A Survey of Possible Uses of Quantum Mechanical Concepts in Financial Economics

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Abstract

In this talk we intend to provide for an overview of the uses we can make in financial economics of several quantum physical concepts. We introduce and briefly discuss the so called information wave function. We discuss how the information wave function can be of use in financial option pricing. We then briefly allude on how the information wave function can be used in arbitrage. We briefly discuss how the information wave concept can be connected to measures of information. We round off the paper on recent work we are doing with Andrei Khrennikov on i) testing probability interference in psychology (Khrennikov and Haven 2006 (I)) and ii) finding uses of 'financial' non-locality and entanglement (Khrennikov and Haven 2006 (II)). *In summary* this talk addresses interference (probability interference), financial entanglement and looks at using the wave function in a variety of ways within an economics/finance – political science – social interaction environment.

Introduction

The notion of information plays an important role in economics. In a recent (2004) survey paper on financial option pricing one of the prominent scholars in option pricing theory, Jerôme Detemple (Broadie and Detemple 2004) indicated that models which incorporate information arrival may be part of a new set of models in financial option pricing (basics of option pricing can be found in (Black 1973) and in (Wilmott 1999). This statement indicates that information, even in such a mature field like financial option pricing can still make inroads. In the potential presentation I would make at the AAAI Spring Symposia – Quantum Interaction meeting (Stanford University), I would have as mission to try to convince the audience that the concept of 'information wave function' and other related quantum mechanical concepts can be used fruitfully in a financial economics context. Any financial economics concepts appearing in this talk (such as option pricing etc..) will be duly explained so the audience can potentially appreciate the possible use of applying quantum physical concepts to this field. For the purposes of the Quantum Interaction meeting, we will use and *expand* on some of the issues raised in invited talks we performed on a similar subject at the following occasions: i) OASIS- Comlab – Oxford University (2006); ii) Foundations of Probability and Physics – 3 (2005) (and 4) (2006) meetings held at the University of Växjö- Sweden; iii) Feynman Festival (2006) – University of Maryland (USA).

Before we go into some detail on what we mean with the information wave function and its applications to financial economics we need to indicate that there has already been some work performed on using quantum physical concepts in social science in general. We mention, in alphabetical order; i) Arfi's (2005) work on quantum games; ii) Busemeyer et al.'s (2006) investigation on whether one can derive "a meaningful model of human information processing from quantum dynamic principles?" (Busemeyer et al.; (p.5)); iii) Baaquie's (2004) work on quantum finance; iv) Choustova's (2001; 2006 (I); 2006 (II)) path-breaking work on using Bohmian mechanics in a financial economics context; v) Haven's (2005) work on applying the WKB approximation technique to solving Black-Scholes pde's with complicated volatility functions and other work using Bohmian mechanics in financial option pricing (Haven (2005 (II); 2006(I)(II)(III)(IV)); vi) Khrennikov and Haven's (2006) (I) work on testing for the existence of probability interference in psychology; vii) Khrennikov's (1999; 2004) world leadership on bringing to light how Bohmian mechanics can be of use in a social science environment; viii) Lambert et al's (2003) work on interpreting the state of an agent as a superposition of potential types; viii) La Mura's (2003) work on using the double slit experiment in economics; ix) Segal's (1998) work on financial option pricing in a quantum context and finally, x) Shubik's (1999) work on quantum economics.

In the next section of this paper we discuss in superficial detail the provenance of the information wave function. We then pursue, in a next section, with discussing how the information wave function can be used in option pricing. We move on to discuss how the information wave function can be used in triggering arbitrage. We then hint onto the possible connections the information wave function may have with information measures, in a next section. We round off this paper with a brief sketch of the probability interference test and the issue of financial non-locality and entanglement.

Provenance of the information wave function

The information wave function concept is based on the polar form of the quantum mechanical wave function. We also make use of Bohmian mechanics. This particular interpretation of quantum physics, thus known under the name of "Bohmian mechanics" was pioneered by David Bohm (1952) and Basil Hiley (1987; 1989; 1993). We use the concept of 'pilot wave' function and we use the interesting property that the pilot wave steers the particle. How does this property come about? Firstly, historically as is remarked in Holland (1993), Louis de Broglie attributed two roles to the wave function, $\Psi(q,t)$ (p. 16):

> ..not only does it determine the likely location of a particle it also influences the location by exerting a force on the orbit.

