Reasoning about Topological and Positional Information in Dynamic Settings

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Abstract

Typical application fields of spatial and spatio-temporal representation formalisms and reasoning techniques include geographic information systems (GIS), mobile assistance systems for route finding, layout descriptions, and navigation tasks of robots interacting with humans. Often these formalisms focus on specific spatial aspects in that they use either topological or positional relations. In this paper we propose a formalism which allows for representing topological and positional relations between disks in the plane. Although this formalism employs a rather abstract representation of objects as disks, it provides an interesting test-bed for investigating typical problems that arise when topological and positional information about objects are combined, or when such combined formalisms are used to represent continuous changes in the considered application scenario.

Introduction

The field of qualitative spatial and spatio-temporal reasoning comprises representation formalisms and reasoning techniques developed over the past 30 years to deal with purely qualitative descriptions of spatial and temporal situations. Since natural languages provide a rich repertoire of qualitative concepts to talk about space and time, qualitative formalisms and related algorithmic methods (subsumed under the term *qualitative calculi*) seem appropriate whenever commonsense knowledge is to be expressed in more formalized contexts.

In recent years combinations of such formalisms have gained more attention in the literature. Such combinations include first combinations of qualitative calculi that deal with different spatial aspects (such as topology, orientation, direction, distance) and, second, so-called temporalizations of qualitative calculi. Examples of the first mentioned line of research include the combination of topological and qualitative size relations (Gerevini & Renz 2002), the combination of Allen's interval relations and relative duration (Pujari, Kumari, & Sattar 1999), the combination of interval and direction relations (Renz 2001), and the combination of topological and directional relations (Li 2007). The second line of research deals with calculi that allow for representing changes of spatial situations in time. A prominent example is the interval temporalization of the RCC8 calculus (Gerevini & Nebel 2002).[1]

In this paper we will focus on very simple geometric objects, namely closed disks in the Euclidean plane. If we describe possible relations from a topological viewpoint, we arrive at the *closed disk algebra* (Egenhofer 1991; Smith & Park 1992; Düntsch 2005), which is closely related to the RCC8 calculus (Randell, Cui, & Cohn 1992). Since disks have a natural "barycenter", namely its geometric center, one can also define positional relations between disks in terms of the positional relations between their center points. For these positional relations we use the relations of Frank's cardinal direction calculus (Frank 1996; Ligozat 1998). In this formalism it is possible to express that a disk is a northern part of another one and that it is connected in the northeast to a third one.

Although topological and positional information are usually assumed to be orthogonal, we will observe that in the presented formalism positional relations between two or more objects will restrict the set of possible topological relations between these objects rather tightly. This aspect is crucial, when composition-based reasoning methods used for the component calculi, namely closed disk algebra and cardinal direction calculus, need to be transferred to their combination.

To model spatial changes in time and to reason with such changes, so-called *neighborhood graphs* (Freksa 1991) have been extensively discussed in the literature on qualitative calculi. Neighborhood graphs present possible transitions between spatial relations and may be used to represent various constraints on the considered objects (kinematics, size persistency, navigation rules, etc.) (Dylla *et al.* 2007). A more expressive variant of these neighborhood graphs are *dominance spaces* (Galton 2000). The combination of such neighborhood graphs is the second topic of this paper.

The paper is organized as follows: In the next section we briefly sketch the underlying component calculi, the closed disk calculus (or: "RCC8 on disks") and the cardinal direction calculus. Then we describe the combined formalism.

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[1] A quite different approach to representing spatial changes in time is followed in the work by de Weghe *et al.* (2005): in the trajectory calculi qualitative relations between trajectories (of moving objects) are considered, while the formalism presented here addresses changes of qualitative (spatial) relations in time.

In particular, we show how a refinement of Allen's interval algebra can be expressed in this framework. Next we outline how the composition table of the combined calculus can be generated from the composition tables of the component calculi by using some restriction rules. Afterwards we describe the neighborhood graph of the combined formalism and finally we summarize the results of the paper and give a short outlook on future research questions.

