

Topological Models for Physical Computing

Richard S. Palmer*
Department of Computer Science
Cornell University
Ithaca, NY 14853
rick@cs.cornell.edu

Vadim Shapiro
Computer Science Department
General Motors Research Laboratories
Warren, MI 48090-9055
shapiro@gmr.com

August 12, 1992

Modeling the interaction of geometric form and physical behavior dominates many engineering and scientific activities. Broadly, such activities can be divided into two categories: (1) simulation and understanding the physical behavior of existing objects and devices, and (2) synthesis of new objects and devices that exhibit a desired physical behavior. Our research is focused on developing a unified theory, computer representations, algorithms, and systems necessary to support a wide range of these activities.

Traditionally, models of the relationship between geometric form and physical behavior have relied on centuries-old classical tools that were intended to be used by human beings. For example, the symbolic forms of vector calculus and differential equations are well suited for human communication and have been crucial in the development and analysis of physical models. Similarly, engineering analysis is often reduced to obtaining a set of values associated with physical quantities in a model, which must then be interpreted to describe qualitative behavior of the model. And, until recently, synthesis has been considered a creative task that could be performed exclusively by humans with the aid of the same tools from classical analysis. Existing tools for modeling, analysis, and synthesis in engineering have evolved from technology that assumed a high degree of human intervention. This assumption is no longer valid; to take full advantage of the power of computation we need new technology.

During the 1960's and 70's, researchers developed methods for characterizing and classifying physical models in terms of *topological* properties of selected physical quantities. These models are consistent with classical methods of modeling (e.g., using differential equations) and analysis (e.g., finite/boundary element methods). In addition, these topological models capture the relationship between form and function at a more fundamental level, explaining analogies and differences between various physical domains, and suggesting new methods for modeling, analysis, and synthesis.

Until now, there has been little emphasis on development of computer representations and algorithms to take advantage of topological models. Our work is a first step in that direction. In particular, we argue that notions from algebraic topology are well suited for abstracting, modeling, representing, manipulating, and combining models of physics. We use algebraic topology to develop new theoretical and computational devices called *Physical Elements*, which are combinatorial structures representing the local behavior of physical phenomena. Physical elements encode the information typically represented by partial differential equations in a manner amenable to automatic computation.

*Supported in part by The Advanced Research Projects Agency of the Department of Defense under Office of Naval Research Contract N00014-88-K-0591.