

Reduced Model Formation for 2D Vortex Interactions Using Machine Learning: Extended Abstract

Haym Hirsh and Tom Ellman and Arunava Banerjee
Computer Science, Rutgers University, Piscataway, NJ 08855

David Dritschel

Applied Mathematics and Theoretical Physics, University of Cambridge, England CB3 9EW

Hongbing Yao and Norm Zabusky

Mechanical and Aerospace Engineering, Rutgers University, Piscataway, New Jersey 08855

Turbulence phenomena have been the focus of considerable recent research in computational fluid dynamics. Computational scientists can, in principle, gain an understanding of turbulence by conducting numerical simulations of the Navier-Stokes equations that govern the dynamics of fluids. Unfortunately, simulation of the full Navier-Stokes equations is too computationally expensive to be feasible in most situations.

Fluid dynamicists typically deal with this problem by developing reduced models of fluids. By “reduced model” we mean a mathematical model of the phenomenon of interest that can be executed by computer in a tractable fashion to yield accurate predictions about the phenomenon. Although the predictions should be accurate, they need not be exact. In general, there will be a range of reduced models, with faster and less accurate models at one end of the spectrum, and slower, more accurate models at the other.

For example, one type of reduced model represents a fluid as a two-dimensional object. In this model fluids are represented in terms of two-dimensional vortices moving and interacting on the surface of a sphere (Dritschel 1993b). As vortices interact with each other, they may undergo substantial changes in shape, split into multiple new vortices, or merge with each other. Although this two-dimensional model is much faster than a full three-dimensional simulation, execution of this model is still computationally expensive.

Our work is aimed at developing tools for automatic or semi-automatic formulation of new reduced models of 2D vortex interactions. In particular, we are aiming for models that strike a balance between the goals of high accuracy and low computational expense. In our context, we measure accuracy according to how well the reduced model predicts the *global* properties of the interacting vortices, such as the total number and mean size of vortices at each point in time, rather than the detailed time-dependent properties of each individual vortex (Carnevale *et al.* 1992).

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For the problem of 2D vortex interactions we begin with two pre-existing models. The *elliptical* or *moment* model (Legras & Dritschel 1991; Melander, Zabusky, & Styczek 1986) approximates regions of vorticity with ellipses, and simulates the movement of vortices by the movement of these ellipses over time (Figure 1). Although tractable, this model is only accurate when vortices are far from one another. When vortices draw near, their shapes and sizes change, and in such cases the elliptical model breaks down and a *contour dynamics/contour surgery* model (Dritschel 1989; 1993a; Zabusky, Hughes, & Roberts 1979) becomes necessary. Contour dynamics represents a vortex by a collection of points along its perimeter, and simulates the movement of vortices by the movement of these points over time (Figure 2). Contour surgery adds and removes points representing the perimeter of a vortex as the vortex’s shape and size changes over time, as well as when vortices split and merge. The contour dynamics/contour surgery model is much more accurate than the elliptical model, but is substantially slower to use.

This paper describes our on-going work exploring two methods for developing reduced models of 2D vortex interactions. These new models fall between the elliptical and contour-dynamics models in terms of tractability and accuracy. Both use the elliptical model until its results would become suspect. The *patched elliptical model* combines the elliptical model with a rule-based simulation system. One set of rules is used to identify situations in which the elliptical model has broken down. Another set of rules is used to correct the elliptical model when vortices split or merge. In combination, the two rule sets are used to simulate the system until reaching a state in which the elliptical model may be resumed. The *hybrid elliptical/contour dynamics model* also uses rules to identify situations in which the elliptical model has become invalid. After identifying a breakdown in the elliptical model, it initiates a simulation using the contour dynamics model. When the resulting vortices return to shapes that are once again well modeled by ellipses, the hybrid model returns to using the elliptical model (switching back

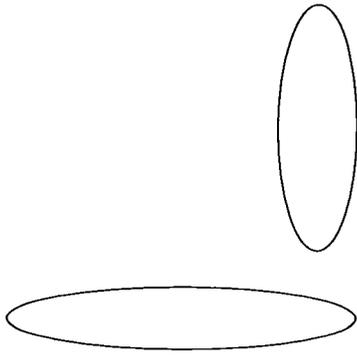


Figure 1: Elliptical Model: A vortex is represented by an ellipse with some area, eccentricity, and orientation.

to contour dynamics whenever necessary). Both the patched elliptical model and the hybrid model are being created by using inductive learning techniques to synthesize the required rule sets.

The Patched Model

The idea of the patched elliptical model is to use the elliptical model, augmented with rules for (1) recognizing when this model breaks down due to vortex interactions and (2) specifying how the resulting incorrect state of the simulation should be updated to reflect the correct outcome. These rules are formed through the use of inductive learning (Quinlan 1993) on data generated from a more accurate 2D contour-dynamics code from which it is possible to identify when objects interact and what the resulting state should be.

