

## HOW TO USE ARISTOTELIAN LOGIC TO FORMALIZE REASONINGS EXPRESSED IN ORDINARY LANGUAGE

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### Abstract

The most efficient, accurate, and fruitful way to communicate reasonings is in natural languages. The proper way to approach the question of how human reasoning is expressed and communicated in natural languages is *not* via typical formal systems or analogous artificial languages, but with Aristotle's syllogistic. The rules of quality and distribution are sound and complete methods for filtering the 24 correct reasonings from the 256 logical possibilities (existential import adopted). The syllogism is fundamental reasoning-wise via complex and complicated reasonings being broken down into syllogistic steps (as polysyllogisms, sorites, and enthymemes illustrate). A 5-quantity syllogistic gives the basic logic of "few", "many", and "most". Syllogistic systems for as many intermediate quantities as you like can be constructed. The infinite-quantity (iQ) syllogistic is modeled on the finite higher-quantity fractional systems. To add relations to syllogistic systems, the *Dictum di Omni* (DDO) is reformulated first as DDO<sup>P</sup> and then DDO<sup>I</sup> is developed to cover iQ syllogisms. Finally, DDO\* results from extending DDO<sup>I</sup> to arguments wherein one or more iQ-categoricals is replaced by a (simple or complex) "basic relational categorical" (BRC). Challenges for further research on DDO\* include iterations, embedded terms due to n-place relations (n>2), VP-modifiers, and other clausal NPs.

\* \* \*

The most efficient, accurate, and fruitful way to communicate reasonings is in natural languages. Use of formal systems, mathematical notations (in which all substantive advanced science appears to be expressed), new artificial languages, projected post-spaceage "information-highway" networks, etc. are not improvements over natural language communication. My aim herein is *not* to argue for or defend this thesis of natural language communicational superiority. Rather, I will just introduce one way to understand it further -- a way which also contributes to showing how "natural language can be viewed as a Knowledge Representation system with its own representation and inferential machinery".

The proper way to approach the question of how human reasoning is expressed and communicated in natural languages is *not* via contemporary applications of formal systems (first and/or higher order predicate calculi) or any analogous artificial language (from APL to PASCAL), but rather with **Aristotle's** syllogistic approach to logic. [And knowledge representation systems will significantly profit from adopting the Aristotelian

approach. For example, it will illuminate and expand the complexity of denial and negation of expressible inferences as introduced by Iwanska (1993). Iwanska's eleven targets for logical negation (in eleven syntactic categories) will find a deeper explanation through the Aristotelian approach (though showing this is not part of my aims herein.)] For the Aristotelian concept of logical forms expressible in ordinary language statements (and strings of them in reasonings) is **directly** relatable to the surface grammatical forms found in many natural languages. It is easy to see that many *English* forms are completely amenable to Aristotelian analysis (as shown below). But remember that the syllogistic was not designed that way, but for Greek (and, through later developments, Latin). I **conjecture** (again, not a topic for explanation or defense herein) that Aristotelian analysis is as easy in (for and about) any natural language as it is in English, Greek, or Latin (or in French, German, or Spanish).

The basic syllogistic system is a method for analyzing the correctness of certain simple reasonings ("arguments") in which exactly three so-called "terms" are crucial, where each term is easily associatable with certain NPs and VPs in surface sentential structure. The idea is that the simplest kind of "move" of reasoning is to *conclude* a connection between two terms via two separate propositions -- where the connection can be one of four types: all-are; none-are; some-are; and some-are-not. The kind of reasoning considered by Aristotle is the forging of a link between two terms (concepts, kinds, classifications, or categorizations of things) by finding some one thing each term can be separately linked to (the third, *middle* term). The form of the links are the again the four styles just mentioned for the conclusion: all-are; none-are; some-are; and some-are-not. One natural way to *represent* this forging of a link (that type of reasoning) is the traditional formal syllogism wherein there are two "premises" (each being one of the propositions linking one of the conclusion terms to the crucial middle term) and the aimed-for (single) conclusion -- as follows:

Premise 1	{ All }	{ P are (not) M }
	{ Some }	{ M are (not) P }
Premise 2	{ All }	{ S are (not) M }
	{ Some }	{ M are (not) S }
Conclusion	{ All }	S are (not) P
	{ Some }	

"Every" and "each" can be substituted for "all", with correlated adjustments of noun-verb number -- when the term (S, P, or M position) that "all" is modifying is a *count*-term (i.e., expressible with a count noun). Also, sometimes nothing at all can be used for either "all" or "some"; e.g., "Whales are mammals" (i.e., all are) and "Elephants aggravate farmers" (i.e., some do). (Also, I simplify tradition by substituting throughout "All-not" forms for "No" forms; cf. Peterson 1988.)

The Aristotelian rules of quality and distribution (rules of quantity being superfluous) are sound and complete methods for filtering out the 24 correct reasonings (valid syllogisms) from the 256 logical possibilities of Aristotelian syllogistic reasonings. Existential import of each term is adopted. That is, a presupposition of expressing a purely syllogistic reasoning is that some instance (at least one) of each of the terms (e.g., some singers for the term "singers") exist, where the presupposition relation is not a logical relation between propositions, but rather a 3-place relation between presupposer, proposition affirmed (or denied) and fact presupposed. Thus, when a presupposition of an assertion (or series of such in an expressed reasoning) is false, no proposition at all is

asserted or expressed -- often contrary to obvious appearances. An interesting detail is that some mass terms (the concrete ones, expressed via non-count nouns in English) also must have "instances" whose nature was not clear until H. Cartwright (1970) explained how they are "quantities" (in her sense of "quantities").

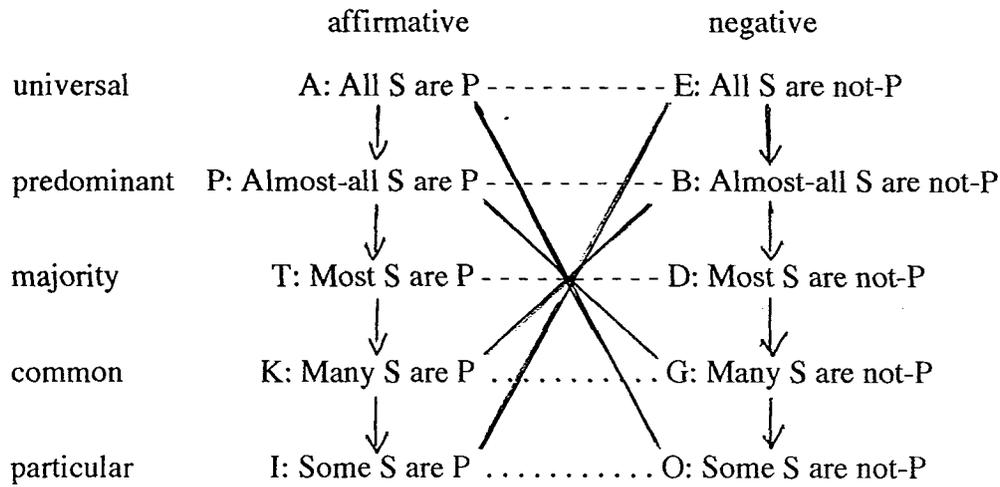
Now it has been often been concluded, and is still widely believed, that the narrow "straitjacket" of forms that the syllogistic system provides is **not** of much utility, practically or theoretically. Most expressed reasoning are just not of that form, and since so few forms (of a fairly small number of candidates, only 256) are valid, it seems very unlikely that the syllogistic methods can be of much use in describing and explaining all the reasonings expressible in natural languages. For one thing, as was long ago urged, syllogisms apparently can't even begin to handle relations -- which predicate calculus (formal system) methods excel at. I disagree -- though I admit that before now not much has been done to show the utility of syllogistic methods for relations (with the almost lone exception of Sommers 1982). (Another reason for the low expectations of syllogistic methods was the *philosophical* confusions about the nature of relations themselves. For a recent example of the continuing philosophical discussion of relations, see Peterson 1990.)