Spatial Constraint Satisfaction Problems

Qualitative reasoning problems are usually cast as constraint satisfaction problems (CSP), i.e., as the problem to determine whether a given finite set of constraints (a *constraint network*) is satisfiable or entailed by another constraint network. Here we focus on binary constraint satisfaction problems: a binary (qualitative) constraint network is a finite set of constraints of the form xRy, where x and y are variables taking values in a given domain D and R is a binary relation R defined on D. The relations allowed for expressing such constraints are the following: one starts from a (finite) set of jointly exhaustive and pairwise disjoint *base relations* on the domain. To represent imprecise knowledge, one allows for arbitrary unions of base relations (written as *sets* of base relations) as constraint relations. Since the considered domains in qualitative reasoning are infinite, constraint networks are manipulated on the symbolic level. That is, given a constraint network one considers its associated (labelled and directed) constraint graph, which is obtained as follows: the node set is the set of variables of the network, each arc in the graph corresponds to a constraint between the considered variables, and each arc label is the set of base relations from which the constraint relation is build up. To refine this constraint graph one often uses the *composition function*, which encodes information about which constraint relation xRy is (semantically) possible given the constraint relations between objects x and z and between z and y, respectively. The composition function is uniquely determined by the composition of base relations (which is usually written down as a table and referred to as the *composition table*). Composition of relations is crucial for applying the path consistency algorithm, which successively refines the labels $R_{x,y}$ between two variable nodes x and y in the constraint graph by the operation

$$R_{x,y} \longleftarrow R_{x,y} \cap (R_{x,z} \circ R_{z,y}),$$

until a fixpoint is reached.

RCC8 on disks. One of the most prominent calculi in the domain of spatial qualitative reasoning is the region connection calculus RCC8. This calculus allows for expressing relations between *regions*, which often are represented as non-void regular closed subsets of some topological space. The set of RCC8 base relations consists of the relations DC ("disconnected"), EC ("externally connected"), PO ("partially overlapping"), TPP ("tangential proper part"), NTPP ("non-tangential proper part"), TPPi ("tangential proper part inverse"), NTPPi ("non-tangential proper part inverse"), and EQ ("equal") (cf. Fig. 1).
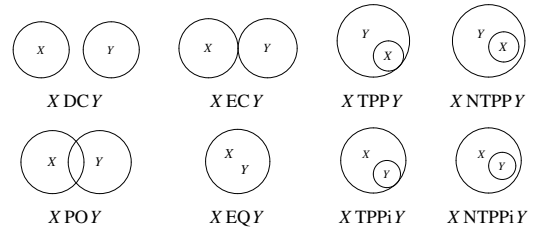

Figure 1: The RCC8 relations

In this paper we restrict the set of possible regions to the set of closed disks in the Euclidean plane. Accordingly, we restrict the RCC8 relations on this domain and obtain a qualitative formalism, which (with some misuse of notation) we will also refer to as RCC8 throughout the paper.[2] Semantically, a *(closed disk) assignment* for the variables in an RCC8 constraint network C is a function $\alpha : V(C) \to \mathbb{R}^2 \times \mathbb{R}^+$ that assigns to each variable d that occurs in C a triple of real numbers $(c, r) \in \mathbb{R}^2 \times \mathbb{R}^+$, where $c = (x, y) \in \mathbb{R}^2$ is the center point and r is the radius of the disk. The model relation w.r.t. an assignment α for C can then be defined in terms of the Euclidean distance d as follows:

$$
\begin{aligned}
\alpha &\models d\,\mathrm{DC}\,d' & &\Longleftrightarrow d(c,c') > r + r' \\
\alpha &\models d\,\mathrm{EC}\,d' & &\Longleftrightarrow d(c,c') = r + r' \\
\alpha &\models d\,\mathrm{PO}\,d' & &\Longleftrightarrow |r - r'| < d(c,c') < r + r' \\
\alpha &\models d\,\mathrm{TPP}\,d' & &\Longleftrightarrow d(c,c') = r' - r > 0 \\
\alpha &\models d\,\mathrm{NTPP}\,d' & &\Longleftrightarrow d(c,c') < r' - r \\
\alpha &\models d\,\mathrm{TPPi}\,d' & &\Longleftrightarrow d(c,c') = r - r' > 0 \\
\alpha &\models d\,\mathrm{NTPPi}\,d' & &\Longleftrightarrow d(c,c') < r - r' \\
\alpha &\models d\,\mathrm{EQ}\,d' & &\Longleftrightarrow d(c,c') = r - r' = 0
\end{aligned}
$$

where $\alpha(d) = (c, r)$ and $\alpha(d') = (c', r')$. An RCC8 *constraint network*, C, is *satisfiable* if there exists an RCC8 assignment for its variables that models all relations of C.

