Fairness in Combinatorial Auctioning Systems

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Abstract

One of the Multi-Agent Systems that is widely used by various government agencies, buyers and sellers in a market economy, in such a manner so as to attain optimized resource allocation, is the Combinatorial Auctioning System (CAS). We study another important aspect of resource allocations in CAS, namely *fairness*. We present two important notions of fairness in CAS, *extended fairness* and *basic fairness*. We give an algorithm that works by incorporating a metric to ensure fairness in a CAS that uses the Vickrey-Clark-Groves (VCG) mechanism, and uses an algorithm of Sandholm to achieve optimality. Mathematical formulations are given to represent measures of extended fairness and basic fairness.

Keywords: fairness, optimality, multi-agent systems, combinatorial auctions

Introduction

Multi-Agent Systems (MAS) have been an interesting topic in the areas of decision theory and game theory. MAS are composed of a number of autonomous agents. In some applications, these autonomous agents act in a self-interested manner in their dealings with numerous other agents. Even in the interactive frameworks of game theory, the decision of one agent often affects that of another. This behavior is seen in the MAS which mainly deal with issues like resource allocation (Bredin & et al 2000; Sycara 1998). In such scenarios, each agent holds different significance over the various possible allocations and hence, concepts like individual rationality, fairness, optimality, efficiency, etc., are important (Chevaleyre, Dunne, & et al 2006). In this paper, we study a framework where optimality is a desirable property but fairness is a required property. An excellent example of such a framework is Combinatorial Auctioning Systems (CAS) where the two most important issues pertaining to resource allocation are *optimality* and *fairness*.

Incorporation of fairness into game theory and economics is a significant issue. Its welfare implications in different systems were explored by Rabin (Rabin 1993). The problem of fair allocation is being resolved in various MAS by using different procedures depending upon the technique of allocation of goods and the nature of goods. Brams and Taylor give the analysis of procedures for dividing divisible and indivisible items and resolving disputes among the self-interested agents (Brams & Taylor Feb 23 1996). Some of the procedures described by them include the "Divide and Choose" method of allocation of divisible goods among two agents to ensure the fair allocation of goods which also exhibits the property of "envy-freeness," a property first introduced by Foley (D.Foley 1967). Lucas' method of markers and Knaster's method of sealed bids are described for MAS comprising more than two players and for the division of indivisible items. The Adjusted-Winner (AW) procedure is also defined by Brams (Brams 2005) for envy-freeness and equatability in two-agent systems. Various other procedures like moving knife procedures for cake cutting are defined for the MAS comprising three or more agents (Brams 2005; Barbanel & Brams 2004).

However, it can also be seen that the perception of fairness varies across the different multi-agent systems, specially with regards to the resource allocations. In some MAS, allocation is perceived to be fair when resources are equitably distributed so that each recipient believes that it has received its fair share. In such a case, each agent likes its share at least as much as that of other agents' share and, hence, it is also known as envy-free division of resources (Brams 2005). But this notion of fairness is not applicable to all the MAS. To explain the different notions of fairness in MAS, we classify fairness into *basic fairness* and *extended fairness* in this paper.

To illustrate these notions of fairness mathematically, we shall use the framework of the Combinatorial Auctioning Systems (CAS). The CAS is a kind of MAS whereby the bidders can bid over combination of items (Nisan 2000; Narahari & Dayama 2005). The CAS approach is being used by different government agencies like the FCC (Cramton 2005) and numerous business applications like logistics and transportation (Caplice & Sheffi 2003; 2005) supply chain formation (W.E. Walsh & Ygg 2000), B2B negotiations (Jones & Koehler 2000), etc. It has been noticed that one of the significant issues in CAS is that of resource allocation. Optimum resource allocation is one of the most desirable properties in a CAS, and deals mainly with the Winner Determination Problem (WDP) (Sandholm 2002;

Narumanchi & Vidal 2005). Determining the winner in a CAS so as to maximize revenue is an NP-complete problem. However, it is seen that besides WDP, fairness is another important objective in many CAS-like government auctions. We realize the significance of fairness in CAS through a quote expressed by Rothkopf in (Rothkopf 2001) that

> Optimal solution to the winner determination problem, while desirable, is not required. What is required is a guarantee that the auction will be fair and will be perceived as fair.

