Ambiguous Landmark Problems in Cognitive Robotics: A Benchmark for Qualitative Position Calculi

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Abstract

In this paper we introduce a task which can serve as a benchmark for qualitative relative position calculi. In this task ambiguous local landmark observations have to be integrated into survey knowledge. We show that the most prominent relative position calculus, Freksa's Double Cross Calculus can solve a specific instance of this task. The observations can be represented in a constraint network and standard constraint propagation solves the ambiguity problem.

However, more general instances of the ambiguous landmark problem cannot be solved using the Double Cross Calculus. Therefore we present an extension to relative position ternary point configuration calculi which uses an adaptable level of granularity. This family of calculi is capable to solve general instances of the proposed benchmark. Thereby robot applications including reasoning about ambiguous perceptions will be made possible.

Introduction

A *qualitative* representation provides mechanisms which characterize central essential properties of objects or configurations. A *quantitative* representation establishes a measure in relation to a unit of measurement which has to be generally available. Qualitative spatial calculi usually deal with elementary objects (e.g., positions, directions, regions) and qualitative relations between them (e.g., "adjacent", "on the left of", "included in").

The constant general availability of common measures is now self evident. However, one needs only remember the example of the history of technologies of measurement of length to see that the more local relative measures, which are qualitatively represented, (for example, "one piece of material is longer than another" versus "this thing is two meters long") can be managed by biological/epigenetic cognitive systems much more easily as absolute quantitative representations. Typically, in Qualitative Spatial Reasoning relatively coarse distinctions between configurations are made only. However, certain configurations can be distinguished more precisely using qualitative methods (Freksa 1991). For example the intersection point of two straight lines can be represented either metrically using real valued Cartesian coordinates or alternatively by using qualitative relations. The qualitative relations would represent that the intersection point lies on both straight lines. In contrast with the metrical, quantitative method and real valued Cartesian coordinates a test for the distance between this intersection point and each of the straight lines may generate the misleading information that the point has a non-zero distance to both lines (due to rounding errors) (Güting 1994). There is a significant loss in the semantics when using metrical information only (Egenhofer et al. 1999).

The two main trends in Qualitative Spatial Reasoning are topological reasoning about regions (Randell, Cui, and Cohn 1992; Renz and Nebel 1999; Egenhofer and Franzosa 1991) and positional reasoning about point configurations (**?**; Schlieder 1995a). Especially positional reasoning is important for robot navigation (Musto et al. 1999). In the next section we give a short introduction about positional calculi. Then we present a benchmark for constraint reasoning with these calculi.

Qualitative Relative Position Calculi

Positional calculi are influenced by results of psycholinguistic research in the field of reference systems (Moratz and Tenbrink 2006). The results point to three different options to give a qualitative description of spatial arrangements of objects labelled by Levinson (Levinson 1996) as *intrinsic*, *relative*, and *absolute*.

We can find examples of all three options of reference systems in the QSR literature. For instance, an intrinsic reference system is used in the dipole calculus (Schlieder 1995b), (Moratz, Renz, and Wolter 2000), a relative reference system in QSR was introduced by Freksa (Freksa 1992b), and finally Andrew Frank's cardinal direction calculus is suitable for an absolute reference system (Frank 1991), (Ligozat 1998).

Qualitative relative position calculi can be viewed as computational models for projective relations in relative reference systems. To model projective relations (like "left", "right", "front", "back") in relative reference systems, all objects are mapped onto the plane. The centers of projected objects can be used as point-like representation of the objects.

Figure 1 shows a simple model for the left/right-dichotomy in a relative reference system, which is given by *origin* and *relatum* (corresponding to Levinson's termi-

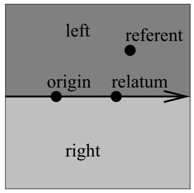

Figure 1: The left/right-dichotomy in a relative reference system

nology (Levinson 1996)). In this figure origin and relatum define the reference axis. The reference axis naturally partitions the surrounding space in a left/right-dichotomy. The spatial relation between the reference system and the *referent* is then described by naming the part of the partition in which the referent lies. In the configuration depicted in Figure 1 the referent lies to the *left*[1] of the relatum as viewed from the origin.

This scheme ignores configurations in which the referent is positioned on the reference axis. Freksa (Freksa 1992b) used a partition that splits these configurations into three sets, corresponding to the relatum: the referent is either behind, at the same position or in front of the relatum. Ligozat (Ligozat 1993) subdivided the arrangements with the referent in front of the relatum in those cases where the referent is between the relatum and the origin, at the same position as the origin, or behind the origin. We then obtain the partition shown in Figure 2. Ligozat calls this the flip-flop calculus. For a compact notation, we use abbreviations for the relation symbols.