This means, that finding an assignment for a constraint network in which only base relations of RCC8 occur is equivalent to solving a system of quadratic inequalities over the real numbers. It is known that the path consistency method does not even decide satisfiability of basic constraint networks of RCC8 on disks, but that its composition table is the same as for RCC8 on regular closed sets (Düntsch 2005).

Cardinal directions for disks. Given two point-like objects in the Euclidean plane, the relative position between these objects can be described by one of the cardinal direction relations *north*, *northeast*, *east*, etc. Accordingly, the complete set of all 9 base relations of the cardinal direction calculus CD is $\{N, NE, E, SE, S, SW, W, NW, Eq\}$.[3] Note that these relative positions are defined with respect to

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[2]It is clear that the set of "regions" considered here is not closed with respect to mereotopological complements. For the complemented disk algebra we refer to (Düntsch 2005) and (Li & Li 2006).

[3]In this paper we focus on the projection-based approach to cardinal directions, which means that the four main directions (north, east, south, west) correspond to half-lines in the Euclidean plane.

a fixed reference frame. Ligozat (1998) examined the computational complexity of the general satisfiability problem (which is NP-complete) and identified tractable subclasses.

Following we define the relative positions of disks in terms of the relative positions of their center points. This means that we do not take into account the size of disks or topological relations between them in order to determine their positional relations. For example, a disk can be southwest of another disk, although it is completely contained in it. This seems conceptually inadequate, but our target formalism (to be described in the next step) will combine topological relations between disks with positional information of their center points.

The semantics of this formalism (again, by misuse of notation, referred to as CD) can be defined as follows: An *assignment* for a CD constraint network C (over disks) is a closed disk assignment α for the variables in C as defined previously Then the model relation w.r.t. α is introduced by:

$$\alpha \models d \, \mathrm{N} \, d' \iff x = x' \text{ and } y > y'$$
$$\alpha \models d \, \mathrm{NE} \, d' \iff x > x' \text{ and } y > y'$$
$$\alpha \models d \, \mathrm{E} \, d' \iff x > x' \text{ and } y = y'$$
$$\ldots$$

where $\alpha(d) = (x, y, r)$ and $\alpha(d') = (x', y', r')$. As mentioned before, the radii do not play any role in the evaluation of formulae.

Combining RCC8 and Cardinal Direction

The calculus RCC8-CD combines constraint relations of RCC8 with constraint relations of the cardinal direction calculus. That is, base relations of the new formalism are tuples (R, S) (where R is an RCC8-relation R and S is a CD-relation) such that $d \, (R, S) \, d'$ holds true if and only if both $d \, R \, d'$ and $d \, S \, d'$ are true.

In more detail, the set of base relations of RCC8-CD is a subset of the Cartesian product of the set of base relations of RCC8 and CD, respectively. It is obvious that the set of base relations is a proper subset: some relation pairs correspond to the empty relation such as $(\mathrm{EQ}, \mathrm{N})$. More precisely, the base relations of RCC8-CD are all pairs of base relations (R, S) that are not contained in the following set:

$$\{(\mathrm{EQ}, x) : x \neq \mathrm{Eq}\} \cup$$
$$\{(\mathrm{DC}, \mathrm{Eq}), (\mathrm{EC}, \mathrm{Eq}), (\mathrm{PO}, \mathrm{Eq}), (\mathrm{TPP}, \mathrm{Eq}), (\mathrm{TPPi}, \mathrm{Eq})\},$$

i.e., RCC8-CD has $72 - 13 = 59$ base relations.

Variable assignments for the combined calculus are just closed disk assignments, i.e., functions $\alpha : V(C) \to \mathbb{R}^2 \times \mathbb{R}^+$ assigning to each variable d in C a triple of real numbers $(c, r) \in \mathbb{R}^2 \times \mathbb{R}^+$. Then we can simply define:

$$\alpha \models d \, (R, S) \, d' \iff \alpha \models d \, R \, d' \text{ and } \alpha \models d \, S \, d'.$$

A projection of the base relations of this calculus naturally defines a refinement of Allen's interval relations (Allen 1983) with 17 base relations. Contrary to Allen's calculus this refinement not only considers start- and endpoints of intervals in the real time line, but also their midpoints (cf. Table 1).