We shall consider a CAS that uses the Sandholm algorithm and the concept of a Generalized Vickrey Auction (GVA) (Narahari & Dayama 2005). Sandholm's algorithm is a method to determine the optimal allocation of resources (Sandholm 2002) in a CAS. The concept of single-round second-price sealed-bid auction is then used to determine the payment made by the winners. According to this, the payment made by a winner is determined by the second-highest bid. In order to achieve fairness in such a CAS, we propose an algorithm that takes into consideration the fair values of resources as perceived by the bidders and the auctioneer in the system. Based upon their estimate of fair values, payments are made by the winners. A detailed analysis is done to highlight some important properties exhibited by this algorithm.

We start by classifying fairness and explaining its different notions. It is followed by our study on CAS and we give the mathematical formulations that are used to represent the measures of basic and extended fairness in CAS. Thereafter, a detail analysis of the scheme that highlights the attractive properties in the new payment scheme is given. We finally conclude the paper by suggesting the ideas for further work along these lines.

Classification of Fairness

To explain the different notions of fairness in various MAS, we classify fairness as *Basic Fairness* and *Extended Fairness*. This section defines the various perceptions about measuring fairness in MAS.

In our analysis, we do not consider agent preferences as being apart from their bids, i.e., if an agent has a higher preference for a good, it is considered to indicate the same by a higher bid, and vice versa. All goods are considered divisible.

Our algorithm creates an allocation that is seen as having fairness (either basic or extended) by all agents in the system.

Basic Fairness

In many MAS, there occurs a need of allocating the resources in an equitable manner, i.e., each agent gets an equitable share of the resources. This happens mainly when every agent holds similar significance for the given set of resources and has a desire to procure it. Thus, it becomes necessary to allocate the resources in an equitable fashion, i.e., such that each agent believes that its share is comparable to the share of other agents. Thus, none of the agents hold preferences over the share of other agents. Hence, we say that every agent believes that the set of resources is divided fairly among all the agents. This concept of fairness is termed as *basic fairness*.

Definition 1. *Basic Fairness is said to be achieved when the resource allocation is done in comparative terms among all the participating agents and the allocation is perceived to be fair by all of them with respect to the share that each of them procures.*

This kind of fairness is required in the applications whereby fairness is the key issue rather than the individual satisfaction of the self-interested agents. In such applications, it becomes necessary to divide a resource set in an equitable fashion so that every agent believes that it is receiving its fair share from the set of resources. Hence, we see that every agent enjoys material equality and this ensures basic fairness among them. In other words, the concept of basic fairness also ensures egalitarian social welfare (Chevaleyre & et al 2005) and envy-freeness (Brams 2005).

An example of such application that pertains to the equitable allocation of resources is given by Lematre (Chevaleyre, Dunne, & et al 2006). It deals with the equitable distribution of Earth Observing Satellite (EOS) Resources. EOS is co-funded and exploited by a number of agents and its mission is to acquire images of specific areas on earth surface, in response to observation demands from agents. However, due to some exploitation constraints and due to large number of demands, a set of demands, each of which could be satisfied individually, may not be satisfiable in a single day. Thus, exploitation of EOS should ensure that each agent gets an equitable share in the EOS resources, i.e., the demands of each agent is given equal weight assuming that agents have equal rights over the resource (we assume that they have funded the satellite equally). Hence, we observe that basic fairness is achieved as the demands of all agents are entertained by the equitable distribution of EOS resources.

Extended Fairness

In every MAS, we observe that each agent intends to procure a resource at a value that is perceived by it to be fair for the procurement. In other words, every agent assigns a fair value to each resource. This value is agent's estimate of the value of the resource in quantitative terms. The fair value attached to each resource can be expressed in monetary terms in most MAS. Thus, an agent intends to procure a resource by trading it with cash which is equal to the fair value attached to the resource by the respective agent. In such cases, each agent believes that it procures the resource at a fair value and, hence, believes the allocation to be fair.

However, it is important to mention that the fair value attached to each resource by an agent does not necessarily reflect the utility value of the resource to it. An agent may hold a higher or lower utility value for a resource irrespective of the fair value attached to the resource by it. Thus, the fair value attached to a resource is an estimate of the actual value of the resource in the system as perceived by an agent in quantitative terms. It means that an agent is always willing to trade a resource at its fair value.

