Fault Propagation Using Fuzzy Cognitive Maps

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Abstract

An algorithm is presented for propagating the effect of single or multiple faults in a system modeled by a cognitive map. The cognitive map is a directed causal graphs that can be used to model complex dynamic system . Also shown is the propagation of variance estimates of uncertainty associated with the presence of a fault and with the arcs, representing strength of causality, in the cognitive map. The proposed approach can provide insights into the dynamic character of the system, and help, explain away intermittent fault behavior.

Introduction

Fuzzy Cognitive Maps (FCM) are directed causal graphs used for modeling complex dynamic systems with feedback[1,2]. They were introduced by Axelrod[3] and were initially used to perform high-level modeling of complex problems. Styblinski and Meyer[4] pointed the similarities between FCMs and Signal Flow Graphs in an application of qualitative analysis for a class of feedback amplifiers and general active R, L, C circuits, leading to a correct qualitative prediction of circuit behavior.

In this paper, the FCM algorithm is shown and proposed as a modeling tool for creating a causal graph between the variables of a diagnosis problem. It is suggested that such algorithm can be used to simulate the effect of a fault on the behavior of the system in a qualitative fashion. The initial values of the fault nodes in the FCM model trigger the propagation of causal activation through repeated updates of the nodes in the map, until a stable solution is obtained. The algorithm is extended to include the propagation of uncertainty estimates for the initial activation values and for the weights of the causal links in the model.

Fuzzy Cognitive Maps

FCM are directed causal graphs with feedback. They are made up of nodes representing concepts and directed arcs representing causation. An arrow points from cause to effect, and it has a sign and magnitude. The sign indicates causal increase or decrease, and the magnitude indicates degree of causality which can be a number on an arbitrary scale [0, 1] or [-1,1] if negative causality is desired.

The nodes in the graph represent concepts that get activated by the flow of causality. The degree of activation or state of each node can be discrete (e.g. {-1 0 1} or continuous on a given arbitrary scale (e.g., [0,1]) and is produced by the weighted sum of the inputs to the node and transformed by a nonlinear function in a way similarly done in a neural network. The causal flow in an FCM is obtained by iterative vector-matrix operations where the vector is a list of the activation states of the nodes, and a connection matrix depicting the causal connectivity of the graph. For a graph of n nodes there is an n-state vector and an n x n connection matrix.

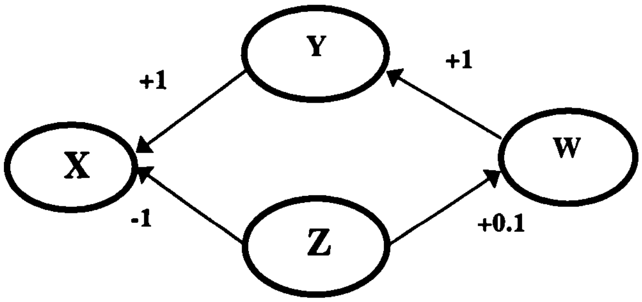


Figure 1 - Cognitive map of a simple model

Starting with an initial state vector C(t) at t_0, the causal flow proceeds iteratively following the equation

$$C(t+1) = T[C(t)\,\mathbf{E}] \qquad (1)$$

where **E** is the connection matrix and T[x] is a vector threshold operation, typically a monotonically increasing nonlinear function such as the sigmoid or logistic equation. The successive state vectors stabilize on a limit cycle, or a fixed point[1] depending on the choice of finite or continuous node states and the choice of the thresholding function. The stable form of the state vector constitutes the inference of the FCM.

Figure 1 shows an example of a simple model where node Z negatively affects node X and positively affects node W, but to a lesser extent. Node W, in turn, positively affects node Y which positively affects X. X is the net balance between Y and Z. In this model we do not assign physical quantities to the variables and their connecting arcs but, rather, a degree of activation for each variable and our belief of the strength of causality between variables represented by the arcs. In this example, Y and Z affect X in opposite ways and to the same degree.

The following illustrates how FCMs work. Defining the state vector C in the order [Z, W, Y, X], the connection matrix for the graph in Figure 1 is

$$\mathbf{E} = \begin{matrix} 0 & .1 & 0 & -1 \\ 0 & 0 & 1 & 0 \\ 0 & 0 & 0 & 1 \\ 0 & 0 & 0 & 0 \end{matrix}$$

Using the logistic equation, $T(x) = 2/(1 + e^{-2x})^{-1} - 1$, as the thresholding function, we get the following sequence of state vectors, when we start with a moderate but permanent increase in Z of degree .2. Starting at t_0 with C(0) = [.2, 0, 0, 0], the following sequence of state vectors is obtained:

$[.2, 0, 0, 0]\ E = [0, .02, 0, -.2]$
$\xrightarrow{T} [0, .02, 0, -.197] = C(1)$

$[.2, .02, 0, -.197]\ E = [0, .02, .02, -.2]$
$\xrightarrow{T} [0, .02, .02, -.197] = C(2)$

$[.2, .02, .02, -.197]\ E = [0, .02, .02, -.18]$
$\xrightarrow{T} [0, .02, .02, -.178] = C(3)$

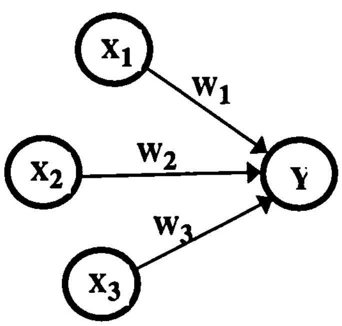
$[.2, .02, .02, -.178]\ E = [0, .02, .02, -.18]$
$\xrightarrow{T} [0, .02, .02, -.178] = C(4)$

The state vector C stabilizes after three iterations. The inference or prediction in this example suggests that a moderate and steady increase in Z results in a very small positive activation of W and Y, and a small net negative activation of X. Note that in each iteration, Z is set to .2 since it is considered a constant policy that is sustained in time, and therefore applies to each iteration.

Uncertainty Propagation

The causal activation associated with each node and the causal links associated with each arc are quantities that are usually provided by experts during the process of building the FCM. Those quantities are soft and therefore subject to uncertainty. If their uncertainty were provided in the form of variance estimates, it could also be propagated through the FCM structure and updated together with the causal activation.

The degree of causal activation of each node is updated in each iteration of equation 1. The activation is the sum of the activations x_i of the input nodes weighted by the causal links w_i associated with each arc, and transformed by the nonlinear function T.



$$y = \sum_{i=1}^{n} w_i x_i$$

$$z = T(y)$$

If both x_i and w_i are independent and uncertain quantities, using the method of moments[5] based on Taylor series expansion, the estimates for the variance of y and z are given by[6]:

$$\mathrm{Var}[y] = \sum_{i}^{n} \left((E[w_i])^2 \mathrm{Var}[x_i] + (E[x_i])^2 \mathrm{Var}[w_i] + \mathrm{Var}[w_i]\mathrm{Var}[x_i] \right) \quad (2)$$

$$\mathrm{Var}[z] = \mathrm{Var}[y]\left[\frac{\partial T}{\partial y}\right]_o^2 \quad (3)$$

These equations can be computed iteratively by implementing them in vector-matrix form such that, given the initial state activation vector with its variance estimates and a connection matrix representing the weights of the causal arcs with their variances, they will update the activation levels and their corresponding variances for every node in the FCM. The following is the iterative form of equations 2 and 3 combined

$$V_C(t+1) = [V_C(t)\mathbf{E}^2 + (C^2(t) + V_C(t))\mathbf{V_E}]\left[\frac{\partial T}{\partial y}\right]_{y(t)}^2 \quad (4)$$

where, $V_C(t)$ is the vector of variances corresponding to each activation state in vector C(t), and $\mathbf{V_E}$ is the matrix of variances corresponding to each causal arc in matrix **E**.

Fault Propagation

FCMs can be used to model causal relations between variables in a diagnosis problem. Such model would include variables that represent faults, variables that represent observed symptoms that result from the occurrence of a fault, and intermediate (or latent) variables that are introduced to facilitate making the connections between a fault and the observed symptom. Causal links between two variables are selected, on an arbitrary scale, to represent the degree of belief in the causal relation between the two variables. After the FCM is completed it can be used as a simulation engine in which to postulate the impact that each of the possible faults may have on the observed

