Adversarial Modeling Using Granular Computing

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Abstract

We look at the problem of adverarial decision making. Our objectivel here is to try to replace the infinite regress inherent in adversarial making decisions problems with the use of knowledge about the adversary. We use granular computing and particularly the Dempster-Shafer belief structure to represent this knowledge. This allows us to turn the problem of adverarial decision making into a problem decision-making.under uncertainy.

1. Introduction

In many situations the outcome one obtains as a result of their decisions is affected by the actions taken by another individual. We shall generically refer to decision-making in this type of environment as adversarial decision-making. Understanding and modeling one's adversary in order to anticipate what actions they will take is an important requirement for attaining one's objectives. Often our knowledge about an adversary is vague, imprecise and highly uncertain. This is particularly the case when an adversary is from a different culture. Here we shall find the capabilities provided by granular computing [1] useful in representation of this type of knowledge.

In its most basic form adversarial decision-making involves two participants, white and black. Each of which chooses an action without knowing the choice of the other. The result of these choices are payoffs to the two participants. Here we suggest a knowledge-based approach that can be used by a participant in this adversarial environment to help them choose their action. When using this approach a player, for example white, will use all its expertise, knowledge and perceptions, about its adversary to construct a knowledge base about the action it believes its adversary will take. Considerable use is made of granular computing technologies in the construction of this knowledge base. This knowledge base can then be converted into an uncertainty profile [2] over the set of actions available to the adversary. Once having this uncertainty profile white's choice of action can be made using the established technology of decision-making under uncertainty [3].

Our overriding goal here is to try to replace the infinite regress inherent in adversarial making decisions problems with the use of knowledge and perceptions about the adversary. This avoids the complexities of mathematics based on fixed-point theorems [4] and replaces it with the simpler mathematics of uncertain decision-making.

2. Knowledge Based Approach

The basic normalized form for adversarial decision-making is expressed by the matrix M shown in figure #1. Here white has a set $X = \{x_1, ..., x_m\}$ of alternative actions from among which it will select one. An adversary, black, has a set of actions $Y = \{y_1, ..., y_n\}$ from among which it will select one. A choice pair of actions $[x_i, y_j]$ results in a payoff (c_{ij}, d_{ij}) where c_{ij} is the amount received by white and d_{ij} is the amount received by its adversary black. Our perspective here is to assist white in making its best decision. Unfortunately in this environment white's choice of action must be made without knowing the action chosen by black. The same is true about black.

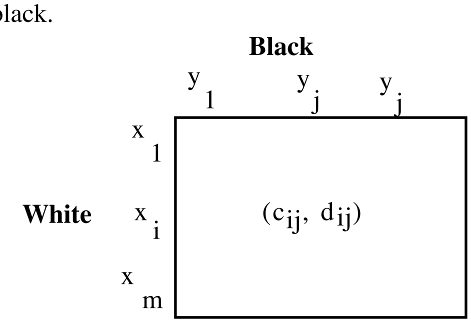

Figure #1. Basic adversarial matrix

While white doesn't know the exact action chosen by black we assume white has available from its experiences and perceptions about black a knowledge base that can be used by white to generate an uncertainty profile over the actions available to black quantifying its anticipation as to the action black will take. In figure #2 we show this situation. White uses its knowledge about black to generate a probability or other uncertainty profile over Y, the set of possible actions available to black.

White then uses this uncertainty profile on Y in conjunction with its own decision objectives to select its action. What must be emphasized here is that white's knowledge base is owned and constructed by white with the purpose of helping it make a better decision. Our objective in this paper is to provide tools to help white represent and use its knowledge base.

