Dynamical Spatial Systems – A Potential Approach for the Application of Qualitative Spatial Calculi*

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Abstract

A dynamical systems approach for modeling changing spatial environments is formalised. The formalisation adheres to the representational and computational semantics of situation calculus and includes a systematic account of all aspects necessary to implement a domain-independent qualitative spatial theory that is applicable across diverse application areas. Foundational to the formalisation is a situation calculus based causal theory and a generalised view of qualitative spatial calculi that encompass one or more spatial domains. Furthermore, aspects considered inherent to dynamic spatial systems are also accounted for and the relevant computational tasks addressed by the proposed formalisation are stated explicitly.

Motivation

Research relevant to the representation and computational modelling of qualitative spatial calculi is mature – there is a general consensus on the underlying properties that qualitative (spatial) calculi should fulfill, to qualify as one per se, and to be efficiently utilisable from a computational viewpoint (Ligozat & Renz 2004; Renz & Nebel 2001). An important next step, therefore, is the application of existing spatial models in application domains such as intelligent systems, cognitive robotics and GIS, to name a few areas where spatial modelling is directly utilized. This applicability issue, closely related to the broader problem of the integration of specialisations such as qualitative spatial reasoning within general logic-based common-sense reasoning frameworks (McCarthy 1977), is fraught with difficulties – if existing spatial theories are to be applied in practical application domains, several key requirements from a specific 'dynamic spatial systems' viewpoint need to be accounted for: (R1). Seamless integration of a domain-independent 'qualitative physics' that is based on existing qualitative theories of space, (R2). Support for modelling and reasoning with dynamic teleological and causal accounts of a system or process in addition to the representation of the underlying (qualitative) physics, (R3). Incorporation of non-monotonic or default forms of inference, which is necessitated by the requirement to model human-like common-sense reasoning patterns, (R4). Investigation of the implications of some of the fundamental epistemological problems (frame, ramification and qualification), which have otherwise assumed a primary significance within the symbolic artificial intelligence domain, for the special case of dynamic spatial systems, and (R5) An account of concurrency and continuity for the specialised spatial domain in the context of existing temporal reasoning approaches.

Notwithstanding that incorporating (R1–R5) within one framework is a difficult proposition demanding unification along ontological, representational and computational levels, the fact that there exist a wide-range of representational apparatus based on mathematical logic, which address the aforementioned requirements from a general viewpoint, needs to be better appreciated within specialised spatial reasoning domain. Albeit in an isolated manner, several formalisations (e.g., *situation calculus* (McCarthy & Hayes 1969), *event calculus* (Kowalski & Sergot 1986), *fluent calculus* (Thielscher 1998)) developed under the umbrella term of 'reasoning about actions and change' address issues related to modelling dynamical systems in general and it is necessary that these be given adequate attention within the spatial domain. We hypothesize that a foundational approach that utilizes a dynamical systems perspective is well-suited for the modelling of temporally varying spatial systems. By a foundational approach, it is implied that key aspects of modelling a dynamic spatial system are thoroughly investigated, keeping in mind the following objectives: (O1). Usability of existing qualitative models of space and/or spatial dynamics is preserved, (O2). Support for basic features or spatial phenomena that would be regarded as inherent in any time-varying spatial system is available, (O3). An explicit statement of the computational tasks that are either desired for specific applications or those that will directly follow from the proposed formalisation is made and finally, (O4). It is also desired that these objectives be achieved in the context of a rigorous formalism that has a well-defined representational and computational semantics that addresses atleast some of the requirements stated in (R1–R5) and also facilitates the achievement of objectives (O3). In this paper, we first present a high-level overview of our approach, with reference to objectives (O1)-(O4), toward modelling dynamical spatial systems. This is followed by the description of

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(the key aspects of) a situation calculus based theory, which is a direct formalisation of the components identified as being necessary to operationalise a 'dynamic spatial systems' perspective. Finally, the relevant computational tasks addressed by the proposed formalisation are stated explicitly and exemplary reasoning patterns are illustrated.

A Dynamical Systems Approach

The notion of dynamic systems used here is closely related to the interpretation in (Sandewall 1994), where a dynamic system is defined as one whose state changes with time and that state transitions in the systems only occur as a result of explicitly defined occurrences, whatever be the ontological status of such occurrences, otherwise the state of the system remains stable.

