

# A Proof-Theoretic Approach to Irrelevance: Foundations and Applications

**Alon Y. Levy**

AT&T Bell Laboratories  
Murray Hill, NJ, 07974  
levy@research.att.com

**Richard E. Fikes**

KSL, Stanford University  
Palo Alto, California 94304  
fikes@cs.stanford.edu

**Yehoshua Sagiv**

Dept. of Computer Science  
Hebrew University, Jerusalem, Israel  
sagiv@cs.huji.ac.il

## Introduction

Control of reasoning is a major issue in scaling up problem solvers that use declarative representations, since inference is slowed down significantly as the size of the knowledge base (KB) is increased. A key factor for the slow down is the search of the inference engine through parts of the KB that are *irrelevant* to the query at hand. The ability of a system to ignore irrelevant information is therefore a key in scaling up AI systems to large and complex domains.

To address this problem we have developed a general framework for analyzing irrelevance and specific algorithms for efficiently detecting irrelevant portions of a knowledge base [Levy, 1993]. Our framework focussed on the following problem. Given a knowledge base  $\Delta$  and a query  $q$ , which parts of  $\Delta$  are irrelevant to  $q$ , and how can the problem solver use this knowledge of irrelevance to improve its performance. Since our main goal in analyzing irrelevance was to speed up inference, we presented a *proof theoretic* analysis of irrelevance, as opposed to attempts to formalize the common sense notion of irrelevance [Keynes, 1921; Carnap, 1950; Gärdenfors, 1978], or a *meta-theoretic* analysis [Subramanian, 1989]. In our analysis we presented a space of possible definitions of irrelevance and analyzed the properties of definitions in the space. We have shown that the proof theoretic analysis yields the necessary distinctions that enable us to address the issues concerning the usage of irrelevance to speed up inferences. The space of definitions provided several insights on the kinds of irrelevancies that arise in inference, on the utility of ignoring irrelevant information, and on problems that seemed previously unrelated.

A key component of our work addressed the issue of developing efficient algorithms for automatically detecting irrelevant parts of a knowledge base [Levy and Sagiv, 1992; Levy *et al.*, 1993; Levy and Sagiv, 1993; Levy *et al.*, 1994a]. Our work yielded solutions to several open theoretical problems, as well as practical algorithms which are now being incorporated into commercial database systems. We have also applied our framework to the problems of automatically creating abstractions [Levy, 1994], creating models for physical

devices (for tasks such as design, simulation and diagnosis) [Iwasaki and Levy, 1994], and gathering information in distributed heterogeneous environments [Levy *et al.*, 1994b].

This paper focuses on the foundations of our framework and outlines the space of definitions of irrelevance. The last section outlines the results concerning automatic determination of irrelevance and describes the applications of our framework.

## Preliminaries

In our discussion, we assume that the theory of the domain is represented by a knowledge base  $\Delta$  of facts which are closed formulas in first-order predicate calculus. We assume that the inference mechanism employs a set of sound inference rules. A derivation  $D$  of a closed formula  $\psi$  from  $\Delta$  is a sequence of closed formulas,  $\alpha_1, \dots, \alpha_n$ , such that  $\alpha_n = \psi$  and for each  $i$  ( $1 \leq i \leq n$ ), either  $\alpha_i \in \Delta$ ,  $\alpha_i$  is a logical axiom, or  $\alpha_i$  is the result of applying an inference rule to some elements  $\alpha_{i_1}, \dots, \alpha_{i_k}$  that appear prior to  $\alpha_i$ . The formulas  $\alpha_{i_1}, \dots, \alpha_{i_k}$  are said to be *subgoals* of  $\alpha_i$ . The set of formulas in  $D$  that do not have any subgoal is called the *base* of the derivation, denoted by  $Base(D)$ . The set  $Base(D)$  represents a "support set" for  $\psi$  in the knowledge base. We consider only derivations in which every  $\alpha_i$  is connected to  $\psi$  through the subgoal relation. A *query* is represented by a closed formula  $\psi$ . The answer to the query is **true** if there is a derivation of  $\psi$  from  $\Delta$ ; otherwise, the answer is **false**.

