Drawing Analogies by Systematic spatial inferencing for manufactured components

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Abstract: In this paper, we describe systematic analogical reasoning using a qualitative spatial model for manufactured components which incorporates both features and mathematical spatial relationships between features [7]. Solving this problem will be of benefit to manufacturing planning. We will discuss the definition and formulation of these computational inferencing tasks, as well as their properties. The utility of these computational inferences within the manufacturing domain is shown.

1 Introduction

The contributions of this paper are the formulation of three spatial inferencing tasks as systematic, efficient computational mechanisms for enhancing and optimizing manufacturing planning. The three tasks addressed here are:

- Generating all interpretations of a component.
- Identifying the symmetric interpretations of a component — interpretations which, although syntactically different, lead to semantically equivalent manufacturing plans.
- Inferring analogies between components and subparts of components which lead to similarities in their manufacturing plans.

Each of these mechanisms provides information about the shape and structure of a component which can enhance planning and manufacturing capabilities.

1.1 Generating interpretations

Informally, an interpretation of a component is a set of features which completely describe the component. Typically, a manufacturing process planner uses an interpretation of a component as input. If the generated plan accounts for each of the features in the interpretation, then the resulting plan will completely produce the component. When the features of a component interact, they create multiple interpretations. The same types of features can often describe a single component in many different, valid, ways.

The merit of a plan generated for a component is directly dependent upon the interpretation used as input to the planner. Generating the multiple interpretations of a component can be beneficial to planning activities by providing alternative input (which correspond to plans with different production strategies), and hence the ability to generate plans having different merits. For example, consider the two different plans shown in Figure 1 for producing the same component using three-axis machining. The first plan, (a), is generated from the interpretation shown in (c). The second plan, (b), is generated from the interpretation shown in (d). The first plan requires three different setups to produce the component, where as the second plan requires only two setups. The number of setups (along with other planning information such as tool changes, etc.) is dependent upon the interpretation of the component. That is, the interpretation in (d) mandates at least three setups, whereas the interpretation in (c) only two setups. Hence, the second plan and its corresponding interpretation are more desirable.

1.2 Drawing Analogies

The motivation for analogical spatial reasoning is to enable the reuse of previously generated plans for a newly introduced component. Consider the two different interpretations (which we will later formally identify as symmetric), for the same component shown in Figures 2 (a) and (b). Although the interpretations are syntactically different (have different symbolic descriptions), these interpretations are semantically equivalent because they generate equivalent process plans. That is, the plans have the same number of setups, tools, tool changes, etc. in the same relative order. Consequently, the merit of these plans will be identical.[1]

With the task of drawing analogies, we demonstrate how to aid the function of a planner by dealing with more than one component, specifically a database of components. With this, a planner can make use of solutions or plans already produced without generating the information from scratch. For example, consider the situation depicted in Figure 3. Here we are introducing a new component. Instead of producing a plan for this

[1]The model does not incorporate information about tolerances between features, or surface finishes. Consequently, when we say identical, we are not considering these aspects.