A1. Space and Occurrences – Ontological Aspects A dynamic spatial system viewpoint is applicable in domains as disparate as cognitive robotics, event-based GIS and possibly other information theoretic exemplars. Therefore, a general view of what constitutes as '*objects*' and '*occurrences*' (or change operators) within potential use-cases needs to be adopted. We operate within a region-based framework involving spatially extended objects and maintain the typical ontological distinction between an object and the region of space that it occupies. *Valid regions* are defined as follows: '*A region is valid if it has a well-defined spatiality, is measurable using some notion of n-dimensional measurability that is consistent across inter-dependent spatial domains (e.g., topology and size) and if the region is convex and of uniform dimensionality*.' Also, we assume the existence of a domain-specific transfer function '$space(object)$', which is essentially a time-dependent mapping from 'object-space' to 'region-space'; however, for brevity, we sometimes refer to spatial relationships directly between objects instead of regions of space. Such a region-based framework is suitable for fine-scale analysis with primitive objects (i.e., their idealized extensions in space) or macro-level analysis with aggregates of entities with or without a well-defined spatiality, i.e., it is possible to define an appropriate spatial semantics for modelling phenomena such as *growth* and *shrinkage* for *regions* which do not have a truly spatial manifestation, a scenario typical of applications related to the modelling spatial diffusion processes (Cliff, Ord, & Versey 1981).

Dynamic Physical Properties and Constraints Distinctions of objects into strictly-rigid (i.e., no interpenetration and change of shape) and non-rigid types are common; such distinctions in turn determine the manner in which changing spatial relationships between objects are interpreted, i.e., either as a result of motion and/or continuous deformation. However, such a coarse distinction into strict-rigidity and non-rigidity is weak – in reality, objects exhibit characteristics of both. For instance, container-class objects or objects that can be treated as locations do not typically grow, shrink or change shape, but they obviously participate in containment relationships with other objects. Similarly, a fluid body is fully flexible, but upon solidification, there is a change in its physical nature which has further implications on the spatial relationships which it may participate in with other objects. Therefore, within a dynamic setup, we not only need an elaborate classification of object properties, but also have to account for the fact that these may be dynamic – for instance: 'A *container* object is completely filled with *water*. In this state, the container (or water) can still contain some other object, lets say, by way of *dropping* a small metal *ball* in the *container*. Now lets say that in a later situation, the *water* is frozen and stays that way for eternity. Consequently, no more containment relationships are possible!'. Given the understanding of a *valid region*, exemplary physical properties that are identifiable include: (P1). *allows_containment*, (P2). *can_deform* (i.e., change of shape, growth, shrinkage), (P3). *rigid*, where neither (P1) or (P2) is possible, and (P4). *non-rigid*, where both (P1) and (P2) are possible. For every physical property, a set of constraints that limit the potential spatial relationships that the object may assume with other existing objects would immediately follow. We refer to these as a '*dynamic physical constraint*' (illustrated later); in general, the following interpretation is applicable: ' *A dynamic physical property is that which characteristically pertains to the physical nature of a material object and which necessarily restricts the range of spatial relationships that the respective object, or class of objects, can participate in with other objects, or class of objects. Physical properties are dynamic in nature, e.g., it is possible for a fully-flexible or non-rigid fluid body to solidify and behave like a solid object while it remains in such a state.*' In addition to the properties identified, if one further assumes that regions are also intrinsically oriented, more properties and resulting constraints are identifiable, offcourse at the expense of generality.

Occurrences: Events and Actions The notion of events being utilised herein is causal in nature and is aimed at characterising explicit causal and (if applicable) teleological accounts of the evolution of a spatial process. This is based on an alternate view of events, where events are identified according to their causes and effects (Davidson 1969). Precisely, the following distinctions are applicable: (a). *Internal events*: These are internal to the system being modelled and have associated occurrence criteria. Internal events are deterministic in the sense that if the occurrence criteria for an internal event is satisfied, the event will necessarily occur. (b). *External events*: These are external to the system and unlike internal events, occurrence criteria for these events are not available, and (c). *Non-deterministic events or actions*: These are agent-centric and are therefore, by definition, volitional in nature. Simply, all pre-conditions for a given action may be satisfied and yet the agent may not perform the action. The last distinction is mainly applicable in scenarios where spatial reasoning abilities of real or simulated agents are being modelled (e.g., robotic control software). Note that at the level of a domain-independent spatial theory, such distinctions are not meaningful – a '*transition*' between two objects from being *disconnected* to *containment* can be interpreted as an action-drive change of location or the result of some event! The distinctions are only applicable at a domain-specific level, where it is possible to characterise *occurrences* as directly affecting the underlying spatial structures being modelled, e.g., a *turn-left* action which has the effect of changing an agent's orientation.