## Defining Irrelevance

Our goal is to express, reason with and automatically derive irrelevance claims, i.e., claims of the form " $X$  is irrelevant to the query  $\psi$  with respect to the knowledge base  $\Delta$ ". To do so, it is essential that we give such claims a formal definition.  $X$  is called the *subject* of the irrelevance claim. Here we consider the case in which  $X$  is a fact or set of facts. Other irrelevance subjects, such as objects, relation arguments and refinements of predicates are discussed in [Levy, 1994].

Broadly, we can take two possible approaches to analyzing irrelevance. The first approach, which has

been pursued by several philosophers ([Keynes, 1921; Carnap, 1950; Gärdenfors, 1978]), is to try to capture our common-sense notion of irrelevance with a formal definition. In that approach, we would consider a formal definition of irrelevance and check whether it satisfies properties that we consider natural for our intuitive notion of irrelevance. We pursue a second approach that focuses on analyzing how irrelevance arises in inference. In this approach, we are most interested in properties of definitions of irrelevance that are informative in designing inference methods that utilize irrelevance. For example, we are interested in whether irrelevance claims can be automatically derived, how the claims change when the KB changes, and what is the utility of removing irrelevant facts. The following example illustrates these properties.

**Example 1:** Consider the following knowledge base  $\Delta_0$ , describing students and the courses in which they can serve as teaching assistants.

$r_1 : \text{attendClass}(X, Y) \Rightarrow \text{pass}(X, Y).$   
 $r_2 : \text{passExam}(X, Y) \Rightarrow \text{pass}(X, Y).$   
 $r_3 : \text{pass}(X, Y) \wedge \text{tookGradCourse}(X) \Rightarrow \text{canTA}(X, Y).$   
 $r_4 : \text{pass}(X, Y) \wedge (Y \geq 300) \Rightarrow \text{tookGradCourse}(X).$   
 $g_1 : \text{attendClass}(\text{Fred}, 101).$   
 $g_2 : \text{passExam}(\text{Fred}, 101).$   
 $g_3 : \text{passExam}(\text{Fred}, 201).$   
 $g_4 : \text{passExam}(\text{Fred}, 301).$

Consider the query  $q = \text{canTA}(\text{Fred}, 101)$ , which can be derived either by using  $g_1, g_4$  and the rules  $r_1, r_3$  and  $r_4$ , or by using  $g_2, g_4$  and the rules  $r_2, r_3$  and  $r_4$ . Hence, each of the ground atoms  $g_1, g_2$  and  $g_3$  is irrelevant to the query when considered alone, because for each one, the answer to the query can be derived without it. However, there are differences between these irrelevance claims. Specifically,  $g_3$  is not used in any derivation of the query. As for  $g_1$  and  $g_2$ , even though the query can be derived without either one of them, it cannot be derived without both of them and, therefore, we cannot remove both.

There are, however, other variants of relevance and irrelevance. For one, even though the ground atom  $\text{canTA}(\text{Fred}, 301)$  is not part of any derivation of an answer to  $q$ , we may still consider it relevant to the query, because it is always entailed by the facts and rules used in any derivation of an answer. To see another type of irrelevance, consider the query  $\text{canTA}(\text{Fred}, 301)$ . The atom  $\text{passExam}(\text{Fred}, 302)$  can be part of a derivation of an answer to this query (if it were in the KB), but such a derivation would not be *minimal*, in the sense that the set of ground atoms that it uses from the KB is not minimal (i.e., since  $g_4$  must be used, along with the rules  $r_2, r_3$  and  $r_4$ , there is no need to use any other ground fact from the KB). Hence, the atom  $\text{passExam}(\text{Fred}, 302)$  may be deemed irrelevant. Finally, rules may also be irrelevant. For example, if we add the rule

$$\text{passExam}(X, Y) \wedge (Y \geq 300) \Rightarrow \text{canTA}(X, Y)$$

it would be considered irrelevant, since answers can be derived without it. However, for some inference mechanisms, it may be the case that this rule will speed inference, since fewer rule applications may be needed to derive answers if this rule is used. ■