The way in which the syllogism could actually be considered fundamental reasoning-wise is not often remembered; viz., that complex and complicated reasonings (about any subject matter whatever) can be broken down into syllogistic steps -- each of which could be a valid syllogism. Two models illustrate what is at issue here -- the "polysyllogism" (wherein several intermediate premises are produced via applying the syllogistic rules to appropriate premises of any many-premised argument and the final step produces a valid conclusion) -- polysyllogisms with the intermediate conclusions unexpressed being "sorites" -- and the "enthymeme" (where new premises needed to produce a valid syllogism must be hypothesized and added to given ones). Of course, there ought to be many other manners of extending the basic syllogistic system for analyzing complicated reasonings (in addition to polysyllogisms and enthymemes), IF Aristotle is right -- that the syllogism is the foundation of human logical reasoning.

Recent developments (Sommers, Englebretsen) have shown that the basic Aristotelian methods **can** be extended to relations. However, before presenting my own variation on how to so extend the syllogism, I will outline an extension of syllogistic methods which I have achieved in recent years -- the extension of the 2-quantity (i.e., traditional universal and particular) system of Aristotle, first, to three more "intermediate" quantities, then, to finite numbers of additional intermediate quantities and, finally, to an *infinite* number of quantities.

The 5-quantity syllogistic -- the basic logic of "few", "many", and "most" (those three quantities added to universal and particular quantity) -- can be summarized in the following squares of oppositions and patterns of valid forms (basic categoricals appear in the squares, where (i) universal negatives are reduced to universals with "internal" negation, (ii) dashes connect contraries, dots connect sub-contraries, and (iii) "Few X are Y" =df. "Almost-all X are not-Y"):

(1) 5-Quantity Squares



(2) 5-Quantity Valid Syllogisms (2Q Traditional Valids Boldfaced)

**AAA**  
 AAP APP  
 AAT APT ATT  
 AAK APK ATK AKK  
**AAI** API ATI AKI **AI**  
**EAE**  
 EAB EPB  
 EAD EPD ETD  
 EAG EPG ETG EKG  
**EAO** EPO ETO EKO **EIO**

Figure 1

**AEE**  
 AEB ABB  
 AED ABD ADD  
 AEG ABG ADB AGG  
**AEO** ABO ADO AGO **AOO**  
**EAE**  
 EAB EPB  
 EAD EPD ETD  
 EAG EPG ETG EKG  
**EAO** EPO ETO EKO **EIO**

Figure 2

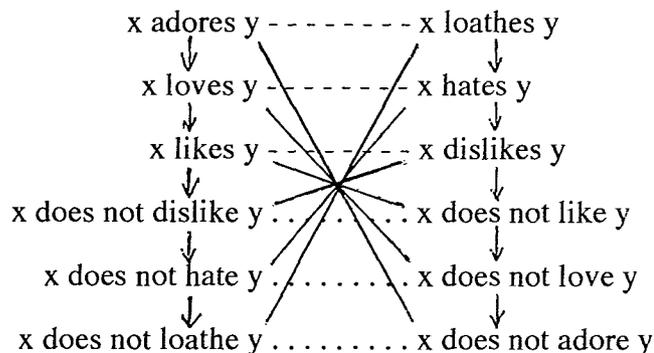
<b>AAI</b>	PAI	TAI	KAI	<b>IAI</b>			
API	PPI	TPI	KPI				
ATI	PTI	TTI			<b>AAI</b>	<b>AAE</b>	<b>EAO</b>
AKI	PKI				PAI	AEB	EPO
<b>AII</b>					TAI	AED	ETO
<b>EAO</b>	BAO	DAO	GAO	<b>OAO</b>	KAI	AEG	EKO
EPO	BPO	DPO	GPO		<b>IAI</b>	<b>AEO</b>	<b>EIO</b>
ETO	BTO	DTO					
EKO	BKO						
<b>EIO</b>							

Figure 4

Figure 3

The valid forms "shadowed" in Figure 3 (embedded arrays) are the logically interesting forms, the rest being *routinely* "intermediate". For full explanation of the 5-quantity syllogistic, see Peterson 1979, Peterson & Carnes ms., Peterson & Carnes 1983, and Peterson 1988.

