Control of switching constrained systems

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Abstract

We consider a particular class of hybrid systems characterized by a finite state machine and a set of discrete-time linear dynamical systems, each corresponding to a state of the machine. The hybrid control problem addressed consists of maintaining the output of the systems in a given subset of the output space, independently of the state of the FSM. Necessary and sufficient conditions are given for the existence of a solution.

Introduction

We consider a hybrid system (see e.g. (Morse 1997)) consisting of a finite state machine (FSM) and a set of dynamical systems, each corresponding to a state of the FSM. We assume that, at each location of the FSM, the actual configuration of the dynamical system is known. The time at which a transition occurs between two different states of the FSM (also called switching time) is not known *a priori* but is determined by an external uncontrollable event. The simpler case where the time of occurrence of the transitions is not completely unknown but can be predicted is also considered. The control problem we solve in this paper consists in maintaining the output of the systems in a given subset of the output space, independently of the state of the FSM. We work in an infinite time horizon framework.

This model can be used to represent a number of control problems of practical interest. In our case, the motivation to study this formulation comes from the engine control problem in automotive applications. Our research group has been involved with the formulation of the engine control problem as a hybrid system control problem and has developed guaranteed performance efficient algorithms for its solution (Balluchi et al. 1997, 1998). The plant to be controlled consists of the engine, modeled as a FSM, and of the powertrain, modeled as a linear dynamical system. A goal of engine control is to ensure comfort during the entire operation of the automobile. Acceleration oscillations caused by torque variations on the powertrain are source of discomfort for the driver. A possible control objective is to maintain such oscillations, which can be expressed as a linear combination of state variables of the powertrain, below a given threshold. We assumed in (Balluchi et al. 1997, 1998) that there is no gear change during the operation of the automobile and, hence, the powertrain model is the same for all the regions of operation of the engine. A gear change has the effect of changing the "structure" of the model of the powertrain dynamics, in the sense that the matrices describing the linear system change. A first step to solve the more general automobile control problem when gear change is considered is to "relax" the hybrid control problem as follows: the plant is the powertrain equipped with a gear change mechanism and the control input is the torque generated by the engine. The plant can then be represented by a hybrid model with an FSM part whose states correspond to a particular gear, and the dynamical system part corresponding to the appropriate powertrain dynamics for that gear. Then, the hybrid control problem described above is an adequate representation for the relaxed control problem.

Switching systems have been considered e.g. in (Marro and Piazzi 1993), (d'Alessandro and De Santis 1996) in the case where the switching instants are known *a priori*. In particular, in (d'Alessandro and De Santis 1996), an optimal solution is derived for systems with linear state constraints and linear cost functional. In (Marro and Piazzi 1993), the problem of robust regulation without error transients is solved. Some interesting control problems are solved in (Sontag 1996) when the transitions between two different states of the FSM are enabled by some guard conditions that may depend on the input and/or on the state of the dynamical system. In this paper, we derive necessary and sufficient conditions for the existence of a controller which maintains the output of the switching system in a given set, in the case where the switching instants are unknown *a priori*. Similar conditions can be obtained using the general results of (Tomlin, Lygeros and Sastry 1998). Our results are less general since they apply to a particular class of hybrid systems but have the advantage of exploiting the structure of the FSM. This allows to simplify the procedure for the determination of a solution and to derive convergence conditions which are unknown in the general case.

The paper is organized as follows. In Section 2, the problem of controlling a switching system is formulated and a solution is given in the case of unknown switching times. In Section 3, the problem is relaxed to the case where the switching times can be predicted. In Section 4, possible extensions of our results are described. Conclusions are offered in Section 5.

Switching constrained systems

The transition structure of an FSM determines the reachability of its states. A connected FSM is such that, for all state bi-partitions, there is always at least a transition from one set of the bi-partition to the other. Without loss of generality, we assume that the FSMs considered in this paper are connected. Consider a connected FSM F with state set $S = \{S_i, i = 1, \cdots, N\}$. To each state S_i of the FSM we associate a discrete time dynamical system (also called for simplicity configuration S_i) described by

$$\begin{aligned} x(t+1) &= A_i x(t) + B_i u(t) \\ y(t) &= C_i x(t) + D_i u(t) \end{aligned} \tag{1}$$

where $t \in \mathbb{N}$, $x \in \mathbb{R}^n$, $u \in \mathbb{R}^m$, $y \in \mathbb{R}^p$, and A_i, B_i, C_i, D_i $(i = 1, \cdots, N)$ are matrices of suitable dimensions.

