Reified Logic for Representing First Order Temporal Constraints

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1 Introduction

The representation of, and reasoning about time play an important role in any intelligent activities. In this paper, we propose an ontology for representing quantitative first order temporal constraints. This logic that we propose in this paper uses instant and interval structures as primitives and has the expressive power of the popular temporal logics of Shoham's [9] and BTK's [4]. The advantage of our logic is that it uses the syntactic structures that explicitly implements the semantics of the temporal structures, such as, true throughout an interval (tt), or true at a point (at). Therefore, developing efficient inference rules and proof procedures for this logic is relatively easy.

This paper is organized as the following: In section 2, we introduce the representation of temporal knowledge. In section 3 we introduce our temporal constraint language providing its syntax and semantics. The paper is concluded with a summary and a discussion in section 4.

2 Representation of Temporal Knowledge

The popular approaches to represent time, within the areas of artificial intelligence, are *time points* and *intervals*. At early stages of the development of ontologies for time, points and intervals are treated as if they were mutually exclusive entities. Allen did not permit the presence of time points in his interval logic [1] even at the delimiting portions of an interval. He viewed the beginning and the ending of an interval as intervals of zero duration. The inability to accommodate points in the logic makes it difficult to handle some specific continuous events as pointed out by Galton [5]. On the other hand, many works on point-based temporal systems treat interval as if it consists of series of discrete points; such works implicitly assume an interval is discrete.

We strongly advocate the needs to have both points and intervals to represent and to reason about temporal knowledge. We follow Antony Galton's [6] approach to represent points and intervals. An interval is a continuous span of time with non zero duration and it is delimited by a pair of points namely the beginning and the ending points of the interval. Thus, an interval I is represented by $[I_b, I_e]$ where I_b and I_e are respectively the beginning and the ending points of the interval I. It is very tempting to define an instant in terms of an interval which has the same beginning and the ending point. For example, a point t may be defined as $[t, t]$. We reject this approach in favor of introducing point structure to represent a fluent at an instant.

Let us examine how points and intervals are related. In the literature it is viewed that an interval consists of points. This view forces one to view the interval as a discrete entity. We take a general approach in which time is linear and continuous. In our representation, one can assert that for any interval there exists a time point that subdivides the interval. That is, $\forall I_b \forall T_e\ I_b < I_e \wedge [I_b, I_e] \rightarrow \exists t\ \ t < I_e \wedge I_b < t \wedge [I_b, t] \wedge [t, I_e]$ which can be equivalently represented by $\exists t\ divide(t, I)$ stating that there is an instant t that subdivides the interval I.

2.1 Representation of Properties

For representing *properties of an object or a fluent* over an interval or at an instant, we introduce reified logic. We use the interval structure $tt([I_b, I_e], P)$ to represent that a fluent P is true throughout the interval delimited by the points I_b and I_e. When P is true throughout an interval, the truth value of P at the end points must be defined precisely. Suppose a pair of intervals I and J such that I meets J. That is, $[I_b, I_e]$ meets $[J_b, J_e]$ and hence, I_e and J_b coincides.

Further, assume that P is true throughout I and false throughout J.

If we assume that a fluent holds at both the beginning and the ending points whenever it holds throughout the interval, we will arrive at an absurd result that P and $\neg P$ are true at I_e. Therefore, we take a position that whenever a fluent holds throughout an interval *it will not hold at the end points unless it is specified explicitly.* In the absence of the explicit specification the fluent values at the end points are not known.

A fluent at an instant is represented using a point structure $at(< instant >, < fluent >)$. For example $at(t, P)$ is interpreted as P is true at the instant t.

The following axioms relates an interval structure and an instant structure.