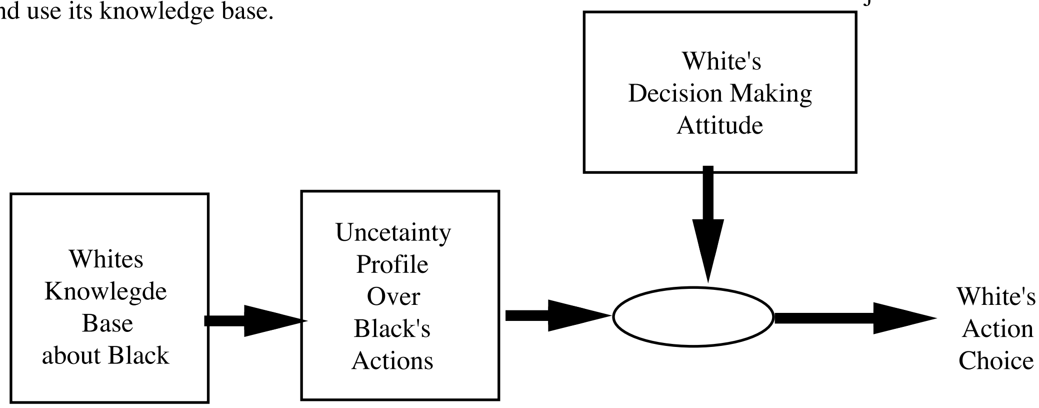

Figure #2. Basic Approach

Here we shall consider one type of knowledge base that can be constructed by white. Fundamental to this type of knowledge base is the concept of an adversarial decision-making strategy, ADMS. By an ADMS we mean some procedure, objective, process or algorithm that black can use to select its action from Y. In order to provide some intuition we describe some simple ADMS, more complex ones will be discussed later. One simple strategy is for black to randomly choose an action from Y. Another strategy that black can use is to treat the decision matrix in figure #1 as an example of decision-making under ignorance. In this case black will select the action y_i that gives it the maximum payoff, that is it selects the action y_{j*} such that $d_{ij*} = \text{Max}_j[\sum_{i=1}^{m} d_{ij}]$, the action with the largest total payoff. Another strategy that black can use is to consider the matrix in figure #1 from a game theoretic point of view and locate the Nash equilibrium [4]. We should note that another strategy could be to directly select an action y_k from Y. Thus the actions themselves can viewed as special strategies, so called atomic strategies.

We emphasize that by a strategy we mean some well understood procedure for selecting an action by black. We note that a strategy can be deterministic, it always leads to some action for the same payoff matrix. A strategy can also be non-deterministic, the final choice of action may be random, it can result in different actions in the same situation.

Let $S = \{S_1, ..., S_q\}$ be a collection of ADMS's generated by white from among which white perceives black will select its actual decision-making strategy (policy). Here white is assuming it knows and understands the possible strategies available to black. By understanding a strategy S_j we mean that given the payoff matrix M white is able to determine an associated well-defined uncertainty profile over the set Y if black uses strategy S_j. Understanding a strategy also implicitly implies specification of the mechanism black uses to select the action having the associated uncertainty profile.

Fundamental to our approach is the assumption that white uses its totality of knowledge about black to construct a knowledge base over the set S reflecting which strategy from S that black will use to select its action. More specifically we shall assume that white's knowledge about how black will make its choice is expressed as a generalized uncertainty profile G over the space S indicating its anticipation of the strategy to be used by black for selecting its action from Y.

In the work that follows our point of departure is the availability of an uncertainty profile G over the space of possible strategies S. We are not concerned here with the process of how white constructs the G from its totality of knowledge about black, however we appreciate the importance and difficulty of this task. In point of fact our formulation of white's knowledge about black's choice procedure over the space S of strategies instead of directly in terms of Y is a concession to the difficulty of formalizing whites knowledge. By using objects of larger granularity, the strategies, rather then actions themselves, we are allowing for less precision, greater granularity, in the formalization of whites knowledge.

Let us now see how this structure of having an uncertainty profile G can be used to support a decision-making methodology for white. For any strategy S_k, using the payoff matrix M, white can generate an uncertainty profile

H_k over Y indicating the anticipation of y_j being black's action under S_k. Using its knowledge base with respect to black's choice of strategy, the uncertainty profile G over S, white can combine the associated uncertainty profiles, the H_k, to obtain a unified uncertainty profile H over Y. White can then use this unified uncertainty profile over Y in coordination with M to make its selection of action from X. In the following figure #3 we provide a schematic with this process.