A2. Appearance and Disappearance of Objects Appearance of new objects and disappearance of existing ones, either abruptly or explicitly formulated in the domain theory, is characteristic of non-trivial dynamic spatial systems. In robotic applications, it is necessary to introduce new objects into the model, since it is unlikely that a complete description of the robot's environment is either specifiable or even available. Similarly, it is also typical for a mobile robot operating in a dynamic environment, with limited perceptual or sensory capability, to loose track of certain objects because of limited field-of-vision. Even within event-based geographic information systems, appearance and disappearance events are regarded to be an important typological element for the modelling of dynamic geospatial processes (Claramunt & Thériault 1995; Worboys 2005). Therefore, we regard that such phenomena, being intrinsic to a typical dynamic spatial system, merit systematic treatment.

A3. Domain Independent Spatial Dynamics In line with objective (O1), one of the key components of the (dynamical systems) approach being adopted in this work is the development of a domain-independent spatial theory. For this purpose, the high-level aspects of axiomatic spatial calculi relevant to differing aspects of space (e.g., topology, orientation) are relevant. Ontological distinctions notwithstanding, the main high-level aspects of such calculi include: a finite set of jointly exhaustive and pair-wise disjoint (JEPD) relations, compositional inference and consistency maintenance and the representation of change on the basis of the continuity or conceptual-neighbourhood of the underlying relation space. We provide a step-by-step generalisation of the manner in which all these aspects of qualitative spatial calculus may be modelled using the proposed approach.

A4. Computational Tasks: Planning and Explanation One basic task with wide-ranging applicability involves deriving a sequence of spatial (and possibly aspatial) actions that will achieve a desired spatial configuration; here, potential applications involve spatial planning and reconfiguration in domains such as cognitive robotics and architectural design. Diametrically opposite is the task of *postdictum* or *explanation*, where given a set of time-stamped observations or snap-shots (e.g., observation of robot or time-stamped GIS data), the objective is to explain which events and/or actions may have caused the resulting state-of-affairs. The overall framework is designed in view of the desired support for these computational tasks; the objective in this paper is to illustrate the manner in which these reasoning tasks may be performed in the context of the proposed formalisation.

Modelling Dynamic Spatial Systems in the Situation Calculus

We describe the components of a causal theory Σ_{causal} (see Fig. 1) that operationalises the dynamical spatial system perspective described in (A1–A4). The foundational part of the theory, Σ_{sit}, is based on a customized version of the situation calculus formalism. The other component, Σ_{space}, constitutes a domain-independent spatial theory that is usable

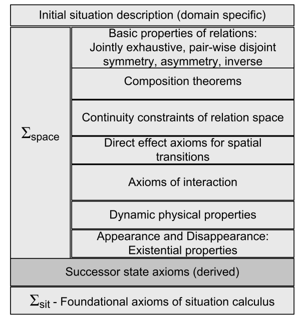

Figure 1: The Causal Framework – Σ_{causal}

in arbitrary application scenarios. The approach to model the underlying qualitative physics of dynamic spatial systems is based on the abstract notion of a 'qualitative spatial calculus', which constitutes a generalized view of a wide-range of spatial calculi that share common semantics. Consequently, (notwithstanding the region-based view adopted herein) any spatial calculus, regardless of the nature of its spatial relationships or its ontological commitments with respect to space (e.g., *point*, *line-segments*), can be instantiated within the proposed causal theory. The theory also accounts for dynamic physical properties of objects as well as their appearances and disappearances, phenomena deemed characteristic to dynamic spatial systems. Detailing the axioms of the theory Σ_{causal} is neither required nor possible in this paper; here, we outline the key aspects of Σ_{causal} so as to provide a broad view of the framework thereby facilitating an intuitive interpretation of the reasoning tasks that follow from the formalisation (see (Bhatt 2007) for elaborations).