Table 1: Mapping RCC8-CD relations to interval relations

(DC, W)	*before*		
(EC, W)	*meets*	(NTPPi, W)	
(PO, W)	*overlaps*	(NTPPi, Eq)	*contains*
(TPP, W)	*starts*	(NTPPi, E)	
(TPPi, W)	*finished-by*	(TPP, E)	*finishes*
(NTPP, W)		(TPPi, E)	*started-by*
(NTPP, Eq)	*during*	(PO, E)	*overlapped-by*
(NTPP, E)		(EC, E)	*met-by*
(EQ, Eq)	*equals*	(DC, E)	*after*

Compositional Reasoning

To derive the composition table for RCC8-CD the composition tables for RCC8 on disks (Düntsch 2005) and CD (Ligozat 1998) alone are not sufficient. This means that the composition of RCC8-CD relations is more fine-grained than what is encoded in the composition tables of the component calculi. Consider, for instance, the composition of $(\mathrm{EC}, \mathrm{E})$ and $(\mathrm{EC}, \mathrm{E})$. The RCC8-composition of EC with EC yields the relation $\{\mathrm{DC}, \mathrm{EC}, \mathrm{PO}, \mathrm{TPP}, \mathrm{TPPi}\}$, and the CD-composition of E with E results in the relation E. Using this information only, we can conclude that the composition of $(\mathrm{EC}, \mathrm{E})$ and $(\mathrm{EC}, \mathrm{E})$ is a subset of $\{(\mathrm{DC}, \mathrm{E}), (\mathrm{EC}, \mathrm{E}), (\mathrm{PO}, \mathrm{E}), (\mathrm{TPP}, \mathrm{E}), (\mathrm{TPPi}, \mathrm{E})\}$. However, if we take the actual semantics of RCC8-CD relations into account, composing the relation $(\mathrm{EC}, \mathrm{E})$ with $(\mathrm{EC}, \mathrm{E})$ results in $(\mathrm{DC}, \mathrm{E})$. An analysis (of the 59 base relations) reveals that, given $A \, (R_1, S_1) \, B$ and $B \, (R_2, S_2) \, C$, there are three topological and four positional base situations to be distinguished. The four topological base situations (without EQ) are:

1. Both objects A and C are outside of B (DC, EC);

2. Both objects A and C are inside of B ($\{\mathrm{TPP}, \mathrm{NTPP}\}$);

3. Object A is inside of B and C is outside of B (or vice versa);

4. Object A or C overlaps with B (PO).

Note that in the second case A and C can overlap even if one relation is, for example, NE and the other is SW, which is not possible in the first case. The four positional base situations are:

A. $S_1 = \mathrm{conv}(S_2)$: the cardinal relations are converse;

B. $\mathrm{Ngh}(S_1, \mathrm{conv}(S_2))$: the cardinal relations are (geometrically) neighbored;

C. $\mathrm{SNgh}(S_1, \mathrm{conv}(S_2))$: S_2 is neighbored to a neighbored relation of S_1;

D. all other cases.

There is a difference in composition if S_1 is a line-like relation (e.g., N, S, E, or W), or if S_1 is a plane relation (e.g., NE, SE, SW, or NW). For example (situation 1): If $R_1 = R_2 = \mathrm{DC}$ (cp. Fig. 2), then
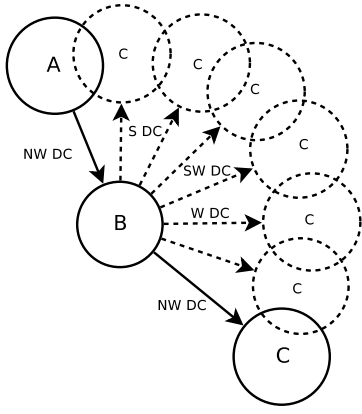
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Figure 2: Possible changes of the position of C effect possible topological relations

- if $S_1 = \mathrm{conv}(S_2)$, then the composition (R_1, S_1) and (R_2, S_2) is *determined component-wisely*, i.e., $(R_1, S_1) \circ (R_2, S_2)$ is the set of all RCC8-CD base relations contained in the Cartesian product of $R_1 \circ R_2$ and $S_1 \circ S_2$;

- if $\mathrm{Ngh}(S_1, \mathrm{conv}(S_2))$, then the composition is determined component-wisely (without $(\mathrm{EQ}, \cdot)$);

- if $\mathrm{SNgh}(S_1, \mathrm{conv}(S_2))$ for $S_1 \in \{\mathrm{NE, NW, SE, SW}\}$, then the composition is determined c.-w. (without $(\mathrm{EQ}, \cdot)$), else the composition is $(\mathrm{DC}, \cdot)$;

- in all other cases, the composition is $(\mathrm{DC}, \cdot)$.