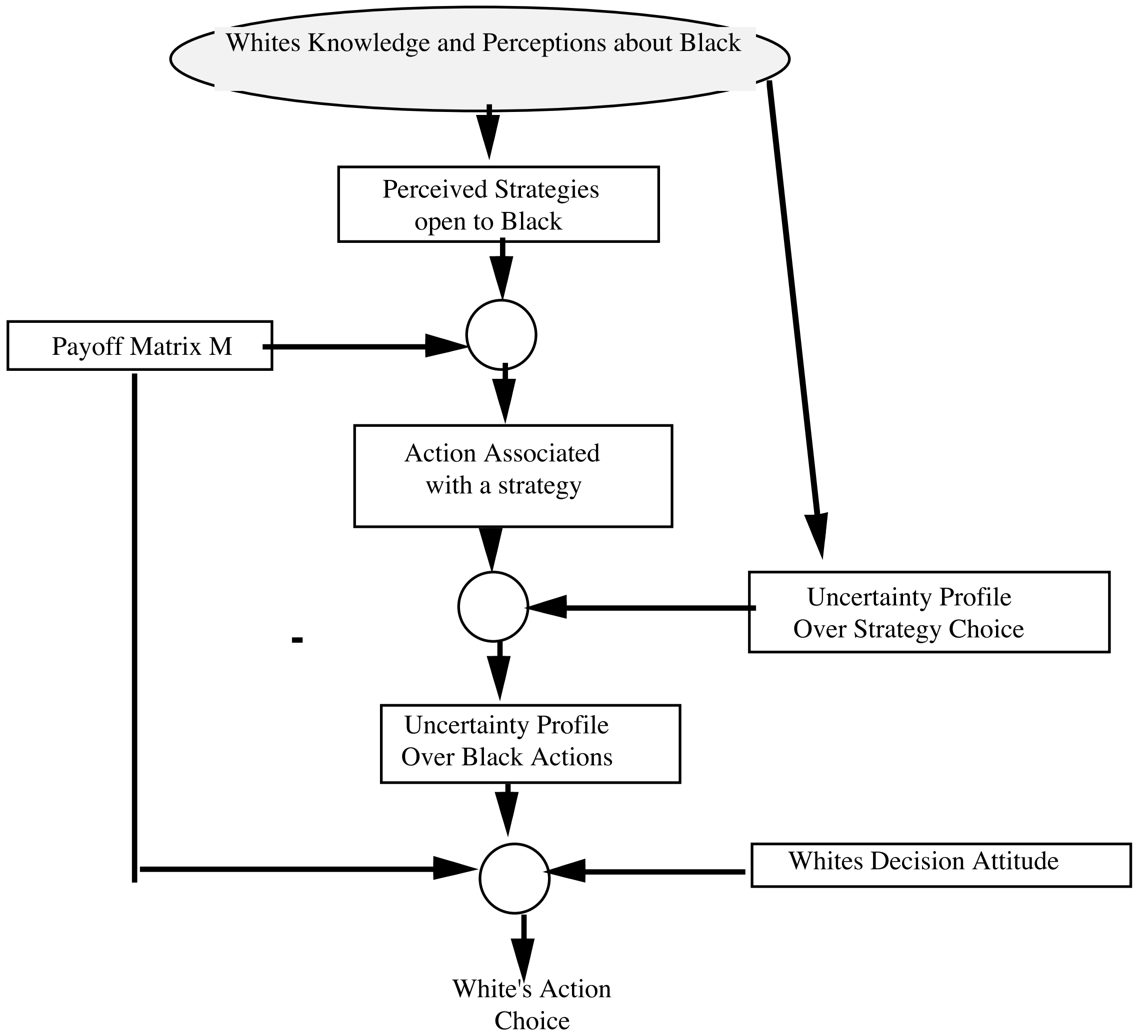

Figure #3. Knowledge Based Adversarial decision-making

In some situations the ability to implement all the required operations are not available. This particularly occurs with the operation of combining the individual H_k guided by G to obtain H. In the light of this we are often faced with a conflict between using a sophisticated uncertainty representation to better model our knowledge and the ability to perform the required operation with the representation used.