(1) If P holds in an interval I, say $[I_b, I_e]$, then it must hold at an instant that subdivides I and throughout the sub intervals of I.

$$\forall I_b \forall I_e \forall t \; tt([I_b, I_e], P) \wedge I_b < t \wedge t < I_e \rightarrow tt([I_b, t], P) \wedge at(t, P) \wedge tt([t, I_e], P) \text{ or}$$
$$\forall I_b \forall I_e \forall t \; tt([I_b, I_e], P) \wedge divide(t, [I_b, I_e]) \rightarrow tt([I_b, t], P) \wedge at(t, P) \wedge tt([t, I_e], P) \text{ or}$$

(2) If P holds in an interval I, say $[I_b, I_e]$, then it must hold at all instants that divides the interval I

$$\forall I_b \forall I_e \forall t \; tt([I_b, I_e], P) \wedge I_b < t \wedge t < I_e \rightarrow at(t, P)$$

2.2 Negation of temporal Structure

Consider an instant structure, say, $\neg at(t, P)$. It is interpreted as saying 'it is not the case that P is true at t'. This is equivalent to saying 'it is the case that not P at t'. This is formally written as

$$\forall t \forall P \neg at(t, P) \leftrightarrow at(t, \neg P)$$

Consider the negation of an interval structure $\neg tt(I, P)$. There are three possible cases for it to be true:

1. $\neg P$ is true throughout I
2. there exists a subinterval of I in which $\neg P$ is true throughout the subinterval, or
3. there exist an instant that subdivides the interval and $\neg P$ is true at the instant..

This is formally stated as following:

$$\neg tt(I, P) \leftrightarrow \begin{array}{l} tt(I, \neg P) \vee \\ \exists I' \; I' \subset I \wedge tt(I', \neg P) \vee \\ \exists t \; divide(t, I) \wedge at(t, \neg P) \end{array}$$

An Example

Let us consider an example of a sequential circuit consisting of two inputs namely in_1 and in_2. The output (out) is at level 1 throughout the interval when in_1 and in_2 are at level 1 and the switch is in a closed position during the interval. The behavior of the circuit is represented using the interval structure tt as following:

$$tt([I_b, I_e], in_1(1)) \wedge tt([I_b, I_e], in_2(1)) \wedge$$
$$tt([I_b, I_e], switch\_status(on)) \rightarrow tt([I_b, I_e], out(1))$$

Further assume the following assertions

The input in_1 is at level 1 throughout the interval 1 to 5.

$tt([1, 5], in_1(1))$

The input in_2 is at level 1 throughout the interval 2 to 7.

$tt([2, 7], in_2(1))$

status of the switch is on throughout the interval 3 to 6.

$tt([3, 6], swith\_status(on))$

2.3 Need for Further Improvements in Representation

The expected behavior of the circuit for the set of given input signals is given as the following: the out1 is at level 1 in the interval $[3, 5]$. Unfortunately, we cannot obtain the expected result by applying deduction with the conventional unification.

3 Definition of the Language

Our approach for solving the problem is based on the concept of (1) separating temporal constraints from the axioms, (2) using new inference rules that incorporate the semantics of the time structure, and (3) generalizing the binary resolution rule that applies to the non-temporal terms. An assertion $tt([1, 5], in_1(1))$ states that in_1 is at level 1 throughout the interval $[1, 5]$. When we separate the temporal constraint, it can be written as $I \subseteq [1, 5] : tt(I, in_1(1))$ indicating that $in_1(1)$ is true for every interval I which is a proper subset of interval $[1, 5]$. Here we use a temporal variable I as a shorthand for $[I_b, I_e]$.

3.1 The Syntax

The sentences of the language consists of temporal constraint reified wffs which takes the form $\phi : \varphi$ where ϕ is the temporal constraint and φ is the reified wff. The temporal constraints consist of *time bounded constraint* only or time bounded constraint with *constraints on temporal terms* which we call *temporal term constraints (TTC).*

Let us introduce the notations which we will be using to describe the syntax of the language.