As an example of situation 2 consider the case that $R_1 = \mathrm{TPP}$ and $R_2 = \mathrm{TPPi}$:

- if $S_1 = \mathrm{conv}(S_2)$, then the composition is $(\mathrm{TPP}, \cdot)$, $(\mathrm{TPPi}, \cdot)$, or $(\mathrm{EQ}, \cdot)$, if $S_1 \in \{\mathrm{N, W, S, E}\}$, otherwise $(\mathrm{PO}, \cdot)$, $(\mathrm{EC}, \cdot)$, and $(\mathrm{DC}, \cdot)$ are also possible;

- in all other cases the composition contains the relations $(\mathrm{DC}, \cdot)$, $(\mathrm{EC}, \cdot)$, and $(\mathrm{PO}, \cdot)$.

For situation 3 consider the case that $R_1 = R_2 = \mathrm{NTPP}$: here composition is determined component-wisely. If $R_1 = \mathrm{PO}$ and $R_2 = \mathrm{PO}$ (situation 4), then:

- if $S_1 = \mathrm{conv}(S_2)$ and if $S_1 \in \{\mathrm{N, W, S, E}\}$, the composition contains $(\mathrm{TPP}, \cdot)$, $(\mathrm{NTPP}, \cdot)$, $(\mathrm{TPPi}, \cdot)$, $(\mathrm{NTPPi}, \cdot)$, $(\mathrm{PO}, \cdot)$, and $(\mathrm{EQ}, \cdot)$; otherwise $(\mathrm{EC}, \cdot)$ and $(\mathrm{DC}, \cdot)$ are also possible;

- if $\mathrm{Ngh}(S_1, \mathrm{conv}(S_2))$, then the composition contains $(\mathrm{PO}, \cdot)$, $(\mathrm{TPP}, \cdot)$, $(\mathrm{NTPP}, \cdot)$, $(\mathrm{TPPi}, \cdot)$, $(\mathrm{NTPPi}, \cdot)$, $(\mathrm{EC}, \cdot)$, and $(\mathrm{DC}, \cdot)$;

- if $\mathrm{SNgh}(S_1, \mathrm{conv}(S_2))$ for $S_1 \in \{\mathrm{NE, NW, SE, SW}\}$, then the composition contains $(\mathrm{PO}, \cdot)$, $(\mathrm{TPP}, \cdot)$, $(\mathrm{NTPP}, \cdot)$, $(\mathrm{TPPi}, \cdot)$, $(\mathrm{NTPPi}, \cdot)$, $(\mathrm{EC}, \cdot)$, and $(\mathrm{DC}, \cdot)$;

- in all other cases the composition contains $(\mathrm{PO}, \cdot)$, $(\mathrm{EC}, \cdot)$, and $(\mathrm{DC}, \cdot)$.

An Example. Stevens and Coupe (1978) report on an experiment in which individuals in San Diego, California,
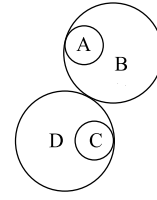

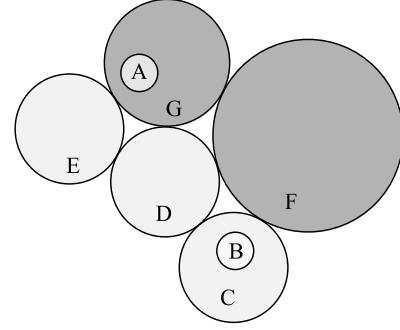Figure 3: A "Reno-San Diego"-like example within the RCC8-CD-framework

Figure 4: An example within the RCC8-CD-framework

were asked to indicate from memory the direction to Reno, Nevada. Most subjects indicated that Reno is northeast of San Diego, while in fact it is northwest. Stevens and Coupe's interpretation is that this distortion is due to a hierarchical organization of spatial knowledge. That is, people tend to store in memory not the location of cities, but rather the relative location of states. Thus, when asked to judge the direction between cities, subjects infer the direction between cities from the spatial relations between hierarchically higher organized objects such as states: Since Nevada is generally east of California, it is wrongly inferred that every city in Nevada is east of each city in California and hence, in particular, that Reno must be east of San Diego. A similar example may be expressed in the language of RCC8-CD (cf. Fig. 3): if D is southwest of B, A is proper part of B and C is proper part of D, then it cannot be concluded that C is southwest of A. A formally correct spatial deduction is provided by the following representation: California is represented by three disks C, D, and E, and Nevada by two disks F and G (cf. Fig. 4). Since A (NTPP, W) G, G (EC, N) D, D (EC, NW) C, and C (NTPPi, S) B, it can be concluded that A (DC, NW) B holds.