The state evolution of the FSM in time is described by the function $s : \mathbb{N} \to S$, so that $s(t)$ denotes the state of the FSM at time t. Let $t_0 \in \mathbb{N}$ be the initial time. Only one switching is allowed at any $t \in \mathbb{N}$ and the switching times are supposed to be not known a priori. The system described by the FSM F, its evolution in time $s(t)$ and the dynamical systems (1) is called switching system.

Our goal is to find under which conditions it is possible to maintain the output $y(t)$ in a given set Ω (where Ω is a region of $\mathbb{R}^p$), for all $t \geq t_0$, i.e. :

$$y(t) \in \Omega, \forall t \geq t_0 \tag{2}$$

More precisely:

Problem 1 *Consider the switching system described by the FSM F and (1), where the switching times are supposed to be unknown. Let t_0 be the initial time and $s(t_0) = S_i$, for some $i = 1, ..., N$. Find the set X of all possible initial states $x(t_0)$ such that the constraint (2) can be satisfied for some control input $u(t)$.*

We consider here linear discrete-time dynamical systems in order to be able to compute the required controls and sets which solve the problem. However the derived structural conditions apply more generally to continuous time and nonlinear systems. To state the results the following definition is needed

Definition 2 *A set $\Sigma \subset \mathbb{R}^n$ is controlled invariant with respect to configuration S_i and constraint (2) if $\forall x \in \Sigma, \exists u \in \mathbb{R}^m : A_i x + B_i u \in \Sigma, C_i x + D_i u \in \Omega$.*

Define the set Ω_{ix} as

$$\Omega_{ix} = \{x : C_i x + D_i u \in \Omega, \text{ for some } u\}$$

Denote by $\mathcal{I}_i(\Lambda)$ the maximal controlled invariant set with respect to configuration S_i and constraint (2) contained in some set Λ.

The solution to our control problem will be given for the following simple cases. Once these cases are solved, we can extend the solution to any connected FSM by repeated application of the conditions for the simple cases.

- Serial case: the FSM is described by the following transitions:

$$S_1 \longrightarrow S_2 \longrightarrow \cdots \longrightarrow S_N \tag{3}$$

where S_i, $i = 1, \cdots, N$, denotes the i-th state of the FSM, being $S_i \neq S_j$ for $i \neq j$;

- Cyclic case: the FSM is described by transitions which are allowed to repeat themselves cyclically, i.e.

$$\begin{tikzcd} S_1 \arrow[r] & \cdots \arrow[r] & S_{N-1} \arrow[dl] \\ & S_N \arrow[ul] & \end{tikzcd} \tag{4}$$

Some states may be repeated, S_i may be equal to S_j for $i \neq j$.

- Star case: the FSM is described by the following transitions:

$$\begin{tikzcd} & S_2 \\ S_1 \arrow[ur] \arrow[r] \arrow[dr] & \vdots \\ & S_N \end{tikzcd} \tag{5}$$

Serial case

Let t_i, $i = 1, \cdots, N-1$, denote the switching time between states S_i and S_{i+1} and $t_0 < t_1 < \cdots < t_{N-1} < \infty$.

We can state the following:

Proposition 3 *Consider the switching system described by (1)-(3) with $s(t_0) = S_1$, then $X = \Sigma_1$ where:*

$$\begin{aligned} \Sigma_i &= \mathcal{I}_i(\Sigma_{i+1} \cap \Omega_{ix}) \qquad i = 1, \cdots, N-1 \\ \Sigma_N &= \mathcal{I}_N(\Omega_{Nx}) \end{aligned}$$

If the set Ω is described by linear inequalities of the form

$$Fy \leq w \tag{6}$$

the sets Ω_{ix} are polyhedra. If we assume that Σ_i are also polyhedra (which is not true in general) described by inequalities of the form

$$G_i x \leq v^i$$

all and nothing but the control vectors that solve the problem satisfy the following:

$$\begin{pmatrix} G_i B_i \\ F D_i \end{pmatrix} u(t) \leq \begin{pmatrix} v^i - G_i A_i x(t) \\ w - F C_i x(t) \end{pmatrix} \qquad t_{i-1} < t \leq t_i \tag{7}$$