P_{props}	a set of primitive propositions
T_{ptsc}	a set of instants or points
T_{ptsv}	instant or point variables
T_{intc}	interval symbols
T_{intv}	interval variables

There are two types of temporal terms: points and interval. The *temporal terms of points type* is denoted by TTP and it is equal to $\{T_{ptsc} \cup T_{ptsv}\}$. Similarly the *temporal terms of the interval type* is denoted by TTI and is equal to $\{T_{intc} \cup T_{intv}\}$.

The *time bound constraints (TBC)* are defined as the followings:

1. If $j \in T_{intv}, t_i \in TTP$ and $t_j \in TTP$ then $j \subset [t_i, t_j]$ and $j \subseteq [t_i, t_j]$ are TBC.
2. If $j \in T_{intv}$ then $j \subseteq [\rhd\lhd]$ is a TBC where $[\rhd\lhd]$ defines an interval of finite duration with undefined end points.
3. If $j \in T_{intv}$ and $j' \in TTI$ then $j \subset j'$ $j \subseteq j'$ are TBC.
4. If $t_i \in T_{ptsv}$ then $t_i \in \{c_1, c_2, ..., c_k\}$ is a TBC where $c_i \in TTP$.

The *temporal term constraints (TTC)* are defined recursively

1. if $t_i \in TTP$ and $t_j \in TTP$ then $t_i\ R\ t_j$ is a TTC where R is one of $\{<, >, \geq, \leq, =\}$.
2. If $j \in TTI$ and $j' \in TTI$ then $j\ R\ j'$ is a TTC where R is one of thirteen relations defined between a pair of intervals.
3. If ϕ is a TTC then $\neg\phi$ is also a TTC.
4. If ϕ and ϕ' are TTC the $\phi \wedge \phi'$ and $\phi \vee \phi'$ are also TTC.

If ϕ' is a time bounded constraint and ϕ'' is a temporal term constraint then $\phi' \wedge \phi''$ is a *temporal constraint.* Any temporal constraint must have the time bounded constraint (ϕ') and an optional constraints on temporal terms (ϕ'').

A *temporal constraint wff* has the form $\phi : \varphi$ where ϕ is a temporal constraint and φ is a reified wiff and it is read as if ϕ is true then φ must be true.

A *reified wff* is recursively defined as the following:

1. If $p \in P_{props}, t_i \in \{T_{ptsc} \cup T_{ptsv}\}$ and $t_j \in \{T_{ptsc} \cup T_{ptsv}\}$ then $tt([t_i, t_j], p)$ is a reified wff.
2. If $p \in P_{props}, j \in \{T_{intc} \cup T_{intv}\}$ then $tt(j, p)\}$ is a reified wff.
3. If $p \in P_{props}, t_i \in \{T_{ptsc} \cup T_{ptsv}\}$ then $at(t_1, p)$ is a reified wff.
4. If φ and φ' are refied wffs so is $\varphi\ R\ \varphi'$. Where R is one of the logical connectives $\{\wedge, \vee, \leftarrow, \rightarrow\}$

3.2 Semantics

For defining the semantics of the logic, we follow the standard approaches as has been taken by Shoham [9], and Gensereth et al. [7].

An interpretation is a mapping between the elements of the language and the elements of the conceptualization. Let us introduce some notations using which we develop the semantics for propositional temporal constraint wffs.

T^c_{pts}	a set of instants or points of the conceptualization
T^c_{int}	a set of intervals of the conceptualization
Const	non temporal constants of the language
Ver	non temporal variables of the language
$\mathcal{D}$	universe of discourse
FC_{temp}	temporal function constants that take temporal terms and maps onto a temporal term.
FC_{nt}	non temporal function constants that take non temporal terms whose mapping are not dependent on time that is , they are not fluent.
FC_{flu}	non temporal function constants that take non temporal terms whose mappings are dependent on time, that is , they are fluent.