Secondly, algebraically by substituting the polar form of the wave function in the Schrödinger equation and separating into real and imaginery parts one obtains two equations. One equation corresponds to the real part and the other equation corresponds to the imaginery part. The details of this derivation are in Appendix A (p.134 of Holland (1993)). The equation corresponding to the real part contains next to the real potential a term, which is known in Bohmian mechanics, as the so called quantum potential. The main ingredient of the quantum potential is the amplitude function of the wave function.

Bohm and Hiley (1993) compare the pilot wave to a radio wave which steers a ship on automatic pilot. In our financial economics context, we can think of the pilot wave as an information function which steers for instance the price of an asset. The particle is an asset price then. We then use the quantum potential as in the papers by Olga Choustova (2001; 2006) and Andrei Khrennikov (1999; 2004) and Haven (2005 (I); 2006 (I); 2006 (III)) as the basis to produce a trend or pricing rule. This will of course be discussed in detail during the talk. There also can be made some useful comparisons with the concept of private information which is often the basis upon which financial arbitrage (the realization of a risk free profit) is based. See Haven (2006) (III); Tirole (1982) and Scheinkman et al. (2004).

The information wave function and option pricing

We can make the argument that arbitrage will occur on the basis of the existence of private information. This information, we believe, can be modeled via the use of an information wave function. We strive to incorporate this information wave function in an option pricing model. For the basics of the option pricing model see Black et al. (1973) and for an excellent overview see Wilmott (1999). In Haven (2005 (I); 2006 (I); 2006 (III)) we explain the option pricing model with the information wave function in much detail. In sketch form, we start out using the Bohm-Vigier (Bohm and Hiley (1993)) equation which indicates the velocity of a particle is function of the gradient of the phase of the wave function and some random factor. We apply this equation directly on the option pricing portfolio and we thereby embed automatically the information wave function in the option price (Haven 2005 (I); 2006 (I); 2006 (III)). However, we would like to extend the underlying Brownian motion used in financial option pricing to contain not only the phase of the wave function but also the amplitude. This leads us into using the so called universal Brownian motion which was first proposed by Nelson (1966; 1967). We then continue in showing how the Universal Brownian motion can be used in i) the financial Sharpe ratio; ii) as a secondary stochastic differential equation besides the stock price Brownian motion. However, in our talk we want to go beyond what Nelson proposed and discuss other avenues which incorporate some of the more recent proposals (Choustova 2006 (II)).

The information wave function and triggering arbitrage

The concept of arbitrage we briefly discussed above is a key concept in financial economics. We can price assets under the non-arbitrage condition. Thus, the theory of option pricing has as underlying condition the non-arbitrage condition. The key theorem is that there will be no arbitrage if and only if there exists a risk neutral probability. This type of probability allows for the discounting of a risky asset at the risk free rate (this rate is the rate of return one receives on an investment which is risk-less). The use of this probability leads to the use of martingales in financial economics. In this talk we want to show that if we use as risk-neutral probability the probability based on the information wave function, we will have a natural model by which we can show that information affects arbitrage. This is detailed in Haven (2006) (II).

The information wave function and information measures

In Haven (2006) (III) we discuss how the information wave function can be used in Rényi's quantity of information measure (1961) and we discuss how Rényi's postulate 9 (1961) can be used with information wave functions. We also discuss how the information wave function can be connected to the Blackwell and Dubins theorem (1961) and we discuss how we can rank levels of information. Nakata (2000) provides for an important economic theory linked to Blackwell and Dubins theorem. This ranking of levels of information is obtained in relation to the existence/absence of the pricing rule (please see section 'Provenance of the information wave function') and in relation to the existence/absence of the drift in the universal Brownian motion (please see section 'The information wave function and option pricing'). We also make use of Blackwell's informativeness criterion (Blackwell 1953). In the talk we will of course provide for all the details.

Probability interference, non-locality and entanglement

In a joint paper Khrennikov and Haven (2006) (I) want to test for the possible existence of probability interference in a psychological setting. An application has been recently made to a funding body to finance the experiment. In the experiment we want to test for the complementarity of two variables: i) time of processing (by experiment participants) of (non-moving) images and ii) the ability (by experiment participants) of recognizing deformations of (non-moving) pictures. We argue why we can not find this complementarity using the Heisenberg Uncertainty Principle. We are careful to discuss the preparation of the experiment. Finally, in joint work, Khrennikov and Haven (2006) (II) are working on a financial interpretation of so called non-locality and entanglement. Non-locality in a Bohmian mechanics environment includes the involvement of the quantum potential. See for instance Holland (1993) for an interesting discussion.

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