Unlike basic fairness, extended fairness concept does not refer to an equitable distribution of resources. A resource is procured by one of the agents. However, it is considered fair by all the participating agents since parameters of the trade are perceived to be fair by all. These parameters of trade may be monetary exchange, or exchange of any tangible or intangible goods.

Definition 2. *Extended fairness is said to be achieved when a resource is procured by a single agent and all the other agents perceive the allocation to be fair with respect to the parameters of trade.*

An example of such a system can be explained through a scenario of job allocations in a multi-national company. Consider a MAS that refers to a company hiring situation, comprising an agent offering the job positions (i.e., the owner agent) and a number of self-interested agents who contend for these jobs. The contending agents express their estimate of the fair value through their curriculum vitae that is submitted to the owner agent, i.e., each contending agent believes that its curriculum vita fulfills the minimum requirements for the job and that it is eligible for the job. Hence, the agents define their perception of the required qualifications for the job through their curriculum vitae and believe it to be sufficient to qualify for the job. The owner agent selects the job-seeker agent that holds at least minimum qualifications required for the job but holds the maximum qualifications among all the contending agents. Thus, the job is allocated to the agent whose curriculum vita matches this criterion. All the agents believe that the curriculum vita of the winning agent was a fair parameter of allocation and, hence, perceive the allocation to be fair.

Thus, when the resources are allocated in a comparable fashion among all the agents, basic fairness is said to be achieved in the system. On the other hand, when fairness is measured with respect to the parameters of trade, extended fairness is said to be achieved.

To explain these notions of fairness mathematically, we shall study a framework where fairness is a required property in resource allocation. However, we also see that resource allocation deals with another key issue of optimality in various MAS. Thus, the best example of resource allocation framework where both optimality and fairness are the key issues is Combinatorial Auctioning Systems (CAS).

Fairness in Combinatorial Auctioning Systems (CAS)

Combinatorial Auctioning Systems are a kind of MAS which comprise an auctioneer and a number of self-interested bidders. The auctioneer aims at allocating the available resources among the bidders who, in turn, bid for sets of resources to procure them in order to satisfy their needs. The bidders aim at procuring the resources at minimum value during the bidding process, while the auctioneer aims at maximizing the revenue generated by the allocation of these resources. Thus, CAS refers to a scenario where the bidders bid for the set of resources and the auctioneer allocates the same to the highest-bidding agent in order to maximize the revenue. Hence, we see that optimality is one of the key issues in CAS. The Sandholm algorithm is used here to attain optimal allocation of resources. It works by making an allocation tree and carrying out some preprocessing steps like pruning to make the steps faster without compromising the optimality (Narahari & Dayama 2005; Sandholm 2002).

However, besides optimality, another key issue desired by some auctioning systems is fairness. To incorporate this significant property in this resource allocation procedure, we propose an algorithm which uses a metric to measure fairness for each agent and determines the final payment made by the winning bidders.

The algorithm that we describe is based upon a CAS that uses the Sandholm algorithm for achieving optimality, and an incentive-compatible mechanism called Generalized Vickrey Auction (GVA) as the pricing mechanism that determines the payments to be given by the winning bidders. The Generalized Vickrey Auction (GVA) has a payoff structure that is designed in a manner such that each winning agent gets a discount on its actual bid. This discount is called a Vickrey Discount, and is defined in (Narahari & Dayama 2005) as the extent by which the total revenue to the seller is increased due to the presence of that winning bidder, i.e., the marginal contribution of the winning bidder to the total revenue.

We give mathematical formulations to show that both kinds of fairness can be achieved in CAS. We show that *extended fairness* is achieved in all cases except in case of a tie, in which case *basic fairness* is ensured.

Mathematical Formulation

Terminology Let our CAS be a multi-agent system which is defined by the following entities:

(i) A set Φ comprising m resources $r_0, r_1, \ldots, r_{m-1}$ for which the bids are raised.

(ii) A set ξ comprising n bidders $b_0, b_1, \ldots, b_{n-1}$. These are the agents among whom the resources are allocated.

(iii) An auctioneer, denoted by λ, is the initial owner of all the resources and invites bids in the auctions.