I. The Foundational Theory We use a first-order many-sorted language, denoted $\mathcal{L}_{sitcalc}$, with equality and the usual alphabet of logical symbols: $\{\neg, \wedge, \vee, \forall, \exists, \supset, \equiv\}$. There are sorts (and corresponding variables) for *events* and *actions* – $\Theta = \{\theta_1, \theta_2, \ldots, \theta_n\}$, *situations* – $S = \{s_1, s_2, \ldots, s_n\}$, *spatial objects* – $O = \{o_1, o_2, \ldots, o_n\}$ and *regions* – $R = \{r_1, r_2, \ldots, r_n\}$. S_0 is a constant symbol that denotes the initial situation and $\Phi = \{\phi_1\ \phi_2, \ldots, \phi_n\}$ is the set of propositional and functional fluents. $\mathcal{L}_{sitcalc}$ consists of 5 foundational elements that are used in the formulation of the meta-theory Σ_{sit} and domain-independent spatial theory Σ_{space}. These elements include: (L1). A ternary relationship of property causation denoted by '$Caused$', (L2). A ternary relationship of situation-dependent property exemplification denoted by '$Holds$', (L3). Action precondition axioms denoted by '$Poss$', (L4). Event occurrence axioms denoted by '$Occurs$', and (L5). A binary function symbol '$Result$' denoting the situation resulting from the happening of an occurrence. Presuming basic familiarity with situation calculus, the usage of these elements will be self-explanatory.

$$Caused(\phi(\vec{r}), \gamma, s) \supset Holds(\phi(\vec{r}), \gamma, s) \tag{1a}$$

$$\begin{aligned} & Poss(\theta(\vec{o}), s) \vee Occurs(\theta(\vec{o}), s) \supset \\ & [\neg(\exists\, \gamma')\, Caused(\phi(\vec{r}), \gamma', Result(\theta(\vec{o}), s)) \supset \\ & Holds(\phi(\vec{r}), \gamma, Result(\theta(\vec{o}), s)) \equiv Holds(\phi(\vec{r}), \gamma, s)] \end{aligned} \tag{1b}$$

Using the language of $\mathcal{L}_{sitcalc}$, the foundational theory Σ_{sit} comprises of: (F1). A property causation axiom determining when fluent values *hold* in situations (1a), (F2). A generic frame axiom that incorporates the principle of inertia (1b), i.e., what does not change when occurrences happen, (F3). Unique names axioms for occurrences, fluents, fluent values and situations, and (F4). Domain-closure axioms for all fluent values, including functional and propositional. Note that (F3–F4) are not included herein.

II. The Spatial Theory This consists of the formalisation of an underlying domain-independent qualitative physics and is founded on the meta-theory Σ_{sit}. It consists of a systematic axiomatisation of all aspects relevant to modelling one or more (possibly inter-dependent) spatial calculi. For illustration purposes, we utilizes the RCC-8 fragment of the region connection calculus (Randell, Cui, & Cohn 1992). Each component of $[\Sigma_{space} \equiv_{def} \Sigma_{cc} \cup \Sigma_{de} \cup \Sigma_{oc} \cup \Sigma_{rc}]$ is elaborated on in the following:

Continuity Constraints (Σ_{cc}) and Direct-Effect Axioms (Σ_{de}) At the domain-independent level, the most primitive means of change is an explicit change of spatial relationship between two objects – let $tran(\gamma,\ o_i,\ o_j)$ denote such a change, read as, o_i and o_j *transition* to a state of being γ. A formalisation of the qualifications for every such spatial transition is necessary – basically, this is equivalent to incorporating the principle of conceptual neighbourhood based change (Freksa 1991). Let '$neighbour$' denote a binary continuity relationship between two spatial relations; given a spatial domain consisting of n distinct spatial transitions that are possible, a total of n transition per-condition axioms of the form in (2a) are required for each spatial domain being modelled:

$$Poss(tran(ec,\ o_i,\ o_j),\ s) \equiv [space(o_i,\ s) = r_i \wedge space(o_j,\ s) = r_j \wedge (\exists\, \gamma')\, Holds(\phi_{top}(r_i,\ r_j),\ \gamma',\ s) \wedge neighbour(ec,\ \gamma')] \tag{2a}$$

$$Poss(tran(ec,\ o_1,\ o_2),\ s) \vee Occurs(tran(ec,\ o_1,\ o_2),\ s) \supset Caused(\phi_{top}(o_1,\ o_2),\ ec,\ Result(tran(ec,\ o_1,\ o_2),\ s)) \tag{2b}$$

Additionally, for every spatial transition within the spatial domain(s) being modelled, a formalisation of their respective direct effects is required, i.e., a total of n direct effects of the form in (2b) for a domain with n base relationships.