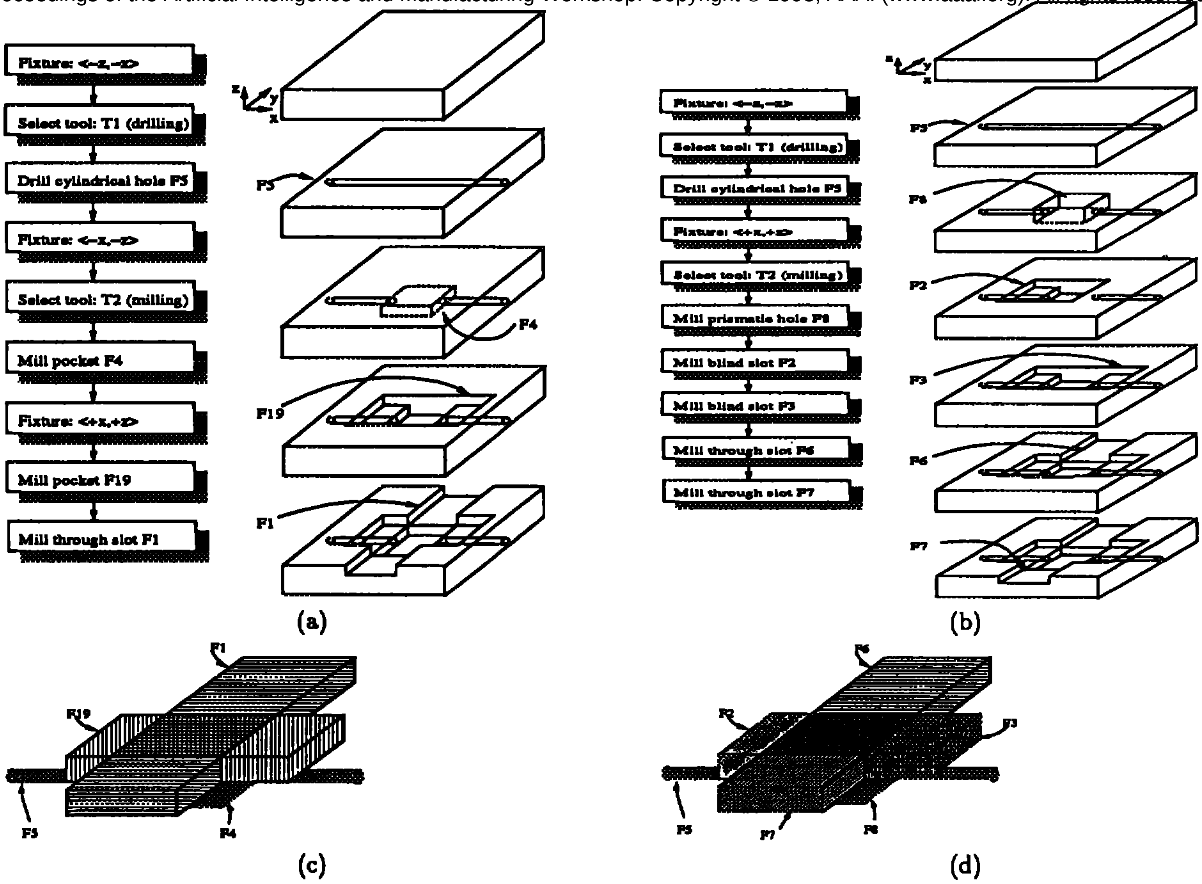


Figure 1: The motivation for generating multiple interpretations. (a) and (b) show two plans for producing the same component (shown at the bottom of both (a) and (b)). (c) and (d) show the interpretations corresponding to the plans of (a) and (b), respectively. Plan (b) as fewer setups than plan (a), and hence is a better plan for manufacturing the component. By generating multiple interpretations, we can identify which plan(s) is(are) optimal for a component.

component from scratch (which would again require the generation of its interpretations, exploration of these interpretations, production of various plans and selection of the optimal plan), similarities between the new component and components from the database enable the planner to reuse plans, either full or partial, stored in the database.

2 The qualitative model

This section discusses construction of a qualitative model for a component from its solid model, more details can be found in [7]. Figure 4 shows an overview of the structure and construction of the model. Figure 4 (a) shows the interrelationship between the different components of the model. Figure 4 (b) shows the steps in the construction of the qualitative model.

2.1 Feature definitions

We represent features through linear sweeps of non-variant 2D surfaces.

Definition 1 *A* **feature** *is a 4-tuple, (T, f, D, I), where T is the* type *(e.g. slot), f is the sweep face, D is the sweep direction and I is the closed sweep interval.*

The sweep direction is represented via a unit vector, the sweep interval through a closed real interval, and the type is represented as a string. The sweep face is defined as:

The scope of this work is limited to sweep faces which are planar. In the scope of the work, edges are either linear — lie within a line and represented through two coordinate points on the line — or circular.

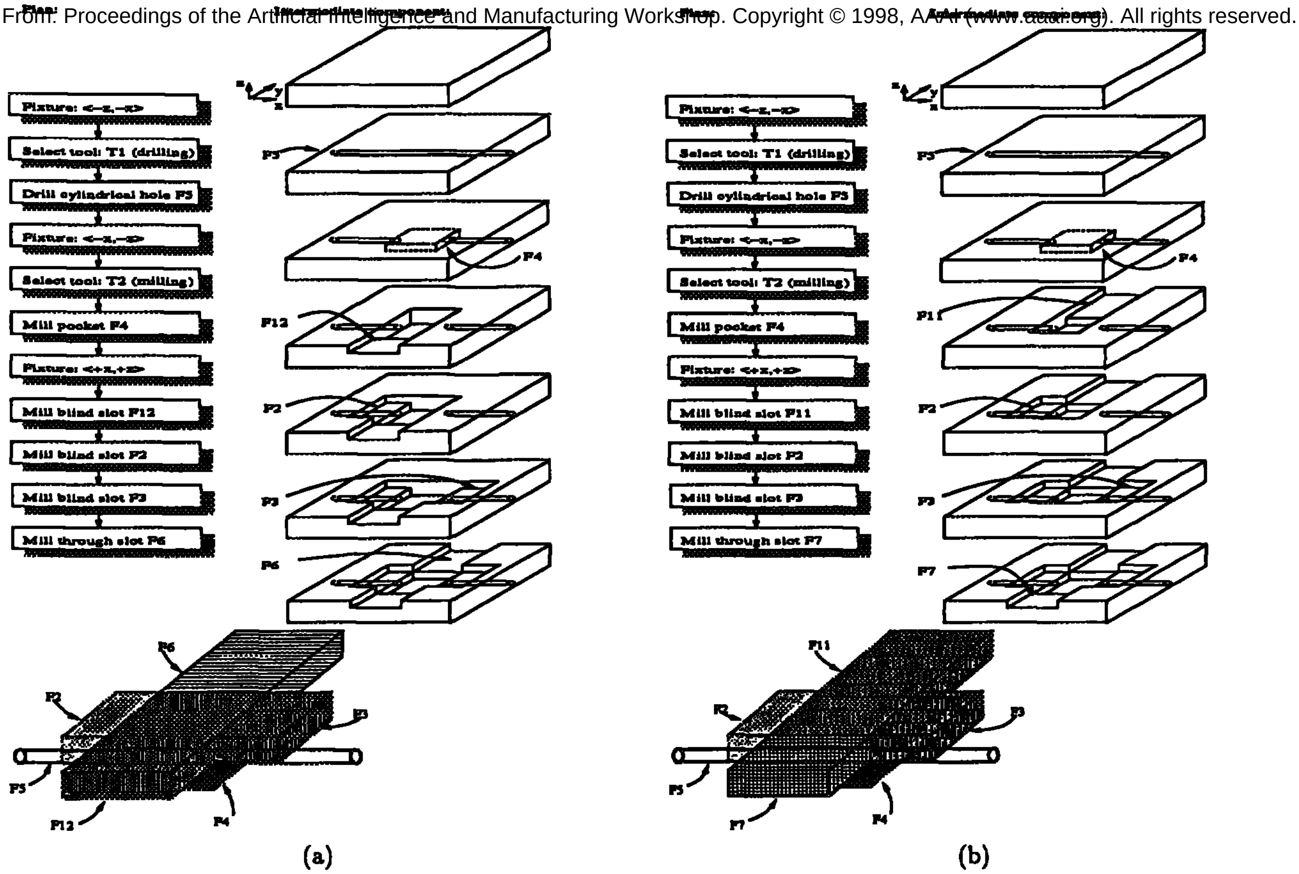


Figure 2: The motivation for identifying symmetric interpretations. (a) and (b) show two semantically equivalent plans generated with *different* interpretations of the same component. These plans both have the same number of setups, tool changes, etc. in the same relative orders. Hence, these plans have the same merit, and the planner need only consider one of them. We dub these interpretations "symmetric."