Let us consider a CAS that comprises three bidders b_0, b_1, b_2, an auctioneer denoted as λ, and three resources r_0, r_1, r_2. Each bidder is privileged to bid upon any combination of these resources. We denote the combinations or subsets of these resources as $\{r_0\}$, $\{r_1\}$, $\{r_2\}$, $\{r_0, r_1\}$, $\{r_0, r_2\}$, $\{r_1, r_2\}$, $\{r_0, r_1, r_2\}$. We shall use the term package to define a set that comprises the subsets of resources won by a bidder. For example, a package for a bidder winning the subsets $\{r_0\}$ and $\{r_1\}$ is defined as $\{\{r_0\}, \{r_1\}\}$.

Assume that the auctioneer and each bidder has fair valuation for each of the individual resource (say, in dollars) as shown in Table 1.

Definition 3. *The fair valuation for an agent represents its estimate of the actual value of the resource.*

Thus, fair valuation by a bidder and an auctioneer for each resource represents their estimate of the actual value of each resource. Thus, a bidder is willing to trade a resource at its fair value and also believes that no loss is incurred by the seller in the trade. Similarly, the auctioneer is willing to sell a resource at the fair valuation described for it by him. Fair value for a combination of resources can be calculated as the sum of the fair value for each of the resources in that combination. The fair valuation for a resource by a bidder does not refer to the utility measure of the resource to the bidder. We shall use the term fair valuation and fair value interchangeably.

	Bidder b_0	*Bidder b_1*	*Bidder b_2*	*Auctioneer λ*
r_0	5	10	10	8
r_1	8	2	5	10
r_2	8	8	10	15

Table 1: Fair valuations for each resource by all bidders

From Table 1, we can see that the bidder b_0 values resource r_0 for \$5, r_1 for \$8 and r_2 for \$8. This means that bidder b_0 is willing to trade resource r_0 with \$5, r_1 with \$8 and r_2 with \$8 and believes that no loss is incurred by the auctioneer in this trade. The fair valuation for the subset $\{r_0, r_2\}$ for the bidder b_0 is calculated as the sum of the fair values for r_0 and r_2 as given by the bidder b_0, i.e., 5 + 8 = \$13. Similarly, fair valuation for a package is the sum of the fair valuation of the comprising sets i.e. for a package $\{r_0\}$, $\{r_1, r_2\}$, the fair value is the sum of the fair values of $\{r_0\}$ and $\{r_1, r_2\}$.

Let the bids raised by the bidders for the individual resource and different combination of resources be as given in table 2. It can be seen that the bids raised by each of the bidder for different sets of resources may or may not be equal to his fair valuation of the respective set of resources. A bidder can put zero bids for the set of resources it does not wish to procure.

	Bidder b_0	*Bidder b_1*	*Bidder b_2*
r_0	0	10	10
r_1	10	5	0
r_2	5	10	15
$\{r_0, r_1\}$	0	30	20
$\{r_0, r_2\}$	20	0	30
$\{r_1, r_2\}$	15	0	0
$\{r_0, r_1, r_2\}$	50	50	30

Table 2: Bids raised by the bidders for different combination of resources

It is assumed that the bidding language used in our system is OR bids, i.e., a bidder can submit any number of bids and is willing to obtain any number of atomic bids for a price equal to the sum of their prices (Nisan 2000; Narahari & Dayama 2005; Sandholm 2002).

The functions and the matrices used in the algorithm are as follows:

(a) A set D which is a subset of the set of natural numbers, i.e., $D \subseteq \mathbb{N}$, describing the possible quantitative values (in dollars) assigned to resources by bidders.

(b) A *fairness matrix*, $\Gamma_{i,[1\times m]}$, for the bidder b_i, and $\Gamma_{\lambda,[1\times m]}$ for the auctioneer, λ, is defined as : $\Gamma_i = [\tau_{i,0}, \tau_{i,1}, \ldots, \tau_{i,m-1}]$, for the bidder b_i.
$\Gamma_\lambda = [\tau_{\lambda,0}, \tau_{\lambda,1}, \ldots, \tau_{\lambda,m-1}]$, for the auctioneer, λ.

where the function τ_i is defined by a bidder, b_i, for a resource, r_j as:

$$\tau_i(r_j) = d, d \in D$$

This function represents a fair valuation of a resource, r_j, by a bidder b_i. From table 1, we have $\tau_0\,(r_1)$ = 8, $\tau_1\,(r_1)$ = 2, etc. Thus, from table 1, we have the following fairness matrices: Γ_0 = [5, 8, 8]; Γ_1 = [10, 2, 8]; Γ_2 = [10, 5, 10]; Γ_λ = [8, 10, 15]

(c) A function $\Upsilon_{i,k}$, known as the *pay function* by a bidder, b_i is defined as:

$$\Upsilon_{i,k}\,(b_i, \Psi_k) = d$$

where $\Psi_k = \{\mu_j | \mu_j \in \text{set of resources won by bidder}, b_i\}$, and $\Upsilon_{i,k}$ is the cost of the package, Ψ_k, to the bidder b_i as calculated from the GVA payment scheme.