Ordinary (Σ_{oc}) and Ramification (Σ_{rc}) Constraints State constraints express temporally invariant laws within the theory – for the present task, these include the basic properties of the underlying spatial calculus being modelled. In general, we need a total of n (ordinary) state constraints of the form in (3a) to express the jointly-exhaustive property of a set of n base relations. Similarly, $[n(n - 1)/2]$ constraints of the form in (3b) are sufficient to express the pair-wise disjointness of n relations. Other miscellaneous properties including symmetry, asymmetry and inverse of the base relations too can be expressed using ordinary state constraints. Such constraints are also used to model '*physical constraints*' that are definable using the exemplary physical properties from (P1–P4) – (3c–3d) constitutes an example from the combinations that are possible even in the simplest case of (P1–P4). Whereas (3c) represents the constraints between semi-rigid and rigid objects, (3d) covers the case of strictly rigid objects.

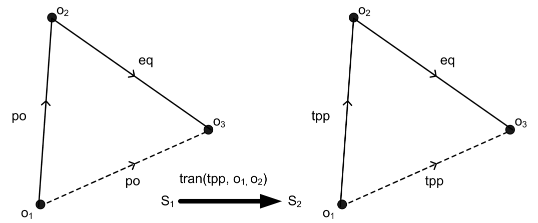

Figure 2: Compositional Constraints and Ramifications

$$\forall s.\ \neg[Holds(\phi_{top}(o_1,\ o_2),\ ec,\ s) \vee Holds(\phi_{top}(o_1,\ o_2),\ po,\ s) \vee Holds(\phi_{top}(o_1,\ o_2),\ tpp,\ s) \vee Holds(\phi_{top}(o_1,\ o_2),\ tpp^{-1},\ s) \vee Holds(\phi_{top}(o_1,\ o_2),\ eq,\ s) \vee Holds(\phi_{top}(o_1,\ o_2),\ ntpp,\ s) \vee Holds(\phi_{top}(o_1,\ o_2),\ ntpp^{-1},\ s)] \supset Holds(\phi_{top}(o_1,\ o_2),\ dc,\ s) \tag{3a}$$

$$\forall s.\ \neg\,[Holds(\phi_{top}(o,\ o'),\ dc,\ s) \wedge Holds(\phi_{top}(o,\ o'),\ ec,\ s)] \tag{3b}$$

$$(\forall o_i,\ o_j,\ s).\ \{allows\_containment(o_j,\ s) \wedge \neg\, can\_deform(o_j,\ s) \wedge rigid(o_i,\ s) \supset [(\exists r_i, r_j)\, space(o_i,\ s) = r_i \wedge space(o_j,\ s) = r_j] \wedge Holds(\phi_{top}(r_i,\ r_j),\ \gamma,\ s)\}\ where\ \gamma \in \{dc,\ ec,\ po,\ eq,\ tpp,\ ntpp\} \tag{3c}$$

$$(\forall o_i,\ o_j,\ s).\ rigid(o_i,\ s) \wedge rigid(o_j,\ s) \supset [(\exists r_i, r_j)\, space(o_i,\ s) = r_i \wedge space(o_j,\ s) = r_j] \wedge Holds(\phi_{top}(r_i,\ r_j),\ \gamma,\ s)\ where\ \gamma \in \{dc,\ ec\} \tag{3d}$$

$$\forall s.\ [Holds(\phi_{top}(o_1,\ o_2),\ tpp,\ s) \wedge Holds(\phi_{top}(o_2,\ o_3),\ eq,\ s) \supset Caused(\phi_{top}(o_1,\ o_3),\ tpp,\ s)] \tag{3e}$$

A second type of state constraints constitutes the so-called ramification or indirect yielding ones – basically, these contain implicit side-effects in them that need to be accounted for whilst reasoning about the effects of events and actions (Lin & Reiter 1994). Consider the example in Fig. 2: here, a change of topological relationship between o_1 and o_2 form po in situation S_1 to tpp in situation S_2 also has an indirect effect on the relationship between o_1 and o_3 in the latter situation. Referring to (Bhatt 2007) for details, here, it suffices to mention that the theory includes one ramification constraint of the form in (3e) for every compositional theorem and axiom of interaction. Assuming only one spatial domain is being modelled (i.e., there are no axioms of interaction), we need a total of $n \times n$ constraints of the form in (3e) for a calculus consisting of n spatial relationships.