As illustrated by the above example, there is no *single* best definition of irrelevance. For example, we can define a formula  $\phi$  to be irrelevant to  $\psi$  if there is some derivation of  $\psi$  that does not contain  $\phi$ , or alternatively, we can require that *no* derivation of  $\psi$  contains  $\phi$ . Therefore, we describe a space of possible definitions of irrelevance, and investigate the properties of various definitions within this space. Our space is based on a proof-theoretic analysis of irrelevance, i.e., on investigating the ways in which formulas can participate in derivations of the query. In contrast, Subramanian [Subramanian, 1989] described a *meta-theoretic* account of irrelevance that considers only the formulas in the KB, not the possible derivations of the query. Consequently, we are able to make finer distinctions than those made in Subramanian's framework.

## A Space of Definitions

Definitions in the space vary along two axes. In the first axis, we consider different ways of defining *derivation irrelevance*, i.e., irrelevance of a subject  $\phi$  to a *single* derivation  $D$  of the query  $\psi$ . Derivation irrelevance is given by defining a binary predicate  $DI(\phi, D)$ . The following are a few examples of how  $DI$  can be defined:

- $DI_1(\phi, D)$  iff  $\phi \notin \text{Base}(D)$ .
- $DI_2(\phi, D)$  iff  $\phi \notin D$ .
- $DI_3(\phi, D)$  iff  $\text{Base}(D) \not\models \phi$ .
- $DI_4(\phi, D)$  iff  $\text{Base}(D) \not\models \phi, \neg\phi$ .

Definition  $DI_1$  requires that  $\phi$  not be in the support set of the derivation  $D$ . Definition  $DI_2$  is stronger and requires that  $\phi$  not be anywhere in  $D$ . Definition  $DI_3$  is even stronger and requires that  $\phi$  not be a logical consequence of the formulas in  $\text{Base}(D)$ , and  $DI_4$  requires that  $\neg\phi$  not be a logical consequence either.

Requiring that  $DI(\phi, D)$  holds for all possible derivations of the query may be too restrictive. Therefore, in the second axis, we consider different subsets of the derivations of the query for which we require  $DI(\phi, D)$  to hold. Formally, given the set  $\mathcal{D}^\psi(\Delta)$  of all possible derivations of  $\psi$  from  $\Delta$ , we consider a subset  $\mathcal{D}_0^\psi(\Delta)$  of  $\mathcal{D}^\psi(\Delta)$  (which may be  $\mathcal{D}^\psi(\Delta)$  itself), and require  $DI(\phi, D)$  to hold only for derivations in  $\mathcal{D}_0^\psi(\Delta)$ . For example, we can require  $DI(\phi, D)$  to hold only for the set of *minimal* derivations of  $\psi$ . As another example, we can consider only the set of derivations bounded by some resource constraint.

Given a choice for  $DI$  and  $\mathcal{D}_0^\psi(\Delta)$ , we give two definitions of irrelevance, depending on whether  $DI$  is required to hold for *all* derivations in  $\mathcal{D}_0^\psi(\Delta)$  or for *some*

derivation in  $\mathcal{D}_0^\psi(\Delta)$ .<sup>1</sup> Formally, a definition of irrelevance in our space is given as follows:

**Definition 1:** Suppose we are given:

1. A knowledge base  $\Delta$ .
2. A closed formula  $\phi$  (the subject).
3. A query  $\psi$ .
4. A predicate  $DI(\tau, D)$  specifying when a formula  $\tau$  is irrelevant to a derivation  $D$ .
5. A subset  $\mathcal{D}_0^\psi(\Delta)$  of  $\mathcal{D}^\psi(\Delta)$  (the set of all derivations of  $\psi$  from  $\Delta$ ). By a slight abuse of notation, we usually denote  $\mathcal{D}_0^\psi(\Delta)$  as either  $\mathcal{D}_0(\Delta)$  or  $\mathcal{D}_0$ .