Cyclic case

We need the following definition:

Definition 4 *A set $\Sigma \subset \mathbb{R}^n$ is controlled invariant with respect to configurations $S_1, \cdots, S_N$ and constraint (2) if:*

$$\forall x \in \Sigma, \forall i = 1, \cdots, N$$
$$\exists u \in \mathbb{R}^m : A_i x + B_i u \in \Sigma, C_i x + D_i u \in \Omega.$$

We can state the following

Proposition 5 *Consider the switching system described by (1)-(4), then $X = \Sigma^*$, where Σ^* is the maximal controlled invariant set with respect to configurations $S_1, \cdots, S_N$ and constraint (2) contained in the set $\Omega_{1x} \cap \Omega_{2x} \cap \cdots \cap \Omega_{Nx}$.*

Proof. The sufficiency is obvious. As for the necessity, let Σ_i^* be the set of states starting from which, if the actual configuration is S_i, all the subsequent constraints can be satisfied. Σ_i^* is controlled invariant with respect to configuration S_i and constraint (2) and moreover

$$\Sigma_i^* \subseteq \Omega_{ix} \quad i = 1, \cdots, N$$
$$\Sigma_1^* \subseteq \Sigma_2^* \subseteq \cdots \subseteq \Sigma_N^* \subseteq \Sigma_1^*$$

Hence, necessarily, $\Sigma_1^* = \Sigma_2^* = \cdots = \Sigma_N^* = \Sigma^*$ and the result follows. ■

In the case of linear constraints, and with the additional assumption that the set Σ^* is a polyhedron described by inequalities of the form:

$$Gx \leq v$$

all and nothing but the admissible control vectors satisfy the following

$$\begin{pmatrix} GB_i \\ FD_i \end{pmatrix} u(t) \leq \begin{pmatrix} v - GA_i x(t) \\ w - FC_i x(t) \end{pmatrix} \quad (8)$$
$$\forall t \text{ such that } s(t) = S_i \quad (9)$$

Remark 6 *A consequence of the last proposition is that solving the problem in the cyclic case is equivalent to solve the problem where the system may switch from one configuration to any other one, without any restriction on admissible transitions.*

If we denote by $\mathcal{C}(S_1, \cdots, S_N)$ the convex hull of the N configurations (1), i.e. the set of systems described by the equations:

$$x(t+1) = Ax(t) + Bu(t)$$
$$y(t) = Cx(t) + Du(t)$$
$$\begin{pmatrix} A & B \\ C & D \end{pmatrix} \in \mathcal{C}\left(\begin{pmatrix} A_i & B_i \\ C_i & D_i \end{pmatrix}, i = 1, \cdots, N\right)$$

and if we assume that Ω is convex, we can state the following

Proposition 7 *Suppose Ω is convex. If some convex set Σ is controlled invariant with respect to configurations $S_1, \cdots, S_N$ and constraint (2), it is also controlled invariant with respect to any system in $\mathcal{C}(S_1, \cdots, S_N)$ and constraint (2).*

Note that this Proposition does not imply robustness of the control with respect to perturbation of the model in $\mathcal{C}(S_1, \cdots, S_N)$, because we are assuming that the exact description of the system is known at each instant of time, and the control law depends on this description.

Star case

Proposition 8 *Consider the switching system described by (1)-(5) with $s(t_0) = S_1$, then:*

$$X = \mathcal{I}_1 \left(\cap_{j=2 \cdots N} \mathcal{I}_j(\Omega_{jx}) \cap \Omega_{1x}\right)$$

General case

A strongly connected FSM is an FSM such that every state is reachable from all other states. As a degenerate case, a single state is strongly connected. It is well known that a connected FSM can be decomposed into its strongly connected components (maximal sets of mutually reachable states) $F_1, F_2, \cdots, F_M$ and that there is a partial ordering among the strongly connected components defined as follows: strongly connected component F_i follows strongly connected component F_k if the states of F_i can be reached from states of F_k. Note that the partial order is well-defined since states of F_k cannot be reached from states in F_i otherwise F_i and F_k would not be maximal sets of mutually reachable states. The strongly connected components of F determine a Directed Acyclic Graph (DAG), T, where the nodes correspond to $F_1, F_2, \cdots, F_M$. The picture in Fig. 1 represents a generic connected FSM, where the nodes represent the states of the automaton and set of nodes inside dashed lines are the strongly connected components.