Algorithm To Incorporate Extended Fairness In CAS

1. Each bidder and the auctioneer define its fairness matrix before the start of bidding process. It is a sealed matrix and is unsealed at the end of bidding process.
2. An allocation tree is constructed at the end of the bidding process to determine the optimum allocation and the winning bidders (Sandholm 2002). Information about all the bidders in a tie is not discarded using some pre-defined criteria.
3. Use GVA pricing mechanism to calculate the Vickrey discount (Narahari & Dayama 2005) and, hence, payments by the winning bidders for their corresponding packages, i.e., calculate Υ_{ij} for the package Ψ_j won by the bidder b_i.
4. Calculate the fair value of the package won by each bidder and denote it as Π_{ij} for the bidder b_i who wins the package Ψ_j.
5. Also calculate the fair value of each package using the fairness matrix of the auctioneer and denote it as $\Pi_{\lambda j}$ for a package Ψ_j.
6. Compare the values of $\Pi_{\lambda j}$ and Υ_{ij} and determine the final payment by the bidder depending upon the following conditions:

Case 1: $\Upsilon_{ij} > \Pi_{\lambda j}$ Bidder pays the amount Υ_{ij} and the auctioneer gains profit equal to $(\Upsilon_{ij} - \Pi_{\lambda j})$ which is distributed among other bidders who bid for the package Ψ_j. The profit is distributed in a proportional manner, i.e., in the ratio of $(\Pi_{kj} - \Pi_{\lambda j})/(\Pi_{\lambda j})$ for a bidder b_k who also bid for Ψ_j but is not a winning bidder.

Case 2: $\Upsilon_{ij} = \Pi_{\lambda j}$ In this case, the bidder pays the amount Υ_{ij} to the auctioneer.

Case 3: $\Upsilon_{ij} < \Pi_{\lambda j}$ Auctioneer suffers a loss of amount ($\Pi_{\lambda j} - \Upsilon_{ij}$). However, loss can be recovered as per the following cases:

(i) $\Pi_{ij} > \Pi_{\lambda j}$ Bidder's estimate of fair valuation is more than Υ_{ij}. Thus, bidder gives the final payment of $\Pi_{\lambda j}$ to the auctioneer.

(ii) $\Pi_{ij} = \Pi_{\lambda j}$ Bidder's estimate of fair value is same as that of auctioneer's estimate and is greater than the value Υ_{ij}. Thus, bidder pays amount Π_{ij} to the auctioneer.

(iii) $\Pi_{ij} < \Pi_{\lambda j}$

(a) $\Pi_{ij} \leq \Upsilon_{ij}$: then bidder's final payment remains the same, i.e., Υ_{ij}

(b) $\Pi_{ij} > \Upsilon_{ij}$: then bidder's final payment is equal to Π_{ij}.

Handling the cases of tie - Incorporating Basic Fairness Unlike traditional algorithms, we do not discard the bids in the cases of a tie on the basis of some pre-decided criterion. We consider these cases in our algorithm to provide *basic fairness* to the bidders.

In cases of a tie, we shall measure the utility value of the resource to each bidder in the tie.

Definition 4. *The utility value of a resource to a bidder is defined as the quantified measure of satisfaction or happiness derived by the procurement of the resource.*

Mathematically, we define utility value for a resource set μ_j as:

$$\upsilon_i(\mu_j) = \nu_i(\mu_j) - \Pi_{ij}$$

where $\nu_i(\mu_j)$ is the bid value of the resource μ_j and Π_{ij} is the fair valuation for the resource set μ_j for the bidder b_i.

The bidders maximize this utility value to quantify the importance and their need for the resource to them. Thus, the higher the utility value, the greater is the need for the resource set.

In such a case, fairness can be imparted if the resource set μ_j is divided among all the bidders in a proportional manner, i.e., in accordance to the utility value attached to the resource by each bidder.