3. Uncertainty Profiles Using Dempster-Shafer Belief Structures

One framework for representing uncertain knowledge is the Dempster-Shafer belief structure [5]. Formally a Dempster-Shafer belief structure is a mapping

$m: 2^S \rightarrow [0, 1]$ such that: **1.** $m(\emptyset) = 0$ and **2.** $\sum_{B \subseteq X} m(B) = 1$. Using the terminology of Shafer [6] we call a subset B for which $m(B) > 0$ a focal element. If we have r focal elements we shall denote them B_j for j = 1 to r. Using this notation $\sum_{B \subseteq X} m(B) = 1$ becomes $\sum_{j=1}^{r} m(B_j) = 1$.

Two important concepts associated with a D-S structure are the measures of plausibility and belief. For any subset A of we define these respectively as

$$Pl(A) = \sum_{B_j \cap A \neq \emptyset} m(B_j)$$

$$Bel(A) = \sum_{B_j \subseteq A} m(B_j).$$

It is well known for $A \neq \emptyset$ that $Pl(A) \geq Bel(A)$.

Let us provide some semantics for the formal structure. A prototypical interpretation for the D-S belief structure is the random set model. Here we have a random experiment in which the outcome instead of being an element is a subset. In this random experiment $m(B_j)$ is the probability that the outcome is in the set B_j. In this framework additional uncertainty stems from the fact that we don't know how the actual element is selected from the set chosen. In our context of using this to represent knowledge about which is the true value of V we can interpret this as follows. We believe with probability $m(B_j)$ that the elements will be selected from the subset B_j of S but we don't know how the actual element is selected from B_j. For example white may believe that there is a 50% chance that black will use a strategy based on viewing the problem just considering the payoffs it will receive, as one of decision-making under ignorance, without being able to make any distinction with regard to blacks preferences among the strategies in this class.

Another interpretation of the D-S formalism is as a generalization of the idea of a probability distribution. With an ordinary probability distribution we have a collection of q probability weights p_j where the p_j are non-negative and sum to one. These weights are assigned to individual elements in the underlying space S. In the D-S framework these weights are assigned to subsets of S, the focal elements, rather individual elements. We don't know how the weights are distributed among the elements in the subset to which they are assigned.

Under this generalized probability interpretation the Dempster-Shafer model can be used to represent situations in which we have imprecise probabilities [2]. In particular given a D-S belief structure it can be shown that for any subset A of S that $Bel(A) \leq Prob(A) \leq Pl(A)$. In this context we call Pl(A) and Bel(A) the upper and lower probabilities. Thus D-S the structure can be used to model situations in which white's assessment of the probabilities associated with black's strategy selection is imprecise. Rather then being specific values they are intervals.

We shall find the following slightly more general expression of Bel(A) and Pl(A) useful. Let $D = A \cap B$ and D(z) indicate the degree of membership of the element z in the intersection of A and B. Using this we define the possibility of V lying in the set A given that it is in the set B as $Poss[A/B] = Max_z[D(z)]$. We also define the degree of certainty that V lies in A given that it is known to lie B as $Cert[A/B] = 1 - Poss[\overline{A}/B]$. Using these concepts we can express

$$Pl(A) = \sum_{B_j} Poss[A/B_j]\, m(B_j$$

$$Bel(A) = \sum_{B_j} Cert[A/B_j]\, m(B_j)$$

Thus if m(B) is seen as the probability of the strategy lying in B_j then Pl(A) is the expected possibility of black's strategy lying in A and Bel(A) is the expected certainty of the strategy lying in A. As we noted these can be seen as upper as lower bounds on the probability that the selected strategy will be in A. One important benefit of this generalization is that it allows us to have focal elements that are fuzzy sets.

Since we are using our uncertainty model to represent white's perceptions about the strategy that black will use the D-S model of uncertainty is be particularly useful in that it allows us to express these perceptions in a very granular way [7]. Using these structures we can allow white to represent this belief about white's choice of strategy as imprecisely as it needs. Thus white is not forced to be more precise then it feels justified, for example it can use imprecise intervals to express its probabilities.