The formula  $\phi$  is *weakly irrelevant* to  $\psi$  with respect to  $\Delta$ ,  $DI$  and  $\mathcal{D}_0$ , denoted by  $WI(\phi, \psi, \Delta, DI, \mathcal{D}_0)$ , if  $DI(\phi, D)$  holds for some  $D \in \mathcal{D}_0(\Delta)$ .

The formula  $\phi$  is *strongly irrelevant* to  $\psi$  with respect to  $\Delta$ ,  $DI$  and  $\mathcal{D}_0$ , denoted by  $SI(\phi, \psi, \Delta, DI, \mathcal{D}_0)$ , if  $DI(\phi, D)$  holds for every  $D \in \mathcal{D}_0(\Delta)$ .

If  $\mathcal{D}^\psi(\Delta)$  is empty (i.e.,  $\psi$  is not derivable from  $\Delta$ ), then  $\phi$  is both weakly and strongly irrelevant to  $\psi$ . ■

In our discussion, we want to refer to irrelevance of a set of formulas. Formally, we define irrelevance of a set of formulas by extending the definition of  $DI$ :

**Definition 2:** If  $\Phi$  is a set of formulas,  $DI(\Phi, D)$  holds if  $DI(\phi_i, D)$  holds for every  $\phi_i \in \Phi$ . ■

The definitions of strong and weak irrelevance remain unchanged. It will also be useful to state irrelevance claims that hold for a set of knowledge bases. For example, in the context of Horn-rule knowledge bases, we may want to know whether a rule is irrelevant with respect to all the knowledge bases that have the same rules (but may have different ground atoms). We extend the definitions as follows:

**Definition 3:** Let  $\Sigma$  be a set of knowledge bases. We say that  $\phi$  is weakly irrelevant to  $\psi$  with respect to  $\Sigma$ , denoted by  $WI(\phi, \psi, \Sigma, DI, \mathcal{D}_0)$ , if  $\phi$  is weakly irrelevant to  $\psi$  with respect to every KB in  $\Sigma$ , i.e., if  $WI(\phi, \psi, \Delta, DI, \mathcal{D}_0)$  holds for every  $\Delta \in \Sigma$  (note that  $\mathcal{D}_0$  is actually a function that returns a subset of  $\mathcal{D}^\psi(\Delta)$  for every  $\Delta \in \Sigma$ ). The definition for strong irrelevance is extended likewise. ■

In Example 1, we can see different kinds of irrelevance claims. The atom  $g_1$  is weakly irrelevant to the query  $q = canTA(Fred, 101)$ , since there is a derivation of  $q$  that does not use  $g_1$  (i.e., it uses  $g_2$  instead). Consequently,  $WI(g_2, q, \Delta_0, DI_2, \mathcal{D}^q(\Delta))$  holds. Similarly for the atom  $g_2$ . The atom  $g_3$  is strongly irrelevant to  $q$ , because none of the derivations of  $q$  uses it. Consequently,  $SI(g_3, q, \Delta_0, DI_2, \mathcal{D}^q(\Delta))$  holds.

<sup>1</sup>We can also consider other ways of quantifying over the set  $\mathcal{D}_0^\psi(\Delta)$ , such as requiring that  $DI(\phi, D)$  holds for some percent of the derivations in  $\mathcal{D}_0^\psi(\Delta)$ . In fact, different ways of quantifying over  $\mathcal{D}_0^\psi(\Delta)$  could be considered a third axis in the space. Here, we consider only universal and existential quantification.

The atom  $q_1 = canTA(Fred, 301)$  is strongly irrelevant to  $q$  if we consider derivation irrelevance based on  $DI_2$ . However, if we consider derivation irrelevance based on  $DI_3$ , the atom  $q_1$  is not strongly irrelevant to  $q$ , since the formulas used to derive  $q$  can also be used to derive  $q_1$ .