Let us consider the same example to explain the concept of basic fairness in our system. From table 2, we observe that the optimum allocation attained through allocation tree comprises the resource set $\{r_0, r_1, r_2\}$ as it generates the maximum revenue of \$50. However, we see that this bid is raised by the two bidders, b_0 and b_1.

Thus, we calculate the fair value of the resource set $\mu_1 = \{r_0, r_1, r_2\}$ for the bidder b_0 and b_1, i.e., Π_{01} = 5+8+8 = \$21 and Π_{11} = 10+2+8 = \$20. Thus, the utility value of the resource set μ_1 for the bidder b_0 and b_1 is as follows:

for bidder b_0, $\upsilon_0(\mu_1)$ = 50 - 21 = \$29, and

for bidder b_1, $\upsilon_1(\mu_1)$ = 50 - 20 = \$30.

Hence, the resource set μ_1 is divided among bidders, b_0 and b_1, in the ratio of 29:30. In other words, bidder b_0 gets 49.15% and bidder b_1 gets 50.85% of the resource set μ_1.

The payment made by the bidders is also done in the similar proportional manner. For example, the bidders, b_0 and b_1, make their respective payments in the ratio of 29:30 to make up a total of \$50 for the auctioneer, i.e., bidder b_0 pays \$24.65 and bidder b_1 pays \$25.35 to the auctioneer for their respective shares.

Thus, we take into account the fair estimates of the auctioneer and the bidders for each resource to ensure that fairness is achieved to auctioneer as well as the bidders. We also see that extended fairness as well as basic fairness are achieved in CAS by using a fairness metric.

We shall do a detailed analysis of this algorithm in the following section.

Analysis

A detailed analysis is done to highlight some important concepts used and the significant properties exhibited by our CAS through the new payment mechanism.

Fairness

In MAS, every agent has its own metric to measure fairness with regards to the allocation of resources. Similarly in CAS, we see that the auctioneer and the bidders have their own estimate of the fairness value attached to each resource. We introduced the concept of fairness matrix to attain the knowledge of the fair value attached to each resource by the auctioneer and each bidder. This matrix is used as a metric to ensure that each allocation of resources is perceived to be a fair allocation by the bidder as well as the auctioneer.

We see that extended fairness is achieved through the algorithm when an agent having the highest bid procures the resource. In such a case, the winning bidder perceives the allocation to be fair as it procures the resource at his estimate of the fair value of the resource. However, other agents also perceive the allocation to be fair since the resource is procured by the agent having the highest bid. Thus, the trading parameter, i.e., the bid value at which the resource is being procured is the highest and hence fair in an auctioning system.

We also see that basic fairness is achieved in our system when there is more than one bidder who has raised equal bid for the same set of resources. In such a case, we divide the set of resources among all the bidders so as to ensure fairness to all the bidders in a tie. However, this division of resources is done in a proportional manner. We intend to divide the resource such that the bidder holding highest utility value to it should get the maximum share. To ensure this, we calculate the utility value (i.e., $\upsilon_i(\mu_j) = \nu_i(\mu_j) - \Pi_{ij}$) of the set of resources to each bidder and divide the set in the ratio of these values among the respective bidders. Thus, we see that each bidder procures its share of the set of resources in accordance to the importance attached by the bidder to it.

Due to the achievement of fairness through our payment scheme, the bidders are expected to show willingness to participate in the auctions.

Rationality

We shall see that the fairness matrix is a metric for fair valuation that forces the bidders and the auctioneer to behave rationally. They attain maximum profits if they describe their fair matrix truthfully. Our system ensures certain behavioral traits of auctioneer and the bidders through which this property of rationality is achieved in our system. These behavioral traits are described in the following:

Proposition 1. *The auctioneer does not state extremely high or low values in its fairness matrix as this does not generate higher revenue.*

Proof. If an auctioneer states very high values in its fairness matrix, then Case 3 follows most of the times. From Case 3, we observe that the auctioneer receives a payment equal to $\Pi_{\lambda j}$ only if this value is comparable to that of Π_{ij} for a bidder b_i. In other words, an auctioneer benefits only if its valuation is not irrationally higher than that of the bidder. On contrary, the auctioneer does not state very low values in its fairness matrix. For such circumstances, Case 1 follows, whereby it seems to be that the auctioneer gains profit which is distributed among the bidders. Hence, the auctioneer does not gain any profit by behaving irrationally. □