In the following we shall indicate some special cases of D-S belief structure and discuss the types of information they represent. Complete certainty about the value of the variable V can be modeled using a D-S belief structure with one focal element B_1 such that $B_1 = \{S_k\}$ and $m(B_1) = 1$. Here white's perception is that the strategy to be used by black is S_k. At the other extreme is the case where we are completely uncertain, this can be modeled with D-S structure having one focal element, $B_1 = S$ and $m(B_1) = 1$. More generally both of these can be seen as special cases of a situation where white believes that the strategy that will be selected is from some subset B. In this case we again have one focal element $B_1 = B$ with $m(B) = 1$. An ordinary probability distribution can be modeled as a special case of D-S belief structure. Here with $S = \{S_1, ..., S_q\}$ we define our focal elements as singletons, $B_j = \{S_j\}$ for j = 1 to q and assign $m(B_j) = p_j$. A possibility distribution can also be represented as a D-S belief structure.

Another important special case is where we have a D-S belief structure such that $m(B) = \alpha$ and $m(X) = 1 - \alpha$. This can provide is model of a situation in which white believes that the probability that black will select its strategy from B is at least α.

Closely related to the preceding D-S structure where $m(B) = \alpha$ and $m(\overline{B}) = 1 - \alpha$. In this case we see $Prob(B) = \alpha$. Here we have a model of whites perception that the probability that black will select its strategy from B is exactly α.

We note that within this framework we can allow the use of fuzzy sets. Thus we can allow the focal elements to be fuzzy subsets. This becomes a special case of what Zadeh called perception based granular probabilities [7]. In particular if the focal elements are fuzzy and A is a fuzzy subsets of S we have

$$Pl(A) = \sum_{B_j} Poss[A/B_j]m(B_j)$$

where

$$Poss[A/B_j] = Max_i[A(S_i) \wedge B_j(S_i)].$$

Here $A(S_i)$ and $B_j(S_i)$ are the membership grades of S_i in A and B_j respectively and $\wedge$ is the Min operator. Furthermore we have $Bel(A) = \sum_{B_j} Cert[A/B_j]m(B_j)$ where $Cert(A/B_j) = 1 - Poss[\overline{A}/B_j]$

As we have indicated the D-S belief structure is very useful for modeling white's perceptions about the strategy that black will use. However there is another useful feature that the D–S approach has for obtaining a representation of the uncertainty profile over black's choice of strategy as seen by white. In addition to providing for the ability to model whites perceptions of the strategy to be used by black a very important feature of the D-S framework is that it has a very convenient methodology for combining these individual perceptions to get the overall uncertainty profile capturing all of whites knowledge. Assume m_i for i = 1 to L are a collection of D-S structures over S. For the belief structure m_i let B_{ij}, j = 1 to η_i be its associated focal elements. Here each m_i can correspond to an perception that white has regarding black's selection of strategy. The most commonly used approach for combining these individual perceptions to get an overall uncertainty profile is to use Dempster's rule of combination [5].

4. Decision-Making with Belief Structures

If we use Dempster-Shafer belief structure to provide a uncertainty profile one task that we have to address is the determination of the expected value under this type of representation of uncertainty. We now describe a methodology for determining an this expected value introduced in [8]. Let **R** be a random variable. Assume our knowledge about the value of this random variable is expressed via a D-S belief structure m having focal elements B_k = 1 to r. Assume each focal element B_k consists of a collection of values $z_{j/k}$ where j = 1 to $|B_k|$. Our problem is to determine the representative or expected value of **R** in this situation.

As discussed in [8] when using the D-S model we are assuming a random selection of focal element with $m(B_k)$ the probability of selecting B_k. Assume we can obtain for each focal element B_k an associated representative value, $Val(B_k)$. Using this value we can obtain the overall valuation of the variable **R** as the expected value of this individual values,

$$Val(\mathbf{R}) = \sum_{k=1}^{r} Val(B_k)\, m(B_k).$$

The issue now becomes the determination of the representative value associated with each of the focal elements. Since we have no information as to how the value will be selected once a focal element is selected the determination of this value can be seen to be equivalent to the valuation of an alternative in decision-making under ignorance. In this case as discussed in [8] we must impose a decision attitude. The choice of decision attitude leads to formulation of $Val(B_k)$. For example an optimistic attitude will lead us to the formulation, $Val(B_k) = Max_j[z_{j/k}]$, the largest value in B_k. At the other extreme is a pessimistic attitude here the formulation is $Val(B_k) = Min_j[z_{j/k}]$, it is the smallest value in B_k. Intermediate to these is a neutral decision attitude where $Val(B_k) = \frac{1}{|B_k|}\sum_{j=1}^{|B_k|} Z_{j/k}$, the average of the elements in B_k.