Finally, suppose that  $q_1$  is the query. If we consider the set of all derivations of  $q_1$ , then the atom  $passExam(Fred, 310)$  would not be strongly irrelevant to the query (had it been in the KB), since it can be used in a derivation of  $q_1$  (to derive  $tookGradCourse(Fred)$ ). However, if we consider only derivations in which  $Base(D)$  is minimal (i.e., there is no subset of  $Base(D)$  that is enough to derive the query), then  $passExam(Fred, 310)$  would not be part of any derivation of  $q_1$  and, therefore, would be strongly irrelevant to the query  $q_1$ .

### Properties of Definitions in the Space

The following theorem summarizes several properties of definitions in our space that are practical interest in using irrelevance for speeding up inference.

**Theorem 1:** *Properties A0–A8 (listed below) hold, given the following notation.*

- $\psi$  denotes a query.
- $\phi, \phi_1$  and  $\phi_2$  denote formulas.
- $\Phi, \Phi_1$  and  $\Phi_2$  denote sets of formulas.
- $\Delta$  denotes a knowledge base.
- $\Sigma, \Sigma_1$  and  $\Sigma_2$  denote sets of knowledge bases.
- $\mathcal{D}_0, \mathcal{D}_1$  and  $\mathcal{D}_2$  denote functions that return a subset of  $\mathcal{D}^\psi(\Delta)$  when given a KB  $\Delta$  and a query  $\psi$ .
- $DI, DI'$  and  $DI''$  denote definitions of derivation irrelevance.
- $DI_1, DI_2, DI_3$  and  $DI_4$  are the definitions of derivation irrelevance from the beginning of Section .

A0. If  $WI(\phi, \psi, \Delta, DI_1, \mathcal{D}^\psi(\Delta))$  holds, then the formula  $\phi$  can be removed from  $\Delta$  without changing the answer to  $\psi$ , i.e.,

$$\Delta \vdash \psi \iff \Delta - \phi \vdash \psi$$

A1. If  $DI'(\Phi, D) \Rightarrow DI''(\Phi, D)$  for all derivations  $D \in \mathcal{D}_0$ , then

$$\begin{aligned} SI(\Phi, \psi, \Sigma, DI', \mathcal{D}_0) &\Rightarrow SI(\Phi, \psi, \Sigma, DI'', \mathcal{D}_0) \\ WI(\Phi, \psi, \Sigma, DI', \mathcal{D}_0) &\Rightarrow WI(\Phi, \psi, \Sigma, DI'', \mathcal{D}_0) \end{aligned}$$

A2. If  $\mathcal{D}_1(\Delta) \subset \mathcal{D}_2(\Delta)$  for all knowledge bases  $\Delta \in \Sigma$ , then

$$\begin{aligned} SI(\Phi, \psi, \Sigma, DI, \mathcal{D}_2) &\Rightarrow SI(\Phi, \psi, \Sigma, DI, \mathcal{D}_1) \\ WI(\Phi, \psi, \Sigma, DI, \mathcal{D}_1) &\Rightarrow WI(\Phi, \psi, \Sigma, DI, \mathcal{D}_2) \end{aligned}$$

A3. The following is always true.

$$SI(\Phi, \psi, \Sigma, DI, \mathcal{D}_0) \Rightarrow WI(\Phi, \psi, \Sigma, DI, \mathcal{D}_0)$$

A4. If  $\Sigma_1 \subseteq \Sigma_2$ , then

$$\begin{aligned} SI(\Phi, \psi, \Sigma_2, DI, \mathcal{D}_0) &\Rightarrow SI(\Phi, \psi, \Sigma_1, DI, \mathcal{D}_0) \\ WI(\Phi, \psi, \Sigma_2, DI, \mathcal{D}_0) &\Rightarrow WI(\Phi, \psi, \Sigma_1, DI, \mathcal{D}_0) \end{aligned}$$