An interpretation which is denoted by $\mathcal{I}$ is a mapping from the elements of the language to the elements of the conceptualization or to the domain of discourse $\mathcal{D}$. If c is an element of the language then the interpretation $\mathcal{I}$ maps c onto the universe of discourse, that is, if $c \in Const$ then $\mathcal{I}(c) \in \mathcal{D}$. Instant symbols and the interval symbols of the language are also mapped onto the instants and the intervals of the conceptualization by the interpretation. That is, if $t_i \in T_{ptsc}$ then $\mathcal{I}(t_i) \in T^c_{pts}$ and $i \in T_{intc}$ then $\mathcal{I}(i) \in T^c_{int}$

A variable assignment $\mathcal{V}$ is a function from variables to an object of the universe of discourse $\mathcal{D}$. That is if $x \in Ver$ then $\mathcal{V}(x) \in \mathcal{D}$. Let the variables x,y and z of the language assigned to constants a,b and c of the conceptualization, then we have $\mathcal{V}(x) = a$, $\mathcal{V}(y) = b$ and $\mathcal{V}(z) = c$. Similarly, the point and interval variables are mapped onto points and the intervals of the conceptualization by variable assignments. If $t_i \in T_{ptsv}$ then $\mathcal{V}(t_i) \in T^c_{pts}$, similarly if $i \in T_{intv}$ then $\mathcal{V}(i) \in T^c_{int}$.

The interpretation $\mathcal{I}$ and the variable assignment $\mathcal{V}$ can be combined into a joint assignment $\mathcal{IV}$ that applied to terms in general and has the following properties

1. Variable assignment does not affect the interpretation of the constant terms (temporal or nontemporal). Taht is,
   (a) If $c \in Const$ then $\mathcal{IV}(c) = \mathcal{I}(c)$.
   (b) If $t_i \in \mathcal{T}_{ptsc}$ then $\mathcal{IV}(t_i) = \mathcal{I}(t_i)$.
   (c) If $i \in \mathcal{T}_{intc}$ then $\mathcal{IV}(i) = \mathcal{I}(i)$.
2. Variable assignments are not affected by the interpretation, that is,
   (a) If $x \in Ver$ then $\mathcal{IV}(x) = \mathcal{V}(x)$.
   (b) If $t_i \in \mathcal{T}_{ptsv}$ then $\mathcal{IV}(t_i) = \mathcal{V}(t_i)$.
   (c) If $i \in \mathcal{T}_{intv}$ then $\mathcal{IV}(i) = \mathcal{V}(i)$.
3. If $f \in FC_{nt}$ and F is a nontemporal function of the form $f(t_1, t_2, ..., t_n)$ where $t_i \in \{Const \cup Ver\}$ then $IV(F) = g(c_1, c_2, ..., c_n) \in \mathcal{D}$ where $\mathcal{I}(f) = g$ and $\mathcal{IV}(t_i) = c_i$ for $i = 1$ to n.

There are two kinds of functions: The first one is called temporal function that takes temporal terms and maps them onto temporal terms and the next one is called non temporal function which takes only non temporal term and maps them onto nontemporal terms. The mapping of the non temporal functions may or may not dependent on time. For example, consider a function *president_of(usa)* whose mapping depends on the time interval we are considering. Such functions are called *fluents*. To define the semantics of a fluent, we have to consider interpretation, variable assignments and the time period. We denote such a combined operator by $\mathcal{IVT}_i$ if the mapping is over an interval, or by $\mathcal{IVT}_p$ if the mapping is at an instance. For example $\mathcal{IVT}_i(president\_of(usa))[1987, 1988] =$ Mr. George Bush and $\mathcal{IVT}_p(president\_of(usa))[t] =$ Mr. George Bush where t is January 31st 1988. Let us define fluent formally.

Consider a k-ary fluent, say f^k such that $f \in FC_{flu}$.