Basic to the Aristotelian approach is the breaking down of negation and denial into two fundamental types -- that labeled "contradictory" and that labeled "contrary" (and "sub-contrary") on the traditional square of opposition. That distinction is repeatedly exploited by all the developments presented herein (and in related research). The fruitfulness of it can be further demonstrated by noticing how it applies to Iwanska's "qualitative scales" (1993, p. 487). For example, consider the love-hate scale reconceived Aristotelianly as follows:



(where, as before, straight lines connect contradictories, dashes connect contraries, dots connect subcontraries, and arrows represent entailments). Making use of such negating characteristics for this kind of qualitative scale will, of course, depend on developing an Aristotelian account of relations (since the love-hate scale is relational), which is my

main topic below. Adjective and adverb modifiers succumb similarly. For example (in sum),

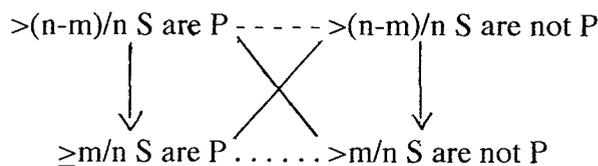
- x is a tall man —\*> x is not a short (anti-tall) man
- x ran slowly —\*> x didn't run fast (anti-slow)
- x is a very tall man —\*> x is not a barely (anti-very) tall man

(where "—\*>" =df. "entails but is not entailed by" and "anti-Ø" is one disambiguation of "not-Ø"). Cf. Peterson 1991 for typical complications due to embeddings.

Syllogistic systems just like the 5-quantity system for as "high" a number of intermediate quantities as you like can be constructed; cf. Peterson 1985. Also, iterations of such quantifiers -- such as expressed by "almost-all of 3/4" and "most of 1/2" -- can be accommodated; cf. Peterson 1991. And some rivals turn out to be unsound and incomplete; cf. Carnes & Peterson 1991.

The **infinite-quantity** (iQ) syllogistic (Peterson 1995a, 1995b; also see Johnson 1994) is modeled on the traditional 2-quantity and the higher-quantity fractional systems. The intermediate quantities in iQ are **rational fractions** (<1) that are **intermediate proportions** between universal and particular. Categorical form is "Q S are(not) P", where "Q" = "≥1" ("every"), ">m/n" (integers n≥m), "≥m/n", or ">0" ("some").

(3) iQ square of opposition (schema for n≥2m):



For "most", "few", and "many", cf. Peterson 1979, 1985. When 2m>n, the schema flips, so that "≥n/n S are P" ("All S are P") is contrary (i.e., ---) to "≥n/n S are not-P" ("All S are not-P").

An argument form of iQ is valid if filtered by the following rules -- extensions of the classical ones which I produced (1995a) by revising (only slightly) R. Carnes' extensions of Aristotelean rules in Peterson & Carnes ms., and 1983.

(4) Rules for iQ Syllogistic

- Distribution ... Rule 1: Sum of *distribution indices* (DIs) of middle terms is >1;**
- Rule 2: No DI of a conclusion term > DI of same term in premises;**

- Quality ..... Rule 3: At least one premise is affirmative;**
- Rule 4: Conclusion is negative if and only if one premise is;**

where DI(T) = explicit quantity of subject T; DI(T)= ≥1 for predicate Ts of negatives, DI(T)= >0 for predicate Ts of affirmatives; and DI(T<sub>i</sub>) + DI(T<sub>j</sub>)=(G<sub>i</sub> +G<sub>j</sub>)(m<sub>i</sub>/n<sub>i</sub> + m<sub>j</sub>/n<sub>j</sub>) for "G" = ">" or "≥" (when (>+>)=>, (>+≥)=>, and (≥+≥)=≥) and G<sub>i</sub>m/n > G<sub>j</sub>u/v iff m/n > u/v or G<sub>i</sub> is > and G<sub>j</sub> is ≥ when m/n = u/v).

