is in the opposite orientation to Θ_{AB} (Algorithm 2) and when Θ_{BC} is in the orthogonal orientation to Θ_{AB} (Algorithm 3). The general case for orientation is treated as an interpolation of the three special ones.

Why use algorithms to derive the composition tables? The resulting sets in the composition tables are made up of consecutive distances, so we are able to give coarse lower and upper bounds. For example, in the case where $\Theta_{BC} = \Theta_{AB}$ it is clear that the lower bound will be at least equal to $\max(d_{AB}, d_{BC})$ and the upper bound would be at most equal to q_n. But finding more precise bounds depends on how the different distances relate to each other. This information is given by the distance system: qualitative distances q_i are mapped to consecutive 1-D geometrical intervals δ_i representing distance ranges, which allows us to compare their magnitude. The algorithms make use of those comparisons to derive more restricted lower and upper bounds. For more details on those algorithms, refer to (Clementini, Di Felice and Hernández, 1997).

Reasoning with distances is easier if the underlying distance system has some homogeneous properties, i.e. if the distance ranges δ_i follow a recurrent pattern. Examples of such properties are:

- Monotonicity: each distance range is at least equal to the previous one, i.e.:
$$\delta_i \geq \delta_{i-1} \forall i > 0$$

- Range restriction: each distance range is greater or equal to the sum of all the previous distance ranges, i.e.:
$$\delta_i \geq \sum_{k=0}^{i-1} \delta_k \forall i > 0$$

- Order of magnitude: some distance range can be considered negligible compared to other ones:
$$ord(q_j) - ord(q_i) \geq p \Rightarrow \delta_j >> \delta_i, \forall i, j \geq 0, i < j$$

where $ord(q_i) = i+1$. For example, if p=2, then $\delta_1 << \delta_3$.

	q0	q1	q2	q3	q4
q0	q0,q1	q1,q2	q2	q3	q4
q1	q1,q2	q1,q2	q2,q3	q3	q4
q2	q2	q2,q3	q2,q3	q3,q4	q4
q3	q3	q3	q3,q4	q3,q4	q4
q4	q4	q4	q4	q4	q4

Table 1. Composition table for five distances, same orientation (order of magnitude restriction)

Table 1 shows an example of a composition table for distances. Clementini, Di Felice and Hernández (1997) show how to construct this table. For instance, if A q_2 B and B q_3 C, then we have A $\{q_2,q_3\}$ C. Similar composition tables can be derived for the case of opposite orientations and orthogonal orientations. If more than four

different orientations are used, the general case is handled by using the best suitable algorithm depending on Θ_{BC} (see Figure 1). We only consider here the case of eight distinct orientations, for which we define the following two sub-ranges for the upper half-plane (the ones for the bottom half-plane are determined by symmetry along the axis AB):

- if $\Theta_{AB} < \Theta_{BC} < \text{orth}(\Theta_{AB})$, the lower bounds given by Algorithm 1 and upper bounds of Algorithm 3 are used
- if $\Theta_{BA} < \Theta_{BC} < \text{orth}(\Theta_{AB})$, Algorithm 3 is used

$\text{orth}(\Theta_{AB})$ is the orthogonal directions to Θ_{AB}, e.g. $\text{orth}(N) = \{W,E\}$. More details can be found in the original paper.
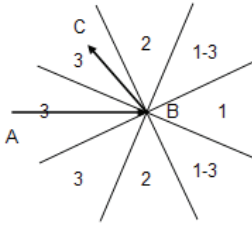

Figure 1. The numbers represent the algorithms used for deriving the composition tables, depending on the orientation of C w.r.t. B

Combining the Composition Tables

The combination of composition tables for topology-orientation and distances is straightforward. Given the relation between A and B and B and C, the principle is to first select the distance composition table to be used, depending on the orientations as previously explained. Then, the algorithm checks for each of the resulting topological-orientation pair whether this pair is still possible when considering the information given by the distance. For example, if $d_{AB} << d_{BC}$, then $\Theta_{AC} = \Theta_{BC}$. All the possible distances are then added to each topological-orientation pair (except if the topology implies some form of contact, in which case the distance should be q_0).

Evaluation

In this section, we first give an example of constraint inference using our framework. We then analyze our model and discuss the positive points as well as the restrictions imposed.