Using the OWA aggregation operator Yager [8] suggested a more general approach which unifies these different formulations. Let $w_{i/k}$ be a collection of $|B_k|$ non-negative weights summing to one. We can view these weights as a vector W_k of dimension $|B_k|$ and denote is as the weighting vector associated with B_k. Let $b_{i/k}$ be the

i^{th} largest of the elements in B_k. Using the OWA operator based method of calculating the $Val(B_k)$ we get

$$Val(B_k) = \sum_{i=1}^{|B_k|} w_{i/k} b_{i/k}.$$

We observe that if $w_{1/k} = 1$ and $w_{i/k} = 0$ for $i \neq 1$ then we get the maximum as the valuation. If we select $w_{n/k} = 1$ and $w_{i/k} = 0$ for $i \neq n$ then we get the minimum as the valuation. If $w_{i/k} = \frac{1}{|B_k|}$ for all i then we get the neutral valuation.

Yager [8] used this formulation to provide an even more general method for the determination of $Val(B_k)$. Here associated with the weighting W_k a feature called its attitudinal character α, which is defined as $\alpha = \frac{1}{|B_k|-1} \sum_{i=1}^{|B_k|} w_{i/k}(|B_k|-i)$. It can be shown that α lies in the unit interval. We further note that if we have the optimistic decision maker where $w_{1/k} = 1$ then we get $\alpha = 1$. If we have the neutral decision maker where $w_{i/k} = \frac{1}{|B_k|}$ for all i then we get $\alpha = 0.5$. For the pure pessimist where $w_{n/k} = 1$ we get $\alpha = 0$. We see that attitudinal character provides a scale indicating the optimism-pessimism of the decision maker.

Inversely we can obtain a set of weights of cardinality $|B_k|$ from a given value of α by solving the following mathematical programming problem. Here we let $|B_k| = n$

$$\text{Max } H(W) = -\sum_{i=1}^{n} w_i \ln(w_i)$$

Such that: **1.** $\frac{1}{n-1}\sum_{i=1}^{n} w_i(n-i)$

2. $\sum_{i=1}^{n} w_i = 1$

3. $0 \leq w_i \leq 1$

We now can summarize our approach to obtaining an expected value in the case of a Dempster- Shafer uncertainty profile. Assume the payoff structure is expressed in terms of a Dempster- Shafer belief structure *m* with focal elements B_k, k = 1 to r.

1. The decision maker provides its desired degree of optimism $\alpha \in [0, 1]$.

2. Using this we obtain a weighting vector of dimension $|B_k|$ for each focal element. This requires solving the preceding MP problem for each focal element.

3. We can then use these weights to obtain the valuation $Val(B_k)$ for each focal element as $Val(B_k) = \sum_{i=1}^{|B_k|} w_{i/k} b_{i/k}$. Here $b_{i/k}$ is the i^{th} largest element in the focal element B_k.

4. Finally we calculate the overall expected value as

$$Val(m) = \sum_{k=1}^{r} Val(B_k)\, m(B_k)$$

We note that for the special but important cases when α= 1, 0.5 or 0 we can directly obtain the weight without having to solve the MP problem.

5. Strategies

Here we discuss the some different strategies that black can use to select its action. Perhaps the most rudimentary strategies that black can use are those in which it uses no other information then the set Y. The most basic of these is where black randomly selects an element from Y. Here the strategy results in $Prob(y_j) = 1/m$, where m is the number of elements in Y. Another example of this type of strategy is where black selects what it did last time. In this case if y_r indicates what it did last time then $Prob(y_r) = 1$ and $Prob(y_j) = 0$ for $j \neq r$. Related to this is the case where black does something different than it did last time. Here $Prob(y_r) = 0$ and $Prob(y_j) = \frac{1}{m-1}$ for $j \neq r$. More generally here we consider strategies that are based simply on some properties of the action itself independent of what alternatives are open to white.