Now to **relations**. The challenge today of extending Aristotle's methods (thus revealing how it can be the foundation for explaining all logically correct reasonings) is **not** just supplying suitable methods for formulating relational propositions for syllogistic (or syllogistic-like) analysis in reasonings in which they occur, but rather doing that for an extension of syllogistic to all possible intermediate quantities. In sum, we want to be able to easily represent reasonings containing all possible kinds of quantifiers. A very good start at this is achieved in the iQ syllogistic -- a single syllogistic system with an infinite number of intermediate quantities (the rational fractions). (And further additions to quantifier-expressions are also addressed in Peterson 1991 and 1995a.) So, I propose that an ambitious attempt at adding relations to syllogistic systems will add them not to the traditional 2-quantity system (or to any other finite-quantity system), but to the iQ system.

The way to begin is with the traditional "*Dictum de Omni*" (DDO). Consider a case like the traditional Darii (AII-1)

	<u>syllogism</u>	<u>DDO description</u>
(P1)	All M are P . . . . .	"is P" is said of <u>every</u> M
(P2)	Some S are M . . . . .	certain Ss are Ms
so,	Some S are P . . . . .	so "is P" can be said of those Ss

The idea is that what "is said" of each one of some kind -- of the Ms in premise 1 (P1) -- can be applied to any particular cases -- the Ss that are Ms in P2. Here is my own re-interpretation (or reworking) of DDO, utilizing classical distribution (D), where "being-distributed" =df. "+D", and "not-being-distributed" = df. "-D". (For a new explication of distribution, see Peterson 1995a where it is identified with quantificational *proportion*.)

(5) **DDO<sup>P</sup>** . . . . . an interpretation of DDO:

	+D	PRED	
(Darii-1)	All M	[are P] . . . . .	the <b>source</b> premise (i.e., M is +D)
		-D	
	Some S are M . . . . .		the <b>target</b> premise (i.e., M is -D)

To generate the valid conclusion, carry out these 3 steps:

1. **SUB**stitute PRED of the **source** for M of the **target** (that is -D),  
which => Some S are [are P]
2. **GRAM**matitalize the result of **SUB**,  
which => Some S are T [wh are P]
3. **RED**uce the result of **GRAM**,  
which => Some S are P

where: (i) *Q-order of source predicate is preserved* through SUB; and  
(ii) If both premises have Ms that are +D, replace one with something it entails that contains M which is -D.

#### VALIDITY

- V1. Any form generated by DDO<sup>P</sup> is valid.
- V2. Any form Y is valid, if it is exactly like a valid form X except that Y's conclusion is entailed by (or distinct but equivalent to) X's conclusion.
- V3. Any form Y is valid, if it is exactly like a valid form X except that one premise of Y only entails (or is distinct but equivalent to) one of X's premises.





(8) BRC Forms

Simple:  $Q_i T_i$  (are)  $\underline{R}$  (to)  $Q_j T_j$  e.g., Every boy loves >3/4 of the girls;

Complex (n-level, here n=2):

$Q_i T_i$  [wh (are)  $\underline{R}$  (to)  $Q_k T_k$ ] (are)  $\underline{R}$  (to)  $Q_j T_j$  [wh  $Q_l T_l$  (are)  $\underline{R}$  (to)]  
 e.g., **Every singer** who loves some poet hates **every musician**  
some critic knows;

BRC forms are generated by the following PS-grammar:

(9) BRC Grammar

$S \rightarrow NP VP$                        $VP \rightarrow V NP (PP)$   
 $NP \rightarrow QT$ ;                          $VP \rightarrow NP V (PP)$   
 $T \rightarrow (T) S^*$ ;                       $V \rightarrow V NP$  (for n-place R, n>2)  
 $S^* \rightarrow wh VP$ ;                       $PP \rightarrow PrepNP$  (ignored herein)  
 $T \rightarrow S, P, M, \text{singer, poet, girl, picture, snow, etc.};$   
 $V \rightarrow R, R', R'', \text{is, loves, hates, saw, touched, heard, found, etc.}$

The terms in syllogisms containing BRC forms are **atomic** (the simple Ts of G(BRC)) and **molecular** -- e.g., forms like (T)(wh every T is R to), and VPs. Note that relations themselves ("R"s above) are not terms even though they relate terms and play various roles in molecular terms.

Necessary adjustments to turn DDO<sup>I</sup> into DDO\* are revealed in the following examples:

(10) DDO\* EXAMPLES

	+D	PRED	
Hamiltoned Darii:	Every M	[is some P]	source
("=", a relation)		-D	
	Some S	is some M	target
	-----		
SUB=>	Some S	is [is some P]	
GRAM=>	Some S	is T[wh is some P]	
RED=>	<b>so,</b>	Some S is (=) some P	

Same form with "drew" and "saw" substituted for "=":

	+D	PRED	
	Every M	[drew some P]	..... source
		-D	
	Some S	saw some M	..... target
	-----		
SUB=>	Some S	saw some [drew some P]	
GRAM=>	Some S	saw some T[drew some P]	
RED=>	<b>so,</b>	Some S saw some -one/-M	who drew some P

(11) **Q-order** example:

Some girl loves every boy. So, Every boy, some girl loves

as in "(SOME-Gx)[(EACH-By) x LOVES y]" entails  
but is not entailed by "(EACH-By)[(SOME-Gx) x loves y]"

An enthymeme

	-D . . . . .	a VP M-term
	Some G [each B loves] . . . . .	target
PRED	+D . . . . .	VP M-term
[Each B loves]	each(one) [each B loves] . . . . .	source

SUB=> [Each B loves] (Some G---) (PRED is first!)

GRAM=> T[each B loves], some girl (is)

RED=> Each B loves some girl

(12) "**Heads of Horses**" Syllogisms:

Easy case . . . . . All horses are animals. So, all heads of horses are heads of animals.

i.e., an enthymeme:

	+D	PRED
	All horses [are animals] . . . . .	<i>source</i>
		-D
	All T[wh is-head-of some horse] is-head-of some horse. . . . .	<i>target</i>
		(suppressed)

SUB => All T[wh is-head-of some horse] is-head-of some [are animals]

GRAM => All T[wh is-head-of some horse] is-head-of some T[wh are animals]

RED => so, All heads of horses are heads of some animals

Harder case . . . Almost-all horses eat some apples.

So, all heads of almost-all horses are heads of something  
that eat apples [some apple eaters]

Enthymeme again:

	+D?	PRED
	Almost-all H [eat some apples] . . . . .	<i>source</i>
		+D?
	All heads of almost-all H are heads of almost-all H . . . . .	<i>target</i>

Like Fig. 3, first adjust second premise:

=> All heads of almost-all H are heads of *some H*

then:

SUB=> All heads of almost-all H are heads of some [eat some apples]

GRAM=> All heads of almost-all H are heads of some T[wh eats some apples]

RED=> so, All heads of almost-all H are heads of something which eats apples  
[some apple eaters]

The challenge of applying DDO\* to all BRCs requires specifying  $\pm D$  (i.e., distributedness, D-value) for all varieties of Ts (terms) -- simple and molecular, at any level of embedding. For potential targets and sources for DDO\* are M-terms (simple and

molecular); i.e., targets contain Ms that are -D and sources Ms that are +D. For an *embedded* term T, its D-value is a function of the Q which immediately precedes it (commands it, grammatically) -- for the most part. The question concerns structures like:

(QQ)  $Q_i T_i [wh + R + Q_j T_j]$  ...e.g., every M which some P chased  
 $Q_i T_i [wh + Q_j T_j + R]$  ...e.g., some M who swallowed each S

For non-embedded T (and/or for T *initially* in some BRC structure),

(DV-1) If QT is "every( $\geq 1$ ) T", D-value of T is +D  
 If QT is "some( $> 0$ ) T", D-value of T is -D  
 If QT is "Gm/n T" ("G"=">" or " $\geq$ ",  $n > m$ ), D-value of T is -D  
 (DV-1 derives entirely from above material)

Generally, the nature of the "higher" T -- i.e.,  $T_i$  with respect to  $T_j$  in (QQ) -- does not over-ride the D-value for the "lower" term,  $T_j$ . So, with respect to  $T_i$  and  $T_j$  of (QQ),

(DV-2) (a) if  $T_i$  is -D and  $T_j$  is +D, then  $T_j$  remains +D (still a source),  
 (b) if  $T_i$  is -D and  $T_j$  is -D, then  $T_j$  remains -D, and  
 (c) if  $T_i$  is +D and  $T_j$  is -D, then  $T_j$  remains -D (-D = targets).

*However,*

(d) if  $T_i$  is +D and  $T_j$  is +D, then  $T_j$  **changes** to -D (even though  $Q_j$  remains  $\geq 1$  ("each"!))

Here are some relational syllogisms illustrating (a), (b), and (c):

- (a) Some G who loves each B is F  
     Each M is B  
 so, [Some G who loves--is F][Each M--]  
 so, Some G who loves each M is F
- (b) Some G who loves some B is H  
     All B are M  
 so, Some G who loves some M is H
- (c) Each G who loves some B is H  
     Each B is M  
 so, Each G who loves some M is H

To illustrate (d), note that (d1) is *invalid* (so, "B" in P1 is no source), and (d2) is *valid* (so, "B" in P1, even with "each", is a target):

- (d1) Each G who loves each B is H (so, "B" no source)  
     Some B is M  
     Each G who loves some M is H                      INVALID
- (d2) Each G who loves each B is H (so, "B" is a target)  
     Each B is M  
     Each G who loves each M is H                      VALID

Interestingly, the (d) alternative --  $T_i$  and  $T_j$  each being +D -- also affects  $T_i$  (the higher T). (d3) is invalid, as is (d4).

(d3)            Each G who loves each B is H  
                   Some G is P  
                   Each lover of each B who is H is [some] P            INVALID

(d4)            Each G who loves each B is H  
                   Each G is P  
                   Each P who loves each B is H                            INVALID

So, "G" in (d4) is not changed to being -D (and so a target), as it was in (d2). Further, if the first quantifier of (d4)'s conclusion is replaced with "some", the inference is still invalid. And since (d5) is *valid*, P1 of (d4) must, evidently, *not entail* P1 of (d5):

(d5)            Some G who loves each B is H  
                   Each G is P  
                   Some P who loves each B is H                            VALID

But it doesn't entail it -- not even with existential import on "G".

The present challenges for further research on DDO\* include the following:

- A. Do iterations (repeated embeddings) satisfy these apparent restrictions -- and/or raise no new difficulties -- for DDO\* (concerning candidates for sources and targets)?
- B. Do **IQs**, and/or **iQ** syllogistic forms, in BRC "syllogisms" **disconfirm** DDO\* as so far described?
- C. Do embedded terms due to n-place relations,  $n > 2$  (via PSG of (9) above) raise any new problems for applying DDO\*?
- D. Do VP-modifiers (presumably adverbial phrases, not included in PSG of (9) above, introducing further clauses with atomic and molecular terms) succumb to these techniques (refinements of DDO\*)? (E.g., "Each athlete ran slower than some race horse trotted.")
- E. Do clausal NPs (not included in PSG of (9) above) succumb to these techniques? (E.g., "The man who saw Mary realized that all the bottles were empty").

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