For any strongly connected component F_j, there always exists a cycle - called for simplicity maximal cycle - containing all the states of F_j. Using the results in Remark 5 every F_j is replaced by its maximal cycle and is still denoted by F_j. Without loss of generality, we assume that the DAGs we consider are *rooted*, e.g., there is only one node that has no incoming arc and we also make the assumption that $s(t_0)$ is in the root node of the DAG. We call *sink* a node of the DAG that has no outgoing arc.

The general solution can be found on the basis of the following two particular cases:

$$F_1 \longrightarrow F_2 \longrightarrow \cdots \longrightarrow F_M \quad (10)$$

and

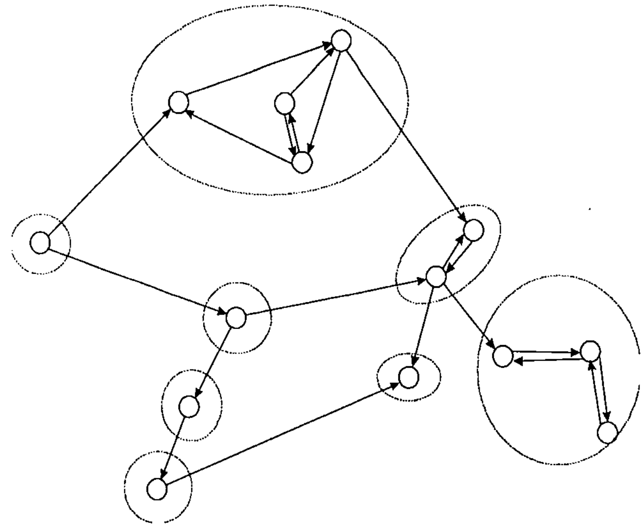

Figure 1: A connected FSM

$$F_1 \begin{array}{c} \nearrow \\ \longrightarrow \\ \searrow \end{array} \begin{array}{c} F_2 \\ \vdots \\ F_M \end{array} \qquad (11)$$

Proposition 9 *Consider the switching system described by (1)-(10), where $s(t_0)$ is a discrete state of F_1, then $X = \Sigma_1'$ where Σ_M' denotes the maximal invariant set computed applying Proposition 5 to F_M and, if $M > 1$, Σ_i' ($i = 1 \cdots M-1$) is the maximal invariant set computed applying Proposition 5 to F_i, with the additional requirement that such invariant set is a subset of Σ_{i+1}'.*

Proposition 10 *Consider the hybrid system described by (1)-(11), where $s(t_0)$ is a discrete state of F_1, then $X = \Sigma_1'$, where Σ_i' , $i = 2 \cdots M$, denotes the maximal invariant set computed applying Proposition 5 to F_i and Σ_1' is the maximal invariant set computed applying the Proposition 5 to F_1, with the additional requirement that such invariant set is a subset of $\cap_{i=2 \cdots M} \Sigma_i'$.*

The following algorithm applies the above propositions reducing successively the DAG to a single node s. The output of the algorithm is the set $\mathcal{L}(s)$ which represents the set of all the initial states which solve the problem in the general case.

Algorithm

MAIN:

Init: Mark all the nodes of the DAG (Directed Acyclic Graph) T as unvisited. Let $\mathcal{L}(v) = \mathbb{R}^n$ for each v in T

1. Start from an unvisited sink of the DAG and find all directed paths, stepping backwards until a node containing two or more "sons" has been found (this last node should not be included in the path). Denote by $\mathcal{P} = \{P_i, i = 1 \cdots \gamma\}$ the set of all such paths.
2. **For** $i = 1 \cdots \gamma$

   Collapse the path P_i to a single node u_i, replace the starting node of P_i with u_i and remove all the others (if any) from the DAG T . Mark u_i as visited. If P_i has at least two elements let $\mathcal{L}(u_i) = Serial(\ P_i\ )$.