2.2 The place vocabulary (PV)

At the higher level of the qualitative model is the place vocabulary (PV)

Definition 2 *The* **place vocabulary** *for a given component is a labeled graph whose node set, V, consists of canonical features, and two nodes are adjacent if and only if the corresponding canonical features intersect. Furthermore, each edge is labeled with the spatial relationships between the two canonical features the edge is incident upon.*

Figure 6 shows the PV partially for the example component in Figure 1. Each node is one of the features seen in Figure 5, each arc is labeled with a spatial relationship. Examples of several spatial relationships are shown at the bottom of the figure.

2.3 Metric diagrams (MDs)

Metric diagrams (MDs) are used to extract the canonical features of a component. A MD is created between every pair of sweep faces found, and it is defined as follows:

Definition 3 *A* **metric diagram** *between two families of features is a 2D-plot such that:*

1. *The axes each represent the length of one of the sweeps. A negative value represents a sweep length in the opposite direction of the sweep direction.*
2. *The points on the plot represent the intersection between the leading sweep faces swept a distance equal to the x of y value of the point itself.*
3. *The maximum (minimum) value of either axis is the length at which the forward (reverse) swept face leaves the stock or intersects the component.*

Figure 7 (b) shows one such MD for our example component. The sweep faces for the MD are shown in Figure 7 (a). The MD is a plot depicting at what sweep lengths the two sweep faces intersect. The reader is referred to [7] for algorithms pertaining to the efficient construction of MDs. However the important properties of MDs are summarized below.

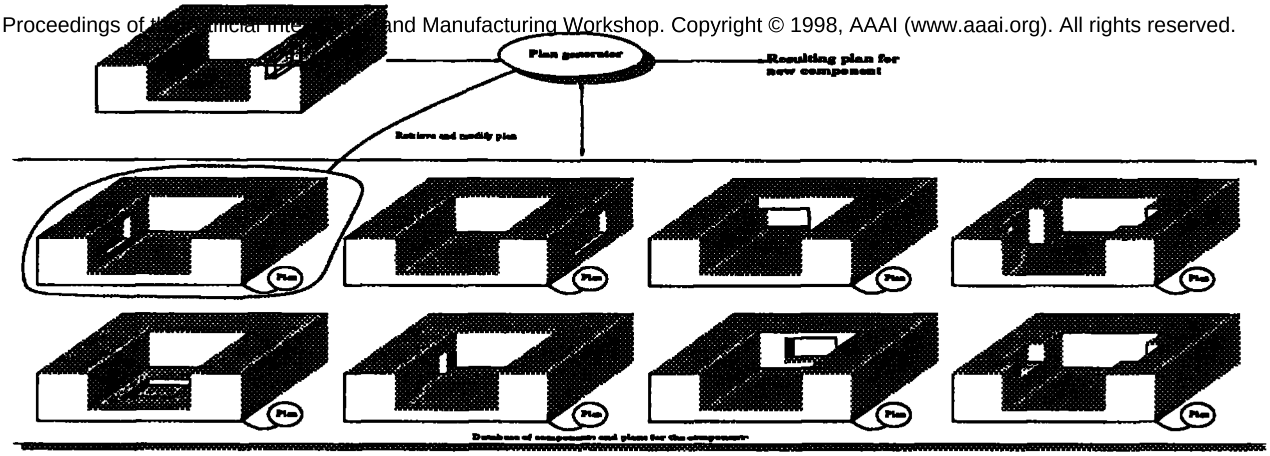
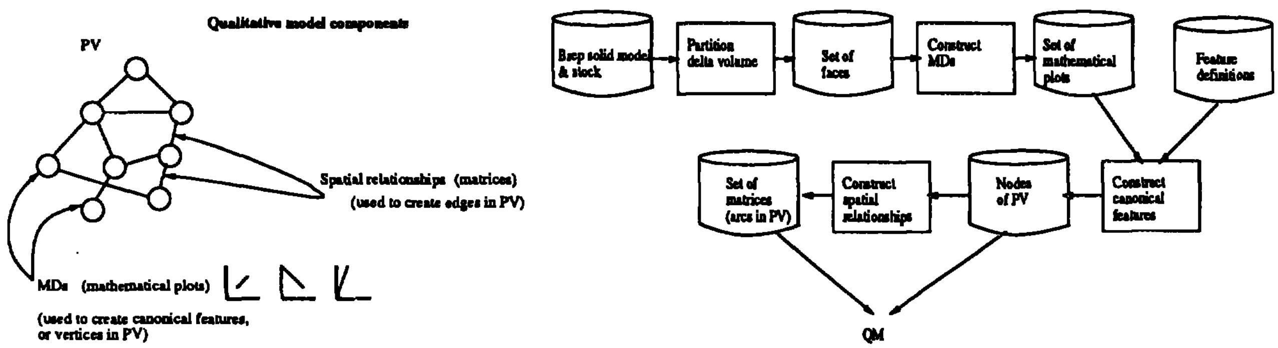


Figure 3: The utility of drawing analogies. As a new component is introduced to a planner, the planner checks if the component is analogous to any previously planned components (stored in the database). If so, then the previous plan is retrieved and modified to fit the new component, rather than generating the new plan from scratch.