Experiment

We tested our model by propagating spatial constraints on a static network of nine spatial objects, representing different countries in Europe: Finland (FI), France (FR), Germany (DE), Greece (GR), Italy (IT), Poland (PL), Portugal (PT), Spain (ES) and United Kingdom (UK). We used two distinctions for topology (t and d), eight for orientation (N, NW, W, SW, S, SE, E, NE) and five

distance symbols (vc, cl, cm, fr, vf) with an order of magnitude restriction. Initially, all relations between the objects are unknown, i.e., there are 48 possibilities for each relation (because a topological relation *t* implies the distance to be *vc*). We then added the following constraints using Allen's algorithm, one by one:

(C1) *FR [d,SE,vc] UK*
(C2) *ES [t,SW,vc] FR*
(C3) *PT [t,W,vc] ES*
(C4) *UK {[d,NW,cl-cm]} IT*
(C5) *DE [d,N,vc] IT*
(C6) *PL [t,E,vc] DE*
(C7) *FI {[d,N,cl-cm]} PL*
(C8) *IT [t,SE,vc] FR*
(C9) *GR {[d,SE,vc-cl]} IT*
(C10) *GR [d,S,cm] FI*

Adding C1 just impacts on the countries directly involved in the relation, but all the other ones have repercussions on the whole network. For instance, adding C2 results in:
UK {[d,N,vc];[d,N,cl];[d,NW,vc];[d,NW,cl]} ES

When C3 is propagated, it leads to:

PT {[d,W,vc];[d,W,cl];[d,SW,vc];[d,SW,cl]} FR
UK "somewhere N, relatively close to" PT
(exact relationships not shown due to space restrictions)

The process is then repeated for all the constraints. Once they have all been added, we have a more precise idea of the relations holding between all the countries. For instance, we then know that:

PT "is somewhere S-SW and cl-cm from" FI
PL "is somewhere NE-E and cl-cm from" ES
GR {[d,SE,cl];[d,SE,cm]} UK

However, some relations are still too imprecise and would need more constraints to reflect accurately the reality. For example,

UK "is somewhere NW-W-SW and has possibly a boundary with" DE

Analysis

Under certain conditions, we are able to infer implicit knowledge from a given knowledge base and to do so efficiently. When adding new constraints, Allen's algorithm checks the consistency of the network, but it does this only for three-node subnetworks. This means that if we consider any three nodes, their relations are consistent, although the whole network might be inconsistent. This is an inherent weakness of Allen's algorithm, which also applies to the algorithm outlined in the previous section. On the other hand, it makes sure that the inference process is tractable, as it limits the complexity of the algorithm to $O(n^3)$, where n is the number of objects in the network.

In addition to these more general considerations, there are other factors to be considered when applying the algorithm:

The Dimension of Objects. This factor plays an important role in deriving distances. In fact, in the original approach for qualitative distances by Clementini, Di Felice and Hernández (1997), only the case of dimension-less points was treated. While using the same distance composition tables for extended objects often seems to work, in some cases it does not. For example, if we know that *FR vc DE* and *DE vc PL*, then we can infer that *FR vc-cl PL*. This is correct, but if we transpose this case to Alaska, Canada and Greenland, we obtain *Alaska vc-cl Greenland*.

We have not found a completely satisfactory way to deal with this problem. Taking into account the size of objects while deriving the distance composition table would be one solution. Restricting the largest objects to be smaller than the smallest distance range (or inversely, restricting the smallest distance range to be larger than the largest object) is another possibility, but it lacks cognitive plausibility: it would imply that "very close" actually refers to a distance range going from East to West Russia in the case of an application at a geographical space level. Yet another solution would be to give the distances w.r.t. the centre of the objects, but once again this is not very cognitively adequate.

The Transitivity of Distances. Another problem concerns the transitivity of distances. How many distances of the same range can we sum up before switching to the next level? Let us consider the following example, using identical orientations and n distance symbols:

$$A_0 \; q_0 \; A_1, \; A_1 \; q_0 \; A_2, \; \ldots, \; A_{m-1} \; q_0 \; A_m$$

Using the composition tables with the monotonicity restriction or the range restriction leads to $A_0 \; q_0\text{-}q_1\text{-}\ldots\text{-}q_n \; A_k$, if $k \geq n$. The question is: what is the largest k such that $A_k \; q_0 \; A_0$?

Applying the orders of magnitude restriction implies that $A_k \; q_p \; A_0$ for all $k \geq p$. However, it would seem normal that by adding q_0 a certain number of times, we finally reach q_n.

Other Related Problems. In our framework, distances have been considered to be symmetric, i.e., $d_{AB} = d_{BA}$. However, in the case of geographic space, this is often not the case, as pointed out in (Egenhofer and Mark, 1995). Defining what *close* or *far* means is also a problem, because people have different perceptions of those notions, depending on the size of the reference object: to be close to the USA does not necessarily mean the same thing as being close to Belgium.

Conclusion

In this paper, we pointed out some deficiencies of existing models for representing qualitative relations between spatial objects and proposed a model to overcome these deficiencies. Our approach draws together two existing frameworks in order to provide a model dealing with the most important aspects of space: topology, orientation and distances. The resulting framework is efficient and effective if all relations use the same frame of reference, distances are symmetric and objects are one-piece objects without holes. In addition, we might have to impose a restriction on their sizes. Further work would include implementing the mechanisms to deal with different frames of reference.

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