   EndFor
3. If all the DAG sinks have been visited continue, else go to 1.
4. Scan the DAG sinks, until a sink v is found such that, if $f(v)$ is the "father" of v, all the "sons" of $f(v)$ have been visited. Denote by V the subtree containing all "sons" of $f(v)$ (v included) and $f(v)$ itself. Let Σ = *Star* (V). If $f(v)$ is the root node of the tree, $s = f(v)$, $\mathcal{L}(s) = \Sigma$. **EXIT.** Otherwise collapse the set V to a single node v' and mark it as unvisited. Let $\mathcal{L}(v') = \Sigma$.
5. Go to step 1.

SUB *Serial*($v_1 \to \cdots \to v_M$)
Return the (invariant) set obtained applying Proposition 8 to the strongly connected components $v_1 \to \cdots \to v_M$ (remember that each node of the DAG represent a strongly connected component in the FSM), with the additional requirement that such a set is a subset of $\mathcal{L}(v_M)$.

SUB *Star*(V)
Return the (invariant) set obtained applying Proposition 4 to the node $f(v)$, with the additional requirement that such a set is a subset of $\cap_i \mathcal{L}(w_i)$, where w_i, $i = 1 \ldots n_v$, are the sons of $f(v)$.

Predictable switching times

In this section we assume that the switching times can be predicted. with advance time τ, i.e. if $\bar{t}$ is a switching time between S_j and S_k, we know at time $\bar{t} - \tau$ that an FSM transition from state S_j to state S_k will take place at time $\bar{t}$. For simplicity we assume here that τ is less than any difference between two consecutive switching times.

The system described by the FSM F, its evolution in time $s(t)$ and the dynamical systems (1), where the switching times are known with advance $\tau \in \mathbb{N}$, is called a predictable switching system.

Problem 11 *Consider the predictable switching system described by the FSM F and (1), where the switching times are supposed to be known with advance τ. Let t_0 be the initial time and $s(t_0) = S_i$, for some $i = 1, \ldots, N$. Find the set X_τ of all possible initial states $x(t_0)$ such that the constraint (2) can be satisfied for some control input $u(t)$.*

Let us define the sets $\Omega_{ix}^{\tau}(\Sigma) \subset \mathbb{R}^n$, $i = 1, \cdots, N$ by means of the recursion:

$$\Omega_{ix}^{\tau}(\Sigma) = \Sigma \cap \Omega_{ix} \qquad \tau = 0$$

$$\Omega_{ix}^{\tau}(\Sigma) = \left\{ \begin{array}{c} x : \exists u : A_i x + B_i u \in \Omega_{ix}^{\tau-1}(\Sigma), \\ C_i x + D_i u \in \Omega \end{array} \right\} \qquad \tau > 0$$

i.e. $\Omega_{ix}^{\tau}(\Sigma)$ is the set of initial states starting from which, at some time t^0, a control sequence exists such that the state at time $t^0 + \tau$ belongs to the set Σ satisfying also the given constraints with respect to the i-th configuration at each time t, $t^0 \leq t \leq t^0 + \tau$.

In the serial case we have

Proposition 12 *Consider the predictable switching system described by (1)-(3) then* $X_\tau = \tilde{\Sigma}_1$ *where*

$$\tilde{\Sigma}_i = \mathcal{I}_i(\Omega_i^{\tau}(\tilde{\Sigma}_{i+1})) \qquad i = 1, \cdots, N-1$$
$$\tilde{\Sigma}_N = \mathcal{I}_N(\Omega_{Nx})$$

It is easy to prove that $\Sigma_i \subseteq \tilde{\Sigma}_i$, $i = 1, \cdots, N-1$, $\tilde{\Sigma}_N = \Sigma_N$. As for the control law in the linear constraints case, the structure is similar to (7).