(1) A fluent mapping over an interval (J)
$\mathcal{IVT}_i(f^k(t_1, t_2, ..., t_k))[J] =$
$\{D_i | \exists j' \subseteq \mathcal{IV}(J) \wedge g^k(t_1', t_2', ..., t_k')[j'] = D_i \wedge \neg(\exists j'' \subset j' \wedge g^k(t_1', t_2', ..., t_k')[j''] \neq D_i)$
where $\mathcal{I}(f) = g$ and $\mathcal{IV}(t_i) = t_i'$ for all i one to k.
(2) A fluent mapping at an instance (t_i)
$\mathcal{IVT}_p(f^k(t_1, t_2, ..., t_k))[t_i] =$
$g^k(t_1', t_2', ..., t_k')[\mathcal{IV}(t_i)] = D_i$
where $\mathcal{I}(f) = g$ and $\mathcal{IV}(t_i) = t_i'$ for all i one to k.

The composite mapping $\mathcal{IVT}$ has the following properties (here $\mathcal{T}$ takes the place of either $\mathcal{T}_i$ or $\mathcal{T}_p$).

1. The interpretation of the constant terms (temporal or nontemporal) does not depend on either the variable assignment or the time. That is,
   (a) If $c \in Const$ then $\mathcal{IVT}(c) = \mathcal{I}(c)$.
   (b) If $t_i \in \mathcal{T}_ptsc$ then $\mathcal{IVT}(t_i) = \mathcal{I}(t_i)$.
   (c) If $i \in \mathcal{T}_intc$ then $\mathcal{IVT}(i) = \mathcal{I}(i)$.
2. A Variable assignment does not depend on either the interpretation, or the time
   (a) If $x \in Ver$ then $\mathcal{IVT}(x) = \mathcal{V}(x)$.
   (b) If $t_i \in \mathcal{T}_{ptsv}$ then $\mathcal{IVT}(t_i) = \mathcal{V}(t_i)$.
   (c) If $i \in \mathcal{T}_{intv}$ then $\mathcal{IVT}(i) = \mathcal{V}(i)$.
3. Non fluent function value does not depend on time, that is, if $f \in FC_{nt}$ and F is a nontemporal function of the form $f(t_1, t_2, ..., t_n)$ where $t_i \in \{Const \cup Ver\}$ then $\mathcal{IVT}(F) = \mathcal{IV}(F) = g(c_1, c_2, ..., c_n) \in \mathcal{D}$ where $IV(t_i) = c_i$.

In a monotonic system each relation in the conceptualization is defined as a tuple. For example, if block A is on another block B and is represented as $on(A, B)$ then the tuple satisfying the relation is $< A, B >$. When we consider a dynamic system where relations among objects changes over time, a tuple satisfying a relation must be attached with an interval over which the tuple is holding. We use cartesian product to attach time with a tuple. For example, suppose block A is on block B during interval I_1 and block B is on A during another disjoint interval I_2 then the tuples satisfying the relation ON would be $\{< A, B > \times I_1, < B, A > \times I_2\}$.

Let us define the semantics of a sentence in terms of the interpretation and the variable assignments as the following:

For a pair of instants, say t_i, t_j, such that
$t_i, t_j \in \{\mathcal{T}_{ptsc} \cup \mathcal{T}_{ptsv}\}$
$\models_I (t_i\ R\ t_j)[\mathcal{V}]$ iff $\mathcal{V}(t_i)\ R\ \mathcal{V}(t_j)$
where R is one of $\{<, >, =\}$.
For a pair of intervals, say i, j, such that
$i, j \in \{\mathcal{T}_{intv} \cup \mathcal{T}_{intc}\}$
$\models_I (i\ R\ j)[\mathcal{V}]$ iff $\mathcal{V}(i)\ R\ \mathcal{V}(j)$
where R is one of the thirteen relations defined for intervals.
For the propositional cases
$\models_I tt(i, p)[\mathcal{V}]$ iff $\exists i'\ \mathcal{IV}(i) \subseteq i' \wedge < i' > \in \mathcal{I}(p)$
$\models_I i \subseteq J : tt(i, p)[\mathcal{V}]$ iff $\exists i'\ \mathcal{IV}(J) \subseteq i' \wedge < i' > \in \mathcal{I}(p)$
$\models_I i \subseteq J : (tt(i, p) \wedge tt(i, q))[\mathcal{V}]$
iff $\models_I i \subseteq J : tt(i, p)[\mathcal{V}]$
and $\models_I i \subseteq J : tt(i, q)[\mathcal{V}]$