Our first set of experiments was aimed at learning rules for predicting when a vortex will split. We used the contour-dynamics model to generate examples of splitting and non-splitting vortices. In particular, we ran the contour-dynamics model until a vortex split occurs. We then fit an ellipse to the description of the vortex as it appeared just before the split occurred. Finally, we described the ellipse in terms of a feature vector suitable for use in an inductive learning system, such as C4.5, treating the vector as a positive example of the concept “vortex-split”. Negative data where there are no splits are generated in a comparable fashion.

Our initial results were disappointing, with our learned rules doing poorly at predicting vortex splits. Subsequent study of our data have pointed to two issues that gave rise to this. The first is a question of suitable data generation. At any point in a simulation there will be many well-behaved vortices, and only occasionally do vortex splits occur. If we generate data from *all* vortices in a simulation, the training data will have a large preponderance of negative data, which is something that current inductive learning systems have difficulty handling. We are currently considering ways to generate data more selectively so that class

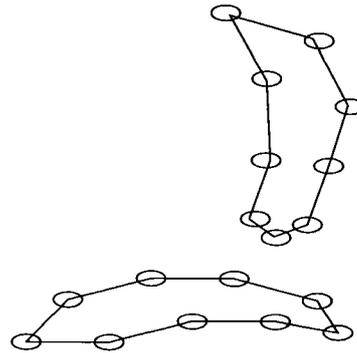


Figure 2: Contour Dynamics Model: A vortex is represented by a collection of points on its perimeter.

sizes will not be skewed and thus learning should be more successful.

The second issue is that the elliptical-model representation of vortices is a very coarse approximation to a vortex, and two very different vortices — one that would split and one that would not — can appear identical in the elliptical representation. Enough information must be represented in the description of a vortex to make it possible to recognize the differences between such cases. Towards this end we are in the process of expanding our representation of vortices in the anticipation that doing so will yield more accurate split-prediction rules. First, our representation will include information not only about the splitting vortex, but also about other vortices in its vicinity, as well as global properties of the system of vortices. Second, the history of a vortex plays an important role in its behavior. For example, whether a vortex will split can often be predicted by whether the vortex was the result of a recent merge of two vortices. We have therefore turned to the task of predicting whether two vortices merge, in the expectation that accuracy in this task will aid in the prediction of vortex splits.

The preceding work addresses the question of predicting when vortices interact, but not what the resulting “patched up” state should be. Addressing this second step awaits further progress on the preceding first step of our approach; at this point we anticipate using regression methods to form functions for each real-valued property that must be updated in the elliptical-model representation of the state.

The Hybrid Model

Our more recent work has also turned to the development of a different type of reduced model, one that switches between the two existing models when appropriate (Dritschel & Zabusky 1995). The hybrid elliptical/contour model is based on the idea of using the elliptical model whenever possible, while switching to the contour dynamics model whenever neces-

sary. In particular, this combined model uses the elliptical model until reaching a situation in which it breaks down. The contour model is then initialized by selecting a collection of points on the boundary each ellipse, and is run until the system returns to a state in which the elliptical model is valid again. The elliptical model is then initialized by fitting an ellipse to each vortex.

The main problem here is to identify when it is appropriate to switch between the models. An initial implementation of this model switches from the elliptical to the contour-dynamics model whenever the aspect ratio of a vortex becomes too large, as well as when certain properties of the underlying physics are observed; it switches back whenever the areas of all vortices in the contour-dynamics representation are closely matched by their elliptical approximations.

These switching rules are somewhat ad hoc and conservative. We plan to explore the use of machine learning to identify rules for recognizing appropriate points for switching, especially as a function of desired accuracy of results. If successful, this hybrid model would behave like the contour-dynamics model if high accuracy is requested, yet degenerate to the elliptical model if accuracy is of no concern.

Prospects for Scientific Discovery

Although this paper has focused on the formation of reduced models for 2D vortex interactions, our interests from an AI perspective are on understanding mechanisms for automated theory formation, viewing this domain as a testbed for our explorations. Reasoning about physical systems requires the use of a hierarchy of models reflecting a range of accuracy and efficiency. The ability to formulate and move between different models of a physical phenomenon will become central to many AI tasks that concern the physical world.

This paper has described on-going work, but we believe it reflects a number of important observations concerning attempts to automate scientific discovery. First, this work is not a rational reconstruction of some pre-existing scientific theory. Rather, it represents an attempt to build a new model where none currently exists. If we are successful, AI will have played a central role in the creation of a new scientific model. Moreover, success in this task will be of interest to fluid dynamicists. This was not a problem designed by AI researchers, but rather one defined by fluid dynamicists that is of independent interest to them.

Further, this work represents an active collaboration between AI researchers and fluid dynamicists, experts in the domain in which our efforts are taking place. Interactions with domain experts has been crucial to our ability to make any progress on this task. Indeed, if we are successful, the results will have been the product of both man and machine, with both playing important roles in the theory formation process. Although we would like to see the day where a computer is able to

discover new scientific models on its own, our efforts on this task has made it all too clear that a far more realistic, yet still very hard problem is to accomplish this task when AI efforts are combined with the efforts of domain experts.

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