(a) (b)

Figure 4: (a) The components of the qualitative model. The place vocabulary (PV) is a graph which models the canonical features and their spatial relationships. The canonical features are found via the metric diagrams, which are mathematical plots. The spatial relationships are matrices based on half spaces. (b) The flow of the construction of the QM.

Observation 1 *If either axis in a MD is divided into intervals based on the first and last points of intersection, these intervals correspond to the intervals of canonical features.*

Figure 7 (c) illustrates this observation. There are two other straightforward properties of MDs that we state here but refer the reader to [7] if more explanation is needed.

Observation 2 *Two unrelated families of features intersect iff their corresponding sweep length intervals both subsume the intersection region on their MD.*

3 Spatial inferencing

3.1 Generating multiple interpretations

Manufacturing planning activities usually use an interpretation of a component. Due to feature interactions, there is often more than one valid interpretation of a single component. These multiple interpretations offer alternative ways in which a component can be viewed and analyzed. Since the particular interpretation used by a planner will effect the resulting plan, it is important, from an optimization viewpoint, to explore the multiple interpretations of a component. For example, in Figure 1 we showed two different plans for the same component. These different plans ((a) and (b)) come about through the use of two different interpretations of the component ((c) and (d)). The importance of this example, is that

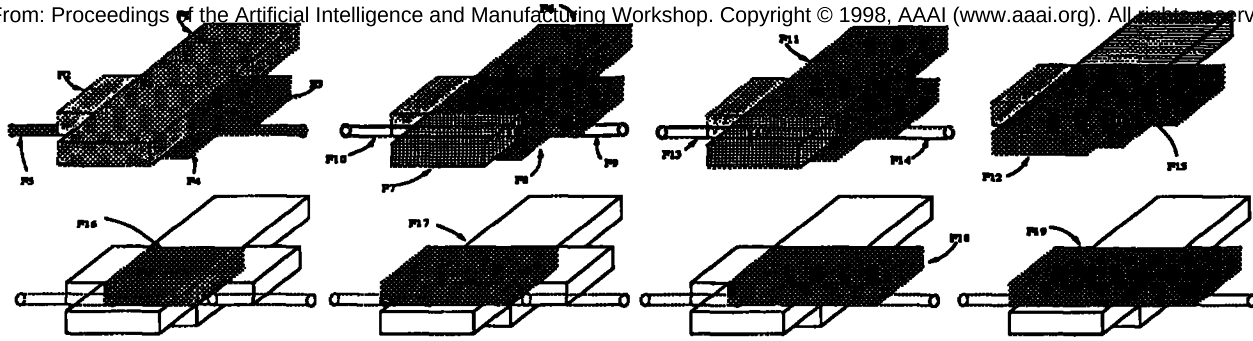


Figure 5: The canonical features for the example component in Figure 1. Notice that all the features are defined by the half spaces of the Δ volume.

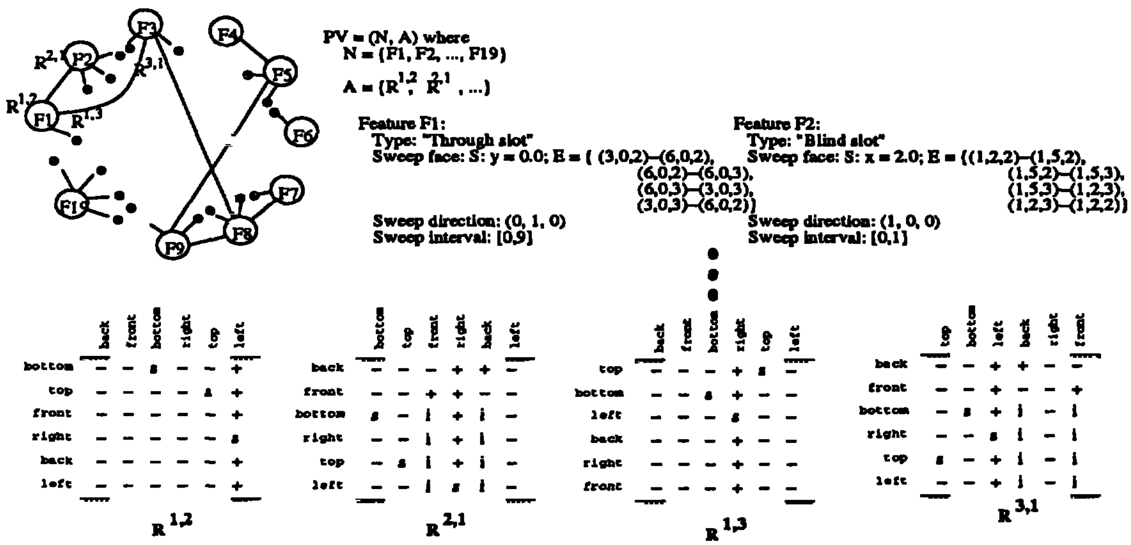


Figure 6: The PV for the example component (Figure 1) shown partially. The nodes of the graph correspond to canonical features of the component. The arcs represent interactions between the features and are labeled with spatial relationships. A couple feature representations and spatial relationships are shown along with the graph.

one plan, (b), is superior to the other, (a), in that it requires fewer fixture setups. If only some of the different interpretations are provided to the planner (*e.g.* say only the interpretation in (c)), then it is not guaranteed that an optimal plan is found (since the excluded interpretations may produce better plans).