A5. If the inference rules are complete,  $\phi_1 \equiv \phi_2$  and  $DI$  is either  $DI_3$  or  $DI_4$ , then

$$\begin{aligned} WI(\phi_1, \psi, \Sigma, DI, \mathcal{D}_0) &\Rightarrow WI(\phi_2, \psi, \Sigma, DI, \mathcal{D}_0) \\ SI(\phi_1, \psi, \Sigma, DI, \mathcal{D}_0) &\Rightarrow SI(\phi_2, \psi, \Sigma, DI, \mathcal{D}_0) \end{aligned}$$

A6. If the inference rules are complete, then

$$\begin{aligned} WI(\phi, \psi, \Sigma, DI_4, \mathcal{D}_0) &\Rightarrow WI(\neg\phi, \psi, \Sigma, DI_4, \mathcal{D}_0) \\ SI(\phi, \psi, \Sigma, DI_4, \mathcal{D}_0) &\Rightarrow SI(\neg\phi, \psi, \Sigma, DI_4, \mathcal{D}_0) \end{aligned}$$

A7. If  $DI(\Phi_1, D) \wedge DI(\Phi_2, D) \Rightarrow DI(\Phi_1 \cup \Phi_2, D)$  for all derivations  $D \in \mathcal{D}_0$ , then

$$\begin{aligned} SI(\Phi_1, \psi, \Sigma, DI, \mathcal{D}_0) \wedge SI(\Phi_2, \psi, \Sigma, DI, \mathcal{D}_0) &\Rightarrow \\ SI(\Phi_1 \cup \Phi_2, \psi, \Sigma, DI, \mathcal{D}_0) & \\ SI(\Phi_1, \psi, \Sigma, DI, \mathcal{D}_0) \wedge WI(\Phi_2, \psi, \Sigma, DI, \mathcal{D}_0) &\Rightarrow \\ WI(\Phi_1 \cup \Phi_2, \psi, \Sigma, DI, \mathcal{D}_0). & \end{aligned}$$

A8. If  $\Delta \vdash \tau$  and  $\Delta$  is consistent, then

$$SI(\Phi, \psi, \Delta, DI_2, \mathcal{D}^\psi) \Rightarrow SI(\Phi, \psi, \Delta \cup \tau, DI_2, \mathcal{D}^\psi)$$

Property A0 shows when we can remove an irrelevant fact without changing the answer to the query. Properties A1–A4 show how the relative strength of irrelevance claims is affected as a result of changing some of the parameters of these claims.

In general, irrelevance claims in our space are not preserved under equivalence of the subject, query or knowledge base. Although preservation under equivalence has been considered natural for a common-sense notion of irrelevance [Gärdenfors, 1978], we believe that it is not necessarily appropriate when analyzing irrelevance for the purpose of speeding inferences. Property A5 identifies some cases in which irrelevance claims are preserved under equivalence. Property A6 is similar in the sense that it shows when irrelevance claims are closed under negation.

Property A7 shows when irrelevance claims can be added up. This property is important when a system needs to determine whether it can use all the irrelevance claims it has or whether using certain ones will falsify others. The same property does not hold for weak irrelevance. In general, adding new formulas to the knowledge base may cause a formula that was irrelevant to become relevant or vice versa. In particular, weak irrelevance claims can change even when the added formulas are logical consequences of the knowledge base. In contrast, as Property A8 shows, strong irrelevance claims do not change when we reason with existing knowledge.

Removing irrelevant facts does not necessarily lead to speedups of inference. In particular, removing a fact that is only weakly irrelevant may not speed inference. In fact, explanation based learning systems do exactly the opposite; that is, they add redundant

rules (which, in our framework, would be considered weakly irrelevant). The utility of adding such rules is a subject of ongoing research (e.g., [Minton, 1988; Greiner and Jurišica, 1992]).