For first order logic
$\models_I tt(i, p(t_1, t_2, ..., t_k))[\mathcal{V}]$
iff $\exists j \;\; \mathcal{IV}(i) \subseteq j \wedge$
$\{< \mathcal{IVT}(t_1), ..., \mathcal{IVT}(t_k) > \times j\} \in$
$\mathcal{IVT}(P(t_1, ..., t_k)[i])$
$\models_I i \subseteq J : tt(i, p(t_1, t_2, ..., t_k))[\mathcal{V}]$
iff $\exists j \; \mathcal{IV}(i) \subseteq j \;\wedge$
$\{< \mathcal{IVT}(t_1), ..., \mathcal{IVT}(t_k) > \times j\} \in$
$\mathcal{IVT}(P(t_1, ..., t_k)[i])$

Example

Let us consider an example to illustrate the representational convenience of our logic. We use the example of a sequential circuit that consists of two inputs namely in_1 and in_2. The output (out) is at level 1 throughout the interval when in_1 and in_2 are at level 1 and the switch is closed. The behavior of the circuit is represented using our ontology as following:

$i \subseteq [\rhd\lhd] : tt(i, in_1(1)) \wedge tt(i, in_2(1) \wedge$
$tt(i, wsitch\_statuson(on)) \rightarrow tt(i, out(1))$
in clausal form
$i \subseteq [\rhd\lhd] : tt(i, \neg in_1(1)) \vee$
$tt(i, \neg in_2(1)) \vee tt(i, \neg switch\_statuson(on)) \vee$
$tt(i, out(1))$
$i \subseteq [1,5] : tt(i, in_1(1))$
$i \subseteq [2,7] : tt(i, in_2(1))$
$i \subseteq [3,6] : tt(i, switch\_status(on))$
Applying the inference rule for the logic will yield the following axiom as expected.
$i \in [3,5] : tt(I, out(1))$

The inference rules, their soundness and proof procedure will be discussed in a separate paper.

4 Discussion and Conclusion

The importance of temporal reasoning for intelligent activities cannot be underscored. Most of the works in temporal reasoning within the context of artificial intelligence focus on propositional temporal knowledge in which the emphasis was on mapping the propositional temporal constraints onto a temporal network and propagating the constraints to the rest of the network for the purpose of obtaining local consistency [1], or minimal labels. [2] Several ontologies were proposed in the literature to represent the first order temporal knowledge.

We use reified logic to represent temporal knowledge. We introduced an interval structure *tt* to represent a fluent throughout the specified interval, and an instant structure *at* to represent a fluent at some specified instant. From our discussion of the syntax of our logic, it is clear that anything that could be expressible using the syntactic constructs of Shoham's logic [9] or BTK's [3] logic can also be expressible in our. Therefore, it is at least as expressible as other logics. The advantage of our logic is the way it uses the temporal constraints. The syntactic structure implements the semantics of the temporal structures *tt* and *at* which make it easier to develop mechanical deductive inference rules [3]. This work is an extension of some of the work done by Suengtaworn [10,8] for his Ph. D. dissertation and it is somewhat similar to the syntactic structure of MacNish et al. [2].

[1] three consistency or path consistency

[2] Obtaining minimal labels or deciding the satisfiability of a interval based temporal constraint network in **NP** hard

Acknowledgement

This work was partially supported by a grant from Louisiana Education Quality Support Fund (LEQSF1993-96RD-A-36).

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