Neighborhood-Based Reasoning

Assume that two disks are related by one of the base relations presented in the previous section. What happens if one moves one of the disks (in very small "steps")? Questions of this type are usually addressed by referring to so-called neighborhood graphs (e.g., Fig. 5). Neighborhood graphs present possible transitions between spatial relations. Obviously, the transitions considered possible depend on general

assumptions on how the spatial properties of the objects can change. For example, if we assume that objects do not underlie any changes in size, the neighborhood graph of RCC8 has the form depicted in Fig. 6.

To make the semantics of these neighborhood graphs more precise, consider rigid objects (disks) d_1 and d_2 at two time points $t_0 = 0$ and $t_1 = 1$ (the fixed radius of each disk is denoted by r_i). Then a *trajectory* of one of the disks can be simply described by a continuous (or smooth) function from $[0,1]$ to $\mathbb{R}^2$ that assigns to each time point $0 \leq t \leq 1$ the position of the center point of the disk at t. Let now R_0 and R_1 be the qualitative spatial relations between d_1 and d_2 at time points t_0 and t_1, respectively. A *continuous transformation* from R_0 into R_1 is a pair of trajectories for objects d_1 and d_2, respectively, such that at t_0, R_0 holds between d_1 and d_2 and at t_1, R_1 holds. We say that a relation R_1 is a *continuous successor of R_0 of type 1* if there exists a continuous transformation from R_0 into R_1 and a time point $t \in (0,1)$ such that at all time points in $[0,t)$, relation R_0 holds between d_1 and d_2 and, at all time points in $[t,1]$, it holds R_1. R_1 is a *continuous successor of R_0 of type 2* if there is a continuous transformation from R_0 into R_1 and a $t \in (0,1)$ such that in $[0,t]$ it holds $d_1 R_0 d_2$ and in $(t,1]$ it holds $d_1 R_1 d_2$. Both types are discerned in the depicted diagrams by different arrows: continuous transitions of type 1 are represented by dashed arrows and those of type 2 by dotted arrows. Note that each node in these graphs is a continuous successor of itself (which is omitted in the diagrams) — thus providing trivial examples for continuous transformations that are both type 1 and type 2. For formal proofs showing the correctness of these graphs under the presented semantics we refer to (Ragni & Wölfl 2006).[4]

The neighborhood graph for RCC8-CD is a subgraph of the product graph of the neighborhood graphs of CD and RCC8, which is obtained by restriction to the base relations of RCC8-CD. That is, a RCC8-CD base relation (R_1, S_1) is a type 1 (resp. type 2) successor of (R_0, S_0) if this holds true component-wisely (R_1 is a type 1 (type 2) successor of R_0 and S_1 is a type 1 (type 2) successor of S_0). For instance, (NTPP, N) is not considered a direct successor of (TPP, NE) (cf. Fig. 7).

Summary and Outlook

In this paper we presented a qualitative formalism, which is based on a simplistic representation of objects as disks.