For strong irrelevance, savings are guaranteed for many cases. For example, if  $SI(\Phi, \psi, \Delta, DI_2, \mathcal{D}^\psi)$  holds (i.e., all derivations of the query are considered), then deriving  $\psi$  from  $\Delta - \Phi$  costs no more than deriving it from  $\Delta$ . As shown in [Levy and Sagiv, 1992; Levy, 1993], it is possible to determine efficiently the facts that are strongly irrelevant to a query and removing such facts yields significant savings in practice. These savings come from several sources. First, removing irrelevant facts prunes branches of the search space. Since much of the cost of reasoning in a large knowledge base is in doing database lookups, removing a large number of irrelevant ground facts at the outset will significantly reduce the cost of each lookup operation, and will also yield significant space savings. Finally, answers to queries do not have to be recomputed after updates that involve only facts that are irrelevant to a query.

## Encompassing Previous Definitions

An important contribution of our space of definitions is that it encompasses definitions of irrelevance previously discussed in the literature and, therefore, enables us to make comparisons among them. For example, Subramanian investigates several definitions of irrelevance; in our framework, all these definitions are instances of weak irrelevance. The main definition investigated in [Subramanian, 1989] is equivalent to  $WI(\phi, \psi, \Delta, DI_3, \mathcal{D}^\psi)$ , assuming that the inference rules are complete. Another definition of Subramanian can be formulated in our space as  $WI(\phi, \psi, \Delta, DI_4, \mathcal{D}^\psi)$ . Couching the definitions investigated by Subramanian in our framework shows how some of the limitations of those definitions can be overcome by using other definitions in the space. In particular, our space identifies definitions of irrelevance which have the property that removing irrelevant facts will lead to speeding up inferences and for which some forms of monotonicity and adding-up hold.

Our framework also sheds new light on the problem of detecting when a query is independent of an update [Blakeley *et al.*, 1989; Elkan, 1990]. In [Levy and Sagiv, 1993], we show that detecting independence is really a form of detecting weak irrelevance (specifically,  $WI(\phi, \psi, \Delta, DI_1, \mathcal{D}^\psi)$ ). In contrast, earlier work on detecting independence only provided algorithms for detecting a form of strong irrelevance, which is a more restricted condition. Identifying the relationship between irrelevance and independence resulted in novel algorithms for detecting independence, based on novel algorithms for strong and weak irrelevance.

Irrelevance is also the underlying notion in certain optimization algorithms for recursive rules. The definition of irrelevance used in [Srivastava and Ramakr-

ishnan, 1992] is equivalent to  $SI(\phi, \psi, \Delta, DI_2, \mathcal{D}^\psi)$ . The notion of irrelevance discussed in [Levy and Sagiv, 1992] can be couched in our framework as  $SI(\phi, \psi, \Delta, DI_2, \mathcal{D}_m)$ , where  $\mathcal{D}_m$  is the set of all minimal derivations of the query.

## Results and Applications

We briefly mention some important results developed in our framework and their applications.

### Automatically Deriving Irrelevance Claims:

We considered the problem of automatically deriving irrelevance claims in detail for Horn rule knowledge bases and some extensions [Levy and Sagiv, 1992; Levy *et al.*, 1993; Levy and Sagiv, 1993; Levy *et al.*, 1994a]. In order to produce algorithms that are of practical interest, we focused on derivation methods that required consideration of only *part* of the KB (e.g., only the rules) and that produced irrelevance claims which are independent of changes made to the other parts. Moreover, our algorithms enforced the semantics of interpreted predicates appearing in the KB (e.g., order and sort predicates). We developed a powerful tool, the *query-tree* that provides a sound and complete inference procedure for strong irrelevance for some classes of Horn knowledge bases. The query-tree also provided solutions to the open problem of completely pushing constraints in Datalog programs. Experiments in [Levy, 1993] show that significant speedups (often orders of magnitude) are obtained by employing the query-tree in inference. The query-tree has provided a basis for a novel algorithm, *predicate move-around* [Levy *et al.*, 1994a], for optimizing SQL queries over relational databases. The predicate move-around algorithm is currently being incorporated into several large scale commercial database systems.