In the cyclic case, we have

Proposition 13 *Consider the predictable switching system (1)-(4). Problem 11 is solvable if and only if there exist N sets* $\hat{\Sigma}_i$ *such that each set* $\hat{\Sigma}_i$ *is controlled invariant with respect to the configuration* S_i *and constraint (2), and the following inclusions hold:*

$$\hat{\Sigma}_1 \subset \Omega_{1x}^{\tau}(\hat{\Sigma}_2)$$
$$\cdots$$
$$\hat{\Sigma}_i \subset \Omega_{ix}^{\tau}(\hat{\Sigma}_{i+1})$$
$$\cdots$$
$$\hat{\Sigma}_N \subset \Omega_{Nx}^{\tau}(\hat{\Sigma}_1)$$

Moreover, if $s(t_0) = S_i$ $(i = 1, ..., N)$, *then* $X_\tau = \hat{\Sigma}_i$

Finally in the star case we have

Proposition 14 *Consider the predictable switching system described by (1)-(5), then:*

$$X_\tau = \mathcal{I}_1\left(\cap_{j=2\cdots N}\Omega_{1x}^{\tau}(\Sigma_j) \cap \Omega_{1x}\right)$$

where:

$$\Sigma_j = \mathcal{I}_j(\Omega_{jx}) \qquad j = 2 \cdots N$$

Extensions

Our results apply to a particular class of hybrid systems but can be extended to a number of cases:

- Constraints on state and/or input variables that are pointwise with respect to time, can be handled in our framework.
- The framework also works when the dynamic matrices in the equations (1) are uncertain, and depend on some unknown parameter, and/or the system is affected by an additive disturbance, i.e. when we have a description of the form

$$x(t+1) = A_i(\pi(t))x(t) + B_i u(t) + D_i \delta(t) \qquad (12)$$
$$y(t) = C_i x(t) + D_i u(t)$$

  where $\pi(t) \in \mathbb{R}^p$ is an uncertain, time varying parameter, and $\delta(t) \in \mathbb{R}^d$ is a time varying disturbance. Using the results in (De Santis 1997) we can analyze the solution obtained in the nominal case (i.e. described by equations (1). In fact once we have computed an invariant set which solves a given problem, we can also give a characterization of the set in parameters-disturbance space, such that the invariance property is preserved, with respect to some configuration (12). In addition we are also able to compute the maximal set of a given shape in the parameters-disturbance space such that a robust control law preserving the invariance exists.
- From the point of view of the computation of a control law that is robust with respect to a given bounded polyhedral set in parameters-disturbance space, the paper (Blanchini 1995) can be generalized to our switching systems problem.
- The more general case in which some of the transitions in the FSM are due to a control action and the others depend on an external event can also be analyzed in the proposed framework. Further generalizations can be obtained when the control action is such that the constraints are satisfied, while a suitable functional, defined on the states of the FSM, is minimized/maximized.

Conclusions

In this paper, we have considered a particular class of hybrid systems characterized by an FSM and a set of discrete-time linear dynamical systems, each corresponding to a state of the machine. The control problem solved in this paper consists in maintaining the output of the systems in a given subset of the output space, independently of the state of the FSM. Necessary and sufficient conditions were given for the existence of a solution. The conditions were expressed in terms of a set of states where the initial condition has to lay. Similar conditions can be obtained using the results of (Tomlin, Lygeros and Sastry 1998) which apply to general hybrid systems. Our results apply to a particular class of hybrid systems but have the advantage of exploiting the structure of the FSM. This allows to simplify the procedure for the determination of a solution and makes possible to derive convergence conditions which are unknown in the general case. Some extensions of our results were also illustrated.

Several new research directions will be pursued using the framework presented in this paper. On the computational aspects, we note that if all the constraining sets are polyhedra, a recursive a gorithm for computing the set can be given that always converges to the *exact* solution, if any. Such strong result has a price in terms of computational complexity: the number of inequalities defining the polyhedron that is converging to the solution increases as the algorithm progresses. This number can grow so large that the problem rapidly becomes computationally intractable. We believe that this drawback could be overcome by relaxing the requirements of obtaining the maximal controlled invariant set and be content with an approximation to it that is computationally appealing.

On the control synthesis aspects, we note that for each configuration, we have in general a static nonlinear state feedback control law. Hence, as the system switches between two configurations, we have to switch from a control law to another. This may be too complex from an implementation point of view. We would like to develop an approach to finding the simplest control law, among the ones that still solve the control problem.

We also believe that our approach can be generalized to the case of uncertainty in the description of the dynamic systems and/or unknown disturbances.

Acknowledgments

This work has been partially supported by CNR and Magneti-Marelli. The authors would like to thank Luca Benvenuti and Stefano Di Gennaro for useful discussions.

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