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[4]Neighborhood graphs constructed from continuous successor relations are closely related to *dominance spaces* (Galton 2000). The idea of dominance can be illustrated as follows: Consider two objects (disks) d_1 and d_2. Assume that at all time points in the interval $[t_0, t_1)$ it holds $d_1 R_1 d_2$ and in the interval $(t_1, t_2]$ it holds $d_1 R_2 d_2$. Furthermore, assume that at t_1 one of both $d_1 R_1 d_2$ or $d_1 R_2 d_2$ is true. If (from the underlying semantics) it follows that one of these relations must hold at t_1, this relation is said to *dominate* the other one. For example, TPP dominates NTPP and N dominates NE. The dominance relation between spatial relations seems similar to the type 2 successor relation: in terms of the given illustration, the main difference can be seen in that the type 2 successor relation does not represent which of the relations R_1 and R_2 *must* hold at t_1, but which of these relations *can* hold at t_1.
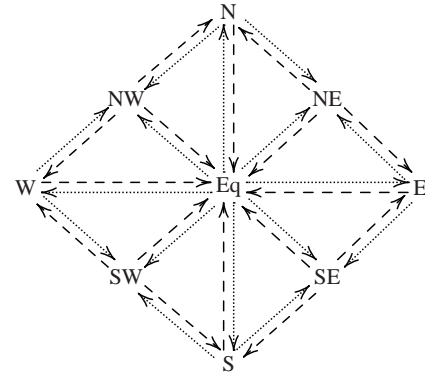
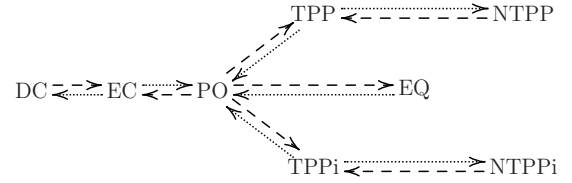Figure 5: The neighborhood graph of the Cardinal Direction Calculus

Figure 6: The RCC8 neighborhood graph under the size persistency constraint

Nevertheless, this calculus provides an interesting test-bed for investigating typical problems that arise when topological and positional information about objects are combined, or when such combined formalisms are used to represent continuous change. We first introduced the combination of the closed disk calculus and the cardinal direction calculus and discussed its composition table, which can be generated from the composition tables of the component calculi by using restriction rules. From a formal point of view, it could be interesting to see if this method can be abstracted and applied to other combinations of calculi. The same holds true for the neighborhood graph of the combined formalism.

Future work will include an analysis of the general satisfiability problem for this calculus. The NP-hardness is obvious, as well as a number of tractability results due to the embedding of the interval algebra, RCC8, and cardinal direction calculus. Combinations of tractable subsets should be addressed in future work as well.

Acknowledgments

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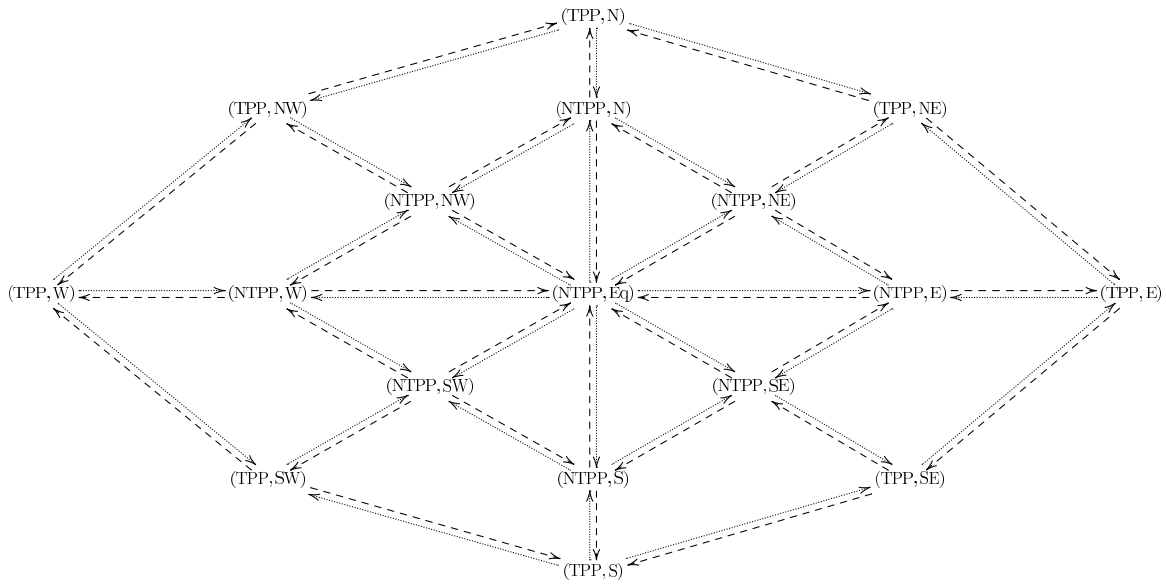de Weghe, N. V.; Kuijpers, B.; Bogaert, P.; and Maeyer, P. D. 2005. A qualitative trajectory calculus and the com-

Figure 7: The neighborhood graph for the relations $\{\text{TPP}, \text{NTPP}\} \times CD$ of RCC8-CD

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