**Automatic Creation of Abstractions:** Often a representation is too complex for a given query because it contains too many distinctions in its domain. Therefore it may be worthwhile to first *abstract* a representation by removing some irrelevant distinctions (e.g. arguments of relations, distinctions between predicates) and then answer the query. In [Levy, 1994] we consider the problem of automatically abstracting a representation by removing distinctions that are irrelevant to a set of queries. This is done by considering other kinds of relevance subjects and devising algorithms for automatically deriving appropriate irrelevance claims. An application of this technique to the problem of automatically choosing models of physical devices is described in [Iwasaki and Levy, 1994].

**Information Gathering in Distributed Heterogeneous Environments:** An important application of future knowledge base systems will be to integrate and provide high level querying facilities for a large number heterogeneous sources (such as databases,

knowledge bases, text files or application programs). Such global information systems are rapidly becoming available to large populations of users via international networks. In order for such systems to answer queries effectively, they must be able to efficiently decide which information sources contain knowledge that is relevant to a given query. In [Levy *et al.*, 1994b] we describe how our framework and algorithms can be applied and extended to such a setting.

## References

- Blakeley, J. A.; Coburn, N.; and Larson, P. A. 1989. Updating derived relations: detecting irrelevant and autonomously computable updates. *Transactions of Database Systems* 14(3):369–400.
- Carnap, R. 1950. *Logical Foundations of Probability*. University of Chicago Press, Chicago.
- Elkan, Charles 1990. Independence of logic database queries and updates. In *Proceedings PODS-90*. 154–160.
- Gärdenfors, Peter 1978. On the logic of relevance. *Sythese* 37:351–367.
- Greiner, Russell and Jurišica, Igor 1992. A statistical approach to solving the EBL utility problem. In *Proceedings of AAAI-92*.
- Iwasaki, Yumi and Levy, Alon Y. 1994. Automated model selection for simulation. In *Proceedings of AAAI-94*.
- Keynes, J. M. 1921. *A Treatise on Probability*. Macmillan, London.
- Levy, Alon Y. and Sagiv, Yehoshua 1992. Constraints and redundancy in Datalog. In *The Proceedings of the Eleventh ACM SIGACT-SIGMOD-SIGART Symposium on Principles of Database Systems (PODS)*. 67–80.
- Levy, Alon Y. and Sagiv, Yehoshua 1993. Queries independent of updates. In *Proceedings of the 19th VLDB Conference, Dublin, Ireland*. 171–181.
- Levy, Alon Y.; Mumick, Inderpal Singh; Sagiv, Yehoshua; and Shmueli, Oded 1993. Equivalence, query-reachability and satisfiability in datalog extensions. In *Proceedings of PODS-93*.
- Levy, Alon Y.; Mumick, Inderpal Singh; and Sagiv, Yehoshua 1994a. Query optimization by predicate move-around. In *Proceedings VLDB-94, Santiago, Chile*.
- Levy, Alon Y.; Sagiv, Yehoshua; and Srivastava, Divesh 1994b. Towards efficient information gathering agents. In *Working Notes of the AAAI Spring Symposium on Software Agents, March, 1994, Stanford, California*. 64–70.
- Levy, Alon Y. 1993. *Irrelevance Reasoning in Knowledge Based Systems*. Ph.D. Dissertation, Stanford University, Stanford, CA.
- Levy, Alon Y. 1994. Creating abstractions using relevance reasoning. In *Proceedings of AAAI-94*.
- Minton, Steve 1988. Quantitative results concerning the utility of explanation based learning. In *Proceedings of the Seventh National Conference on Artificial Intelligence*.
- Srivastava, Divesh and Ramakrishnan, Raghu 1992. Pushing constraint selections. In *Proceedings of the PODS92*.
- Subramanian, Devika 1989. *A Theory of Justified Reformulations*. Ph.D. Dissertation, Stanford University, Stanford, CA.