3.1.1 A new notion of interpretation

The standard definition of an interpretation for a component has been a set of features that cover the Δ volume of a component [3, 5, 8, 13]. The Δ volume of a component is the volume produced by subtracting the component from the raw stock from which it is produced (see Figure 1 (c) or (d)). We extend this definition by formulating an interpretation as a set of features as well as the spatial relationships between these features.

Definition 4 *Given a component, and its canonical features, $\{F_1, F_2, \cdots, F_n\}$, an* **interpretation of the component** *is a subgraph of the component's PV, (F, R), where:*

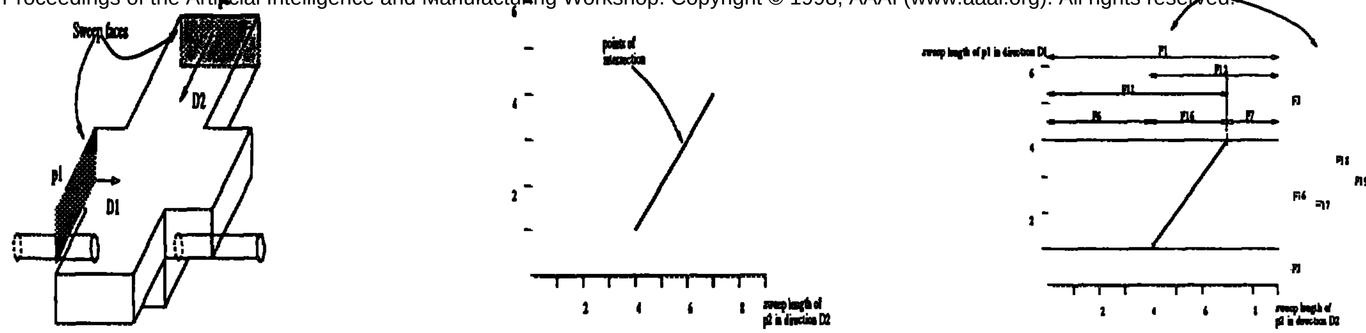
$F \subseteq \{F_1, F_2, \cdots, F_n\}$, the union of the feature volumes in F cover the Δ volume of the component, and for distinct i and j, $F_i, F_j \in F$ iff F_i and F_j are not (volumetrically) redundant, and

$R = \{\mathcal{R}^{i,j} \mid \mathcal{R}^{i,j}$ is in the PV$\}$

3.2 Identifying symmetric interpretations

3.2.1 Defining symmetry

Our use of the term "symmetric" comes from the spatial notion of symmetry. Intuitively, two interpretations are symmetric if they are isomorphic, *i.e.* there is a



(a) (b) (c)

Figure 7: (a) Two sweep faces from our example component. (b) The metric diagram between the two sweep faces, p_1 and p_2 shown in (a). (c) The same metric diagram is now labeled to indicate its important property. Namely, when each axis is divided into intervals based on the intersection region, these intervals correspond to canonical features.

direct mapping between the feature types in each interpretation, and for each spatial relationship in the first interpretation, the corresponding spatial relation in the second interpretation is equivalent.

Definition 5 *Given two interpretations, $I_1 = (F_1, R_1)$ and $I_2 = (F_2, R_2)$, we say I_1 is* ***symmetric*** *to I_2 if:*

- $\exists \phi : F_1 \rightarrow F_2$ *and ϕ is one-to-one and onto, such that $\forall f \in F_1 : type(f) = type(\phi(f))$, and*
- $\forall f_1, f_2 \in F_1$, *with corresponding relationships,* $\mathcal{R}^{1,2} \in R_1$ *and* $\mathcal{R}^{\phi(1),\phi(2)} \in R_2$, *implies* $\mathcal{R}^{1,2}$ *is* equivalent *to* $\mathcal{R}^{\phi(1),\phi(2)}$.

This definition requires, and hinges on, the notion of two spatial relationships being equivalent. We formally define equivalence between spatial relationships as follows:

Definition 6 *Given two spatial relationships, $\mathcal{R}^1_{n\times m}$ and $\mathcal{R}^2_{n\times m}$, we say $\mathcal{R}^1$ and $\mathcal{R}^2$ are* ***equivalent*** *if:*
$\exists P^1_{n\times n}, P^2_{m\times m}$, *permutation matrices, such that* $\mathcal{R}^1 = P^1\mathcal{R}^2P^2$.

where a permutation matrix is[2]:

Definition 7 *An $n \times n$ matrix, P, is a* ***permutation matrix*** *if it can be obtained by any number of row exchanges on I_n.*

Therefore, the problem with identifying symmetric interpretations is one of performing an inexact match. Specifically, it is necessary to match the features for types, although they need not be the same features, and the spatial relationships for equivalence, although not necessarily equality.

[2] If we have a spatial relationship, $\mathcal{R}_{n\times m}$, and a matrix, $A_{l\times n}$, we cannot write $A\mathcal{R}$ since $\mathcal{R}$ is a matrix of characters. Therefore, mathematically, we need to represent each of the four characters in $\mathcal{R}$ with an ordinal value before multiplying.

3.2.2 The search for symmetry

Given the interpretations of a component (from its envisionment), we wish to determine which of these are symmetric. We formulate this task on a pairwise basis.

We saw that the problem of determining symmetry was two fold: *(i)* determine if there is a match between the feature sets, and *(ii)* determine if there is a match between spatial relationships. From this, we can formulate a search space that is hierarchical.