Proposition 2. *Bidders do not state extremely high or low values in the fairness matrix as it does not help them procure the resources at lower values.*

Proof. We see that the Case 3 deals with the fairness values of the bidder b_i. In case $\Upsilon_{ij} < \Pi_{\lambda j} < \Pi_{\lambda j} < \Pi_{ij}$, the bidder pays the amount $\Pi_{\lambda j}$, i.e., higher than his fair estimate of the package Ψ_j. Otherwise if $\Upsilon_{ij} \leq \Pi_{ij} \leq \Pi_{\lambda j}$, the bidder pays the amount equal to Π_{ij}. In both the cases, we see that the value to be paid is higher than the bid value. However, if the bidder is in a tie for a resource set, then its utility value falls negative if $\Upsilon_{ij} \leq \Pi_{ij}$. Hence, the bidder does not get the profits which are distributed among other bidders in a tie. Thus, a bidder undergoes a loss if the value of Π_{ij} is very high. On contrary, the bidder does not state lower values in the fairness matrix. In this case, a loss is perceived by the bidder under Case 3, condition (iii), part (a). □

Proposition 3. *Bidders raise their bids truthfully.*

Proof. Bidders gain by bidding truthfully. On bidding truthfully, they can maximize the Vickrey Discount on their bids. Secondly, in the cases of tie, they can maximize the profit earned ($\upsilon_i(\mu_j) = \nu_i(\mu_j) - \Pi_{ij}$), i.e., for a given value of Π_{ij}, profit can be maximized by raising the bids truthfully. □

Incentive Compatibility

The payment mechanism described in our system is incentive compatible in certain cases. In the cases, when payment value for a package, as calculated from the VCG mechanism, is greater than the fair valuation of the auctioneer for the same package, then Case 1 follows, i.e., the auctioneer gets an amount higher than its fair valuation for that package. It means that the auctioneer gains the profit equal to ($\Upsilon_{ij} - \Pi_{\lambda j}$). This profit is distributed in the proportional manner among the bidders who bid for the same package as explained in Case 1.

Thus, it also forces the bidders to bid truthfully so as to gain monetary incentives from the auctioning system.

Efficiency

The cases of a tie are handled in such a way so as to ensure basic fairness. In such a case, we divide the resource in proportion to its utility value to a bidder. Thus, a resource is allocated in accordance to the wishes of the consumers and, hence, the net benefit attained through its use is maximized. In other words, we can say that our system is allocatively efficient as the resources are allocated to the bidders who value them most and can derive maximum benefits through their use. Hence, we achieve allocative efficiency by handling the cases of tie in an efficient manner.

Optimality

Optimality is a significant property that is desired in a CAS. We ensure this property by the use of Sandholm algorithm in our system. It is used to obtain the optimum allocation of resources so as to maximize the revenue generated for the auctioneer. Thus, output obtained is the most optimal output and there is no other allocation that generates more revenues than the current allocation.

Conclusion

Thus, we have shown that fairness is incorporated in CAS, whereby all the agents receive their fair share if they behave rationally. Extended fairness as well as basic fairness is attained through our payment mechanism. Optimal allocation is obtained through the Sandholm algorithm and the other significant properties like allocative efficiency and incentive compatibility are also achieved. This is an improvement because in the existing world of multi-agent systems, there do not seem to be many studies that attempt to incorporate optimality as well as fairness. The present paper addresses this lack in a specific multi-agent system, namely, the CAS.

However, this work can be extended towards achieving a generalized framework suitable for all, or at least many, multi-agent systems, rather than just CAS.

The framework described can also be extended in several ways: one is to de-centralize the suggested algorithm, to avoid use of a single dedicated auctioneer. Especially in distributed computing environments, it would be best to have a method to implement the suggested algorithm (or something close to it) without requiring an agent to act as a dedicated auctioneer.

A second important extension would be to find applications for the work. Some applications that suggest themselves include distribution of land (a matter of great concern for governments and people the world over) in a fair manner. In land auctions where a tie occurs, no pre-defined or idiosyncratic method need be used to break the tie; rather, the allocation can be done fairly in the manner suggested.

Fairness is also an important and pressing concern in the computing sciences and information technology, particularly, in distributed computing (Lamport 2000). It is therefore also of interest to see how our method for achieving fairness could be applied in such contexts.

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