Appearance and Disappearance The problem of identity maintenance is beyond the scope of this work – we presume object identity whilst modelling appearances and disappearances. Our focus has been on the model-theoretic implications of (achieving the effect of) modifying a *fixed* domain of discourse when objects, previously unknown, come into existence. The solution involves maintaining the existential status of every object by a propositional fluent, namely $exists(o)$. Additionally, two special external events – $appearance(o)$ and $disappearance(o)$ – are definable in domain specific ways. Further, appropriate precondition and effect axioms (of the form in (2)) that govern the dynamics of the existential fluent are defined – e.g., "$appearance(a)$ causes $exists(a)$ to be $true$ in situation s" – and a form of non-monotonic reasoning is applied to infer that the new object does not $exist$ in the situation-based history of the system. Finally, axioms are also introduced to

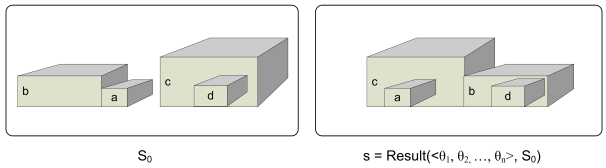

Figure 3: Spatial Re-configuration

consistently maintain the spatial relationship of the new object with other previously existing objects in the present as well as past situations. Offcourse, it is also ensured that an object that has disappeared cannot participate in spatial relationships with any other object until such a future *situation* when it re-appears. The set of axioms modelling such phenomena is elaborate and therefore excluded from this paper.

Derivation of Successor State Axioms Successor state axioms (SSA) specify the causal laws of the spatial theory being modelled, i.e., what changes as a result of various occurrences in the system being modelled. We utilise the seminal approach of (Reiter 1991) for the derivation of SSA's (for solving the Frame problem) and the extensions thereof by (Lin & Reiter 1994) for incorporating the presence of ramification yielding state constraints, which, in so far as the domain-independent level is concerned, are represented by Σ_{rc}.

$$\begin{aligned} & Poss(\theta,\ s)\ \vee\ Occurs(\theta,\ s)\ \supset\ [Holds(\phi_{top}(o_i,\ o_j),\ tpp, \\ & Result(\theta,\ s))\ \equiv\ \{\theta\ =\ tran(tpp,\ o_i,\ o_j)\}\ \vee \\ & \{(\forall\,\gamma^{'})\ Holds(\phi_{top}(o_i,\ o_j),\ tpp,\ s)\ \wedge\ \theta\ \neq\ tran(\gamma^{'},\ o_i,\ o_j)\}\ \vee \\ & \{(\exists o_k)\ Holds(\phi_{top}(o_i,\ o_k),\ tpp,\ s)\ \wedge\ Holds(\phi_{top}(o_k,\ o_j),\ eq,\ s)\}] \end{aligned} \tag{4a}$$

The derivation involves minimizing, using circumscription (McCarthy 1980), the extensionality of the ternary '$Caused$' relation in order to derive causation axioms determining what changes, i.e., what is forced to change, given the direct effects of the known occurrences and the ramification constraints within the axiomatisation. The resulting causation axioms, not included here, are instrumental in obtaining a SSA for every fluent within the system. The SSA in (4a) is presented as one example – here, it may be verified that this axiom formalises every possible way in which two objects establish a 'tpp' relationship. The conjunction of all SSA's with the set of formulae introduced so far results in the final theory Σ_{causal}, which is then directly usable for reasoning purposes.

Spatial Reasoning within the Causal Framework

Given the structure and semantics of the situation calculus based theory Σ_{causal}, fundamental reasoning tasks involving projection and explanation can be directly represented (Reiter 2001; Shanahan 1993). In the spatial domain, these translate to spatial planning/re-configuration and causal explanation of dynamic spatial phenomena. In the following, we focus on illustrating the structure of these reasoning tasks for the theory Σ_{causal}:

Spatial Re-configuration Spatial re-configuration is a form of spatial planning where the objective is to derive a sequence of spatial transitions that will achieve the desired objective; here, an objective is specifiable by a desired

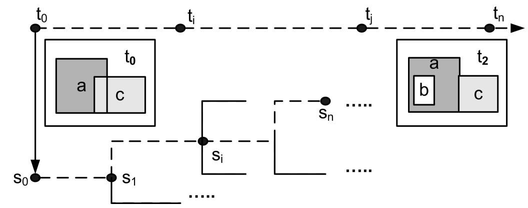

Figure 4: Abductive Explanation

configuration of the objects of the domain. For instance, given the following: the domain-independent spatial theory in Σ_{space}, the foundational axioms of the situation calculus in Σ_{sit}, a (partial) initial situation and a goal-state description in Ω_{ini} (5a) and $\Omega_{goal}[s]$ (5b) respectively (see Fig. 3), the re-configuration task essentially involves deriving the entailment in (5c)[1] – what needs to be done is to derive a legal-binding for the (only) free situation term s in (5c) as a side-effect of a theorem-proving task; this approach, where plans are synthesized as a side-effect of theorem-proving being a standard account of planning in the situation calculus (Reiter 2001).