Another class of strategies that black can use are those which simply use the payoff matrix M and make no supposition regarding the action that will be taken by white. In this case black is using what is called in the literature decision-making under ignorance. Within this class a number of special cases are well known. One is the Maxi-Min strategy. Using this strategy black determines its action using the following procedure. For each action open to it, each column of the matrix M, it will calculate the smallest payoff it can receive. Let us denote these as $D_j = Min_i[d_{ij}]$. Under this strategy the action that will be taken by black is to select y such that $y_{j*} = Max_j[D_j]$. In the case of Maxi-Min strategy white is able to uniquely determine the action to be taken by black.

Another strategy in this class is one where black would calculate for each action y_j the sum of the payoffs to it, $D_j = \sum_{i=1}^{m} d_{ij}$. Using this black would then select the action that maximizes this sum.

Another strategy within this class is the Maxi-Maxi strategy. In this case black would calculate for each action y_j open to it, each column in M, the largest of the payoffs it can receive, $D_j = \text{Max}_i[d_{ij}]$ Using this it again would select the action with largest of these.

Implicit in the preceding is an assumption by white that black's selection of strategy is based upon a desire to maximize its payoff. This may not be the case. In particular white must include in the set S of a strategies that can possibly be used by black those in which blacks fundamental underling imperative is not its own gain. White must consider the possibility that blacks underlying objective is to minimize the payoff that white will receive. White can determine the action that black will take in this case as follows. For each action y_j open to black it calculates $E_j = F(c_{1j}, c_{2j}, \ldots, c_{mj})$. We point out the that the E_j's are based on the payoffs to white, the c_{ij}'s, not as in preceding where we determined D_j using the payoffs to black, the d_{ij}. Using the E_j the selection of black is y_{j*} such that $E_{j*} = \text{Min}_j[E_j]$. In this case F as in the preceding can be $\text{Min}_i[c_{ij}]$, $\sum_{i=1}^{m} c_{ij}$ or Max $_i[c_{ij}]$. The most conservative of these would be the Min-Max where $E_j = \text{Max}_i[c_{ij}]$. Here black is finding out the best that white get under its action and then black will select the action that gives white the minimum of these

The important point again here is that in this framework of black just using M, decision-making under ignorance, even under the assumption that black's underlying imperative is to deny satisfaction to white, malicious, white can determine what exact action black will take.

Another underlying decision imperative that can be held by black is to try to do much better than white. In this case the determination of the action taken by black can be obtained as follows. For each component of the matrix M we calculate $\Delta_{ij} = d_{ij} - c_{ij}$. For each y_j using these we calculate $\Delta_j = F(\Delta_{1j}, \Delta_{2j}, ..., \Delta_{mj})$. The action selected by black would then be y_{j*} where $\Delta_{j*} = \text{Max}_j[\Delta_j]$. Here again is this case F can be obtained as, $\text{Min}_i[\Delta_j]$, $\sum_{i=1}^{m} \Delta_{ij}$ or the $\text{Max}_i[\Delta_{ij}]$

We shall refer to these three different underlying decision imperatives that can be associated with black respectively as acquisitive, malicious and competitive.

Another class of strategies open to black are those in which in the process of choosing its action in addition to using the information contained in M black reflects upon what action might be taken by white. The field of Game Theory is very much concerned with this type of problem. The possibilities here are very diverse and rich and are strongly context dependent. Investigating and describing these in detail is beyond our current scope, however we shall just mention one important example of this. Consider the case where black's objective is to maximize the difference between its payoff and that of white, it wants to $\text{Max}\Delta_{ij}$. In this case black can view this situation as a zero sum game [4]. Applying the well-known game theory technology developed for this problem, black's decision would be to select its action based on a probability distribution over the set of actions. Thus under the assumption that black is using this type of strategy white's knowledge about black's action would be a probability distribution.

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