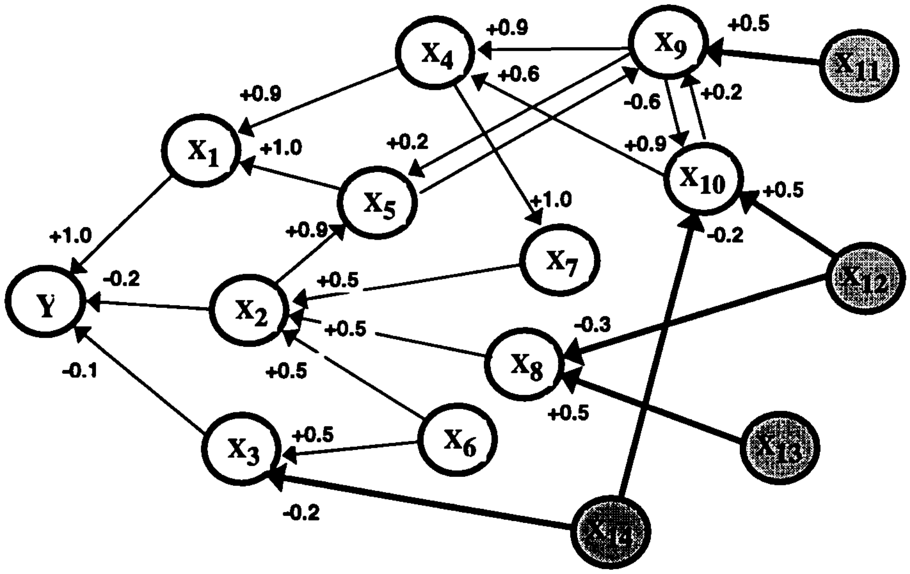


Figure 2 - A fuzzy cognitive map representing a diagnostic model

symptoms. The uncertainty propagation algorithm provides bounds that can be used to assess the relative impact that each fault may have on the observed symptoms.

Figure 2 shows an FCM that represents a particular diagnostic model. Causal weights are associated with each arc and for each weight a corresponding variance estimate of uncertainty is given. The grayed nodes X_{11}, X_{12}, X_{13}, and X_{14}, represent four faults, and node Y represents a particular observed symptom.

A recommended procedure for fault isolation is to simulate the impact of each fault on the possible observed symptoms. A table is then constructed that ranks the faults based on the belief that they could account for the observed symptom.

The procedure begins with all nodes in their neutral state (i.e., zero in a scale [-1,1]), except for the node that represents the fault in question. The fault (grayed) node has an initial level of activation of, say, 1 with associated uncertainty of .1 as its standard deviation. The causal flow starts by applying equations 1 and 4, iteratively, and ends when all nodes obtain stable activation levels. The answer sought is the degree of belief that the fault may cause the activation of the observed symptom.

The use of this procedure is aimed at providing general parameter trends. Once the relative impact of each fault is assessed, a ranking is obtained from the stable activation level resulting at the symptom node (node Y in figure 2).

As an example of the use of this procedure, an activation level of 1 with variance .1 is applied to the FCM for each of the fault nodes (X_{11}, X_{12}, X_{13}, and X_{14}). The resulting activation for symptom Y due to the impact of each fault are shown in Table 1 with the resulting standard deviation estimates. Figure 3 shows the connection matrix where the non zero values are the weights of the arcs in the model of figure 2. We also assume that the links have uncertainties with standard deviation of .1. The final ranking is obtained when the activation level of node Y and its standard deviation achieve stable values within a specified significant figure.

Fault Nodes	Activation of symptom Y	Standard Deviation σ_Y
X_{11}	0.516	0.087
X_{12}	0.333	0.100
X_{13}	-0.025	0.084
X_{14}	-0.014	0.081

Table 1 - Activation of Symptom Y due to presence of faults Xi

$$V_F = \begin{matrix}
0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 \\
1 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 \\
-.2 & 0 & 0 & 0 & 0 & .9 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 \\
-.1 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 \\
0 & .9 & 0 & 0 & 0 & 0 & 0 & 1 & 0 & 0 & 0 & 0 & 0 & 0 & 0 \\
0 & 1 & 0 & 0 & 0 & 0 & 0 & 1 & 0 & -.6 & 0 & 0 & 0 & 0 & 0 \\
0 & 0 & .5 & .5 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 \\
0 & 0 & .5 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 \\
0 & 0 & .5 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 \\
0 & 0 & 0 & 0 & .9 & .2 & 0 & 0 & 0 & 0 & .9 & 0 & 0 & 0 & 0 \\
0 & 0 & 0 & 0 & .6 & 0 & 0 & 0 & 0 & .2 & 0 & 0 & 0 & 0 & 0 \\
0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & .5 & 0 & 0 & 0 & 0 & 0 \\
0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & -.3 & 0 & .5 & 0 & 0 & 0 & 0 \\
0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & .5 & 0 & 0 & 0 & 0 & 0 & 0 \\
0 & 0 & 0 & -.2 & 0 & 0 & 0 & 0 & 0 & 0 & -.2 & 0 & 0 & 0 & 0
\end{matrix}$$

Figure 3 - Connection Matrix for model in figure 2

From Table 1 one concludes that fault X_{11} is believed to be the most responsible for the observation of symptom Y, followed by X_{12}, X_{14}, and X_{13}.

Conclusions

An algorithm is presented for propagating the effect of single or multiple faults in a system modeled by a cognitive map. The algorithm was extended to allow for the propagation of variance estimates of uncertainty associated with the initial activation levels for each fault and the weights of the arcs in the fuzzy cognitive map. A proposed procedure was shown for using the propagation algorithms to rank the possible faults associated with an observed symptom. An example was shown to illustrate the use of the procedure.

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