$$\begin{aligned} \Omega_{ini}\ \equiv\ & [\,Holds(\phi_{top}(a,\ b),\ ec,\ S_0)\ \wedge\ Holds(\phi_{top}(d,\ c),\ tpp,\ S_0)\ \wedge \\ & Holds(\phi_{top}(a,\ c),\ dc,\ S_0)\ \wedge\ Holds(\phi_{top}(b,\ c),\ dc,\ S_0)\ \wedge \\ & Holds(\phi_{ort}(a,\ c),\ r,\ S_0)\ \wedge\ Holds(\phi_{ort}(b,\ a),\ r,\ S_0)] \end{aligned} \tag{5a}$$

$$\begin{aligned} \Omega_{goal}[s]\ \equiv\ & [\,Holds(\phi_{top}(a,\ c),\ tpp,\ s)\ \wedge\ Holds(\phi_{top}(d,\ b), \\ & tpp,\ s)\ \wedge\ Holds(\phi_{top}(b,\ c),\ ec,\ s)\ \wedge\ Holds(\phi_{ort}(c,\ b),\ r,\ s)\,] \end{aligned} \tag{5b}$$

$$\Sigma_{causal}\ \cup\ \Omega_{ini}\ \models\ [\,(\exists s).\,Legal(s)\ \wedge\ S_0\ \leq\ s\ \wedge\ \Omega_{goal}[s]\,] \tag{5c}$$

$$\begin{aligned} Legal(s)\ \equiv_{def}\ & (\forall\,\theta,\ s^{'}).\,[Result(\theta,\ s^{'})\ \leq\ s]\ \supset \\ & [Poss(\theta,\ s^{'})\ \vee\ Occurs(\theta,\ s^{'})] \end{aligned} \tag{5d}$$

$$\left\{\begin{aligned} & s\ =\ Result(<\theta_{61},\ \theta_{62},\ \theta_{63},\ \theta_{64},\ \theta_{65}>,\ Result(\vec{\theta_5}, \\ & \quad Result(\vec{\theta_4},\ Result(\vec{\theta_3},\ Result(\vec{\theta_2},\ Result(\vec{\theta_1},\ S_0)))))) \\ & \vec{\theta_1}\ =\ [\,tran_{11}(rf,\ a,\ c),\ tran_{12}(f,\ a,\ c)\,] \\ & \vec{\theta_2}\ =\ [\,tran_{21}(po,\ d,\ c),\ tran_{22}(ec,\ d,\ c),\ tran_{23}(f,\ d,\ c)\,] \\ & \vec{\theta_3}\ =\ [\,tran_{31}(fl,\ d,\ b),\ tran_{32}(f,\ d,\ b)\,] \\ & \vec{\theta_4}\ =\ [\,tran_{41}(ec,\ d,\ b),\ tran_{42}(po,\ d,\ b),\ tran_{43}(tpp,\ d,\ b)\,] \\ & \vec{\theta_5}\ =\ [\,tran_{51}(ec,\ a,\ c),\ tran_{52}(po,\ a,\ c),\ tran_{53}(tpp,\ a,\ c)\,] \\ & \vec{\theta_6}\ =\ [\,tran_{61}(rf,\ b,\ c),\ tran_{62}(f,\ b,\ c),\ tran_{63}(lf,\ b,\ c), \\ & \quad tran_{64}(l,\ b,\ c),\ tran_{65}(ec,\ b,\ c)\,] \end{aligned}\right\} \tag{5e}$$

For simplicity, assume a simple intrinsic orientation system with labels $left(l)$, $front(f)$, $front-left(fl)$ and so forth. Also assume that all object in Fig. 3 always have their respective '$fronts$' facing the same direction. Although a proof cannot be included here, it is worth highlighting that for this particular example, the binding for the free situational term 's' takes the form of a situation-based history (5e) that is rooted in the initial situation 'S_0' – i.e., the derived sequence of spatial transitions achieves the desired re-configuration.