Definition 8 *Let $I_1 = (F_1, R_1)$ and $I_2 = (F_2, R_2)$ be two interpretations. The* hierarchical state-space search *to determine if I_1 and I_2 are symmetric is the triple, (SS_h, SS_l, A), where:*

- *(higher level state-space): $SS_h = (N_h, A_h, S_h, G_h)$ is a state-space search as follows:*
  - *(higher level nodes): N_h = all possible mappings $\phi : F_1 \rightarrow F_2$.*
  - *(higher level arcs): A_h where for $n_{h_1}, n_{h_2} \in N_h$ then $(n_{h_1}, n_{h_2}) \in A_h$ iff n_{h_1} and n_{h_2} differ in the mapping of exactly two features.*
  - *(higher level start nodes): S_h = the mapping from element i in F_1 to element i in F_2.*
  - *(higher level goal nodes): G_h = the set of mappings that* (i) *maintain feature type consistency and* (ii) *that have all adjacent arcs in A incident upon satisfied lower level search spaces.*
- *(lower level state-space): $SS_l = \{(N_{l_{1,1}}, A_{l_{1,1}}, S_{l_{1,1}}, G_{l_{1,1}}), (N_{l_{1,2}}, A_{l_{1,2}}, S_{l_{1,2}}, G_{l_{1,2}}), \ldots, (N_{l_{|R_1|,|R_2|}}, A_{l_{|R_1|,|R_2|}}, S_{l_{|R_1|,|R_2|}}, G_{l_{|R_1|,|R_2|}})\}$, where for each search we have:*
  - *(lower level nodes): $N_{l_{i,j}}$ = all possible permutations of element i in R_1.*
  - *(lower level arcs): $A_{l_{i,j}}$ where for $n_{l_1}, n_{l_2} \in N_{l_{i,j}}$, $(n_{l_1}, n_{l_2}) \in A_{l_{i,j}}$ iff n_{l_1} and n_{l_2} differ in the exchange of exactly one row or column.*

  - *(lower level start nodes): $S_{l_{i,j}}$ = the node with no row or column exchanges.*
  - *(lower level goal nodes): $G_{l_{i,j}}$ = all nodes equal to element j in R_2.*

- *(high-to-low arcs): A where for $n_h \in N_h$ and $n_{l_{i,j}} \in N_{l_{i,j}}$, $(n_h, n_{l_{i,j}}) \in A$ iff element i in F_1 maps to element j in F_2.*

On the higher level, the search space contains all possible mappings between the two sets of features. On the lower level, which is formed once a solution to the first level is found, there will be a search space for each pair of spatial relationships determined from the feature mapping and interpretation. Each search space contains the permutations between a pair of spatial relationships. The higher level and lower level in Figure 11 show examples of the heirarchical state space for the interpretations shown in Figure 2

3.3 Drawing analogies

If a database of components is maintained, drawing analogies provides the ability for planners to reuse previous plans by identifying which components from the database (and hence which plans) are analogous to a given component. Drawing component analogies is a non-trivial task since the geometric models, and even the feature models, of the two components will be different.

3.3.1 Defining analogies and subparts

The motivation for analogies is to retrieve previous planning information that is likely applicable to some current situation. It is important, therefore, to define analogies so that two components are analogous if their planning information is similar. However, since we do not store any planning information, we cannot use similarity of plans as a definition. With symmetric interpretations, if both the features and spatial relationships in two interpretations were equivalent, then their corresponding plans would likely be structurally equivalent. Extending this idea, we can define analogous components as having the same feature types and equivalent spatial relationships. Formally then:

Definition 9 *Two components are* **analogous** *if their corresponding PVs are isomorphic. Here corresponding spatial relationships within the PV must be equivalent and corresponding features must have the same types.*

Notice two important consequences from this definition. First, it is essential to include, and compare, spatial relationships in this matching.

Second, we have developed methods to compare two interpretations. Interpretations, however, are just subgraphs of the PV. Therefore, we can use all of the notions from the previous section with interpretations replaced with PVs to solve the problem of analogies between components.

We cannot ignore spatial relationships since they can create important differences in partial plans.

3.3.2 Efficient analogies with maximal features

In this section we will provide the details on how to efficiently solve the problem of drawing analogies between components. Here we take advantage of a new concept called the maximal-feature subgraph of a component's PV. The maximal-feature subgraph provides a significant reduction in computational effort for the task of drawing analogies between components as well as a basis for the systematic extraction of subparts of a component.

To clarify this inferencing task, consider Figure 8 which shows the overall structure of the problem and solution for drawing analogies. At the highest level, the task of drawing analogies between components is to perform graph isomorphism on the PVs of the components. Rather than carry through with such an expensive process, we propose extracting a subgraph of the PV which is unambiguous. For two components, these subgraphs can then be compared. The figure also illustrates how we extract subparts of components. Subparts are found directly from the maximal-feature subgraph. These subparts are subgraphs of the maximal-feature subgraph, and hence are also subgraphs of the PV. Again, once the correct subparts (subgraphs) are identified, we can compare them with the routines developed for symmetric interpretations.

Now then, the maximal-feature subgraph is defined as follows:

Definition 10 *Let the nodes of a PV be $N = \{1, 2, \ldots, n\}$, and the arcs be A. Then the* **maximal-feature subgraph** *is a subgraph whose nodes are $\phi(i), 1 \leq i \leq n$, where $\phi(i) = i$ if there is no $\mathcal{R}^{i,k}$ of type* subsumes *in A, $1 \leq k \neq i \leq n$. Furthermore, $\phi(i)$ is adjacent to $\phi(j)$ in the maximal-feature PV iff i and j are adjacent in A.*

For the example component, the maximal-feature subgraph is shown in Figure 9. The only features that remain are those not subsumed by any other feature. This graph can be systematically found by searching for edges of type *subsumes*. Notice that for the example component, the maximal-feature subgraph is a (4,4) graph, in contrast to the PV which is a (19,93) graph. Hence when checking for isomorphism (as in symmetric interpretations), we have significantly reduced the amount of work necessary.