Occurrence Driven Causal Explanation Explanation, in general, is regarded as a converse operation to temporal projection essentially involving reasoning from effects to causes, i.e., reasoning about the past (Shanahan 1989). In

[1] '$S_0\ \leq\ s$' denotes that s includes S_0 in its sub-history.

the context of the situation calculus formalism, an abductive approach to explanation has been proposed by (Shanahan 1993), and causal explanation is treated as such in this work. The objective in an explanation task is as follows: given a set of temporally-ordered snap-shots (e.g., observations of a mobile-robot or time-stamped GIS data), derive a set of events and/or actions that may have caused the observed state-of-affairs. In the following, we outline the structure of the causal explanation task without going into the details of the underlying/supporting axiomatisation: 'consider the illustration in Fig. 4 – the situation-based history $< s_0,\ s_1, \ldots,\ s_n >$ represents one path, corresponding to a actual time-line $< t_0,\ t_1, \ldots,\ t_n >$, within the overall branching-tree structured situational space. Furthermore, assume a simple system consisting of objects 'a', 'b' and 'c' and also that the state of the system is available at time-point t_i and t_j. Note that the situational-path and the time-line represent an actual as opposed to a hypothetical evolution of the system. From the viewpoint of this discussion, two auxiliary predicates, namely $HoldsAt(\phi,\ t)$ and $Happens(\theta,\ t)$, that range over '*time-points*' instead of '*situations*' are needed to accommodate the temporal extensions required to map a path in the situation-space to an actual time-line; complete definitions can be found in (Pinto 1994). Given an initial situation description as in Φ_1 (see (6)), where 'b' does not exist and 'a' and 'c' are partially overlapping, in order to explain an observation sentence such as Φ_2, a formula of the form in Δ needs to be derived'.

$$\left\{\begin{array}{l}
\Phi_1 \ \equiv\ HoldsAt(\phi_{top}(a,\ c),\ po,\ t_1)\\
\Phi_2 \ \equiv\ HoldsAt(\phi_{top}(a,\ c),\ ec,\ t_2)\ \wedge\ HoldsAt(exists(b),\ true,\ t_2)\\
\qquad \wedge\ HoldsAt(\phi_{top}(b,\ a),\ ntpp,\ t_2)\\
[\Sigma_{sit}\ \wedge\ \Sigma_{space}\ \wedge\ \Phi_1\ \wedge\ \Delta]\ \models\ \Phi_2, where\\
\Delta\ \equiv\ (\exists\, t_i,\ t_j,\ t_k).[\,t_1\ \leq\ t_i\ <\ t_2\ \wedge\ Happens(appearance(b),\ t_i)]\\
\wedge\ [\,t_i\ <\ t_j\ <\ t_2\ \wedge\ Happens(tran(b,\ a,\ tpp),\ t_j)]\ \wedge\\
[t_k\ <\ t_2\ \wedge\ Happens(tran(a,\ c,\ po),\ t_k)]\ \wedge\ [\,t_k\ \neq\ t_i\ \wedge\ t_k\ \neq\ t_j]
\end{array}\right\} \tag{6}$$

The derivation of Δ primarily involves non-monotonic reasoning in the form of minimising change ('$Caused$' and '$Happens$' predicates), in addition to making the usual default assumptions about inertia; the details are beyond the scope of this paper.

Discussion and Outlook

Based on a customised situation calculus formalism, a dynamical systems approach for modelling a domain-independent qualitative spatial theory has been proposed. The approach is primarily aimed at operationalising qualitative spatial calculi toward the representation of useful computational tasks that involve planning, explanation and simulation in arbitrary spatial scenarios. Key aspects considered inherent to '*dynamic spatial systems*' (e.g., dynamic physical properties, appearance and disappearance) as well as epistemological issues related to the frame and ramification problems that arise whilst modelling changing spatial systems are also accounted for. Property persistence, i.e., the spatial relationship between two objects '*typically*' remains the same, is one default reasoning pattern connected to the frame problem that was encountered here. Similarly, the ramification problem arises whilst modelling constraints containing indirect effects (e.g., compositional constraints, axioms of interaction). It is also evident that several forms of non-monotonic inference are useful (e.g., in deriving the successor state axioms and also in explanation tasks), and in some cases even necessary (e.g., compositional constraints), when reasoning about changing spatial relationships between objects. Clearly, in addition to aspects already accounted for, a closer look at application-level use-cases that may benefit from default or non-monotonic reasoning patterns is an important next step and this work is geared towards that. The issue of the best implementation strategy for the proposed formalisation – within or outside the the context of existing situation calculus based languages such as Golog, Indigolog – is also an important next step. At a broader level, comparative studies with other formal techniques (e.g., event or fluent calculus) are also essential; here, evaluation parameters, for instance, include representational parsimony, computational complexity and so forth.

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