3.3.3 Subparts from the maximal features

Consider all of the subgraphs of the maximal-feature subgraph in Figure 9. The node sets would be $\{F_1\}$, $\{F_5\}$, $\{F_8\}$, $\{F_{19}\}$, $\{F_1, F_5\}$, $\cdots$, $\{F_1, F_5, F_8, F_{19}\}$. We

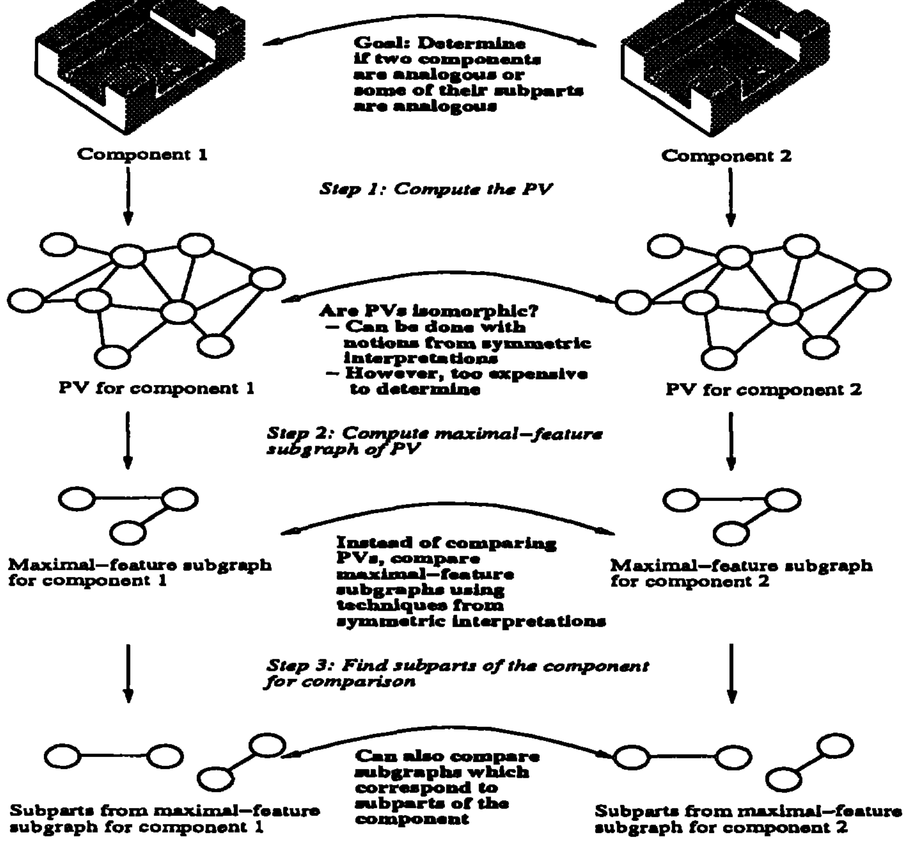


Figure 8: The utility of the maximal-feature subgraph. The goal (top) is to draw analogies between two components. From each component we can construct a PV. These PVs can be compared, but only at a high computational cost. Therefore, we first find an unambiguous subgraph (the maximal-feature subgraph) of the PV and compare these instead. The maximal-feature subgraph also aids in the extraction of subparts of the component.

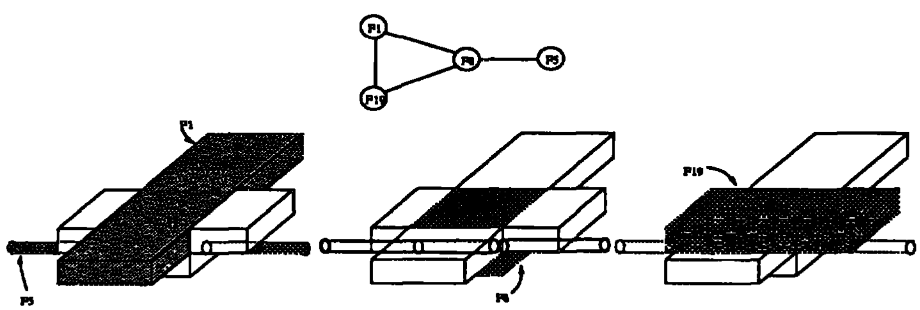


Figure 9: The maximal-feature subgraph for the example component. Only the features not subsumed by another other feature remain in the graph. The features themselves are shown below the graph.

could choose to identify each of these as a subpart, but this will not always lead to situations of applicable planning information. Instead, the problem is to identify which of these subgraphs should be considered. Since we noticed that we need to not only consider features, but also spatial relationships, it follows that the solution should also consider spatial relationships. Hence we make the following observation:

Observation 3 *XS The blocks of the maximal-feature subgraph of a component correspond to subparts of the*

component which will likely have applicable manufacturing planning information for analogous subparts.

A block of a graph is a maximal non-separable subgraph. Such a notion suits our formulation of subgraphs since it includes both features (maximality) and spatial relationships (non-separability). Blocks correspond to subparts of a component which are sets of maximal features that all interact. For the example component, the blocks are shown in Figure 10 along with the corresponding subparts. These subparts contain maximal features which all interact, making them likely to have applicable planning information for other similar subparts.

Finally, to conclude this section, Figure 11 shows the overall search space for the problem of drawing analogies. This space is hierarchical, containing three levels. The top level has the maximal-feature subgraph and its blocks. Each of these can be compared to subparts of other components through the mechanisms developed for symmetric interpretations (the bottom two levels). The search mechanism for drawing analogies is summarized in the following steps:

1. Given two PVs, PV_1 and PV_2, compute their maximal-feature subgraphs, MFS_1 and MFS_2, respectively.
2. Using MFS_1 and MFS_2, invoke the higher level algorithm for symmetric interpretations to determine if they are analogous.
3. for each block in MFS_1
   (a) Compare this block to each block in MFS_2 using the higher level algorithm for symmetric interpretations.

4 Conclusion

In this paper we developed and presented systematic spatial inferencing using our qualitative model of a component [7]. The tasks discussed were the generation of the multiple interpretations of a component and drawing analogical inferences between both components and subparts of components. By formulating these problems as tasks based on a unique qualitative model, we have provided an integrated framework for spatial inferencing about a component.

The mathematical definition of the spatial relationships between features has been a key to all of the spatial inferencing tasks presented. Most previous notions of spatial relationships consisted of a finite vocabulary that covered enough distinctions between feature relationships to serve one particular task. The model can also be extended to other activities requiring spatial inferencing.

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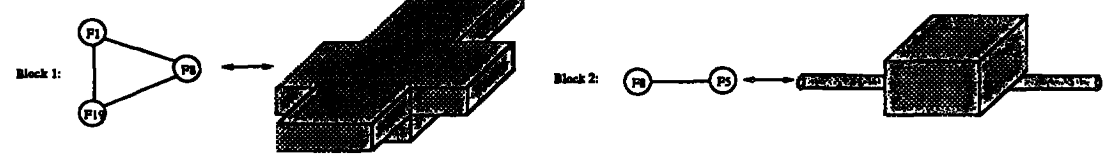

Figure 10: The blocks of the maximal-feature subgraph for the example component. Shown along side the block are the corresponding subparts of the component. Since these subparts are composed of set of maximal features which all interact, their corresponding planning information will likely be applicable to other similar subparts.

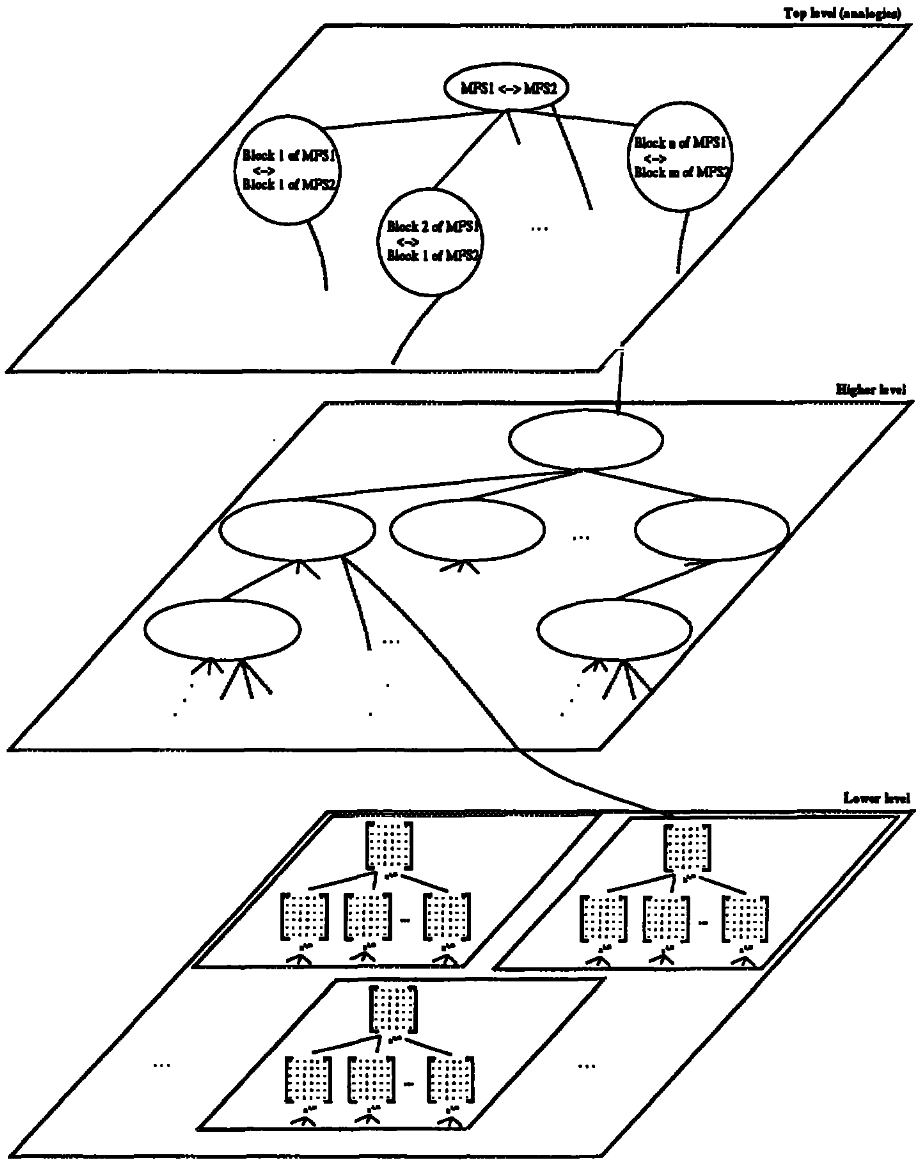

Figure 11: The hierarchical search space for the task of drawing analogies. The top level contains both the maximal-feature subgraph and its blocks. Each of these can be compared to subparts of other components through the mechanisms developed for symmetric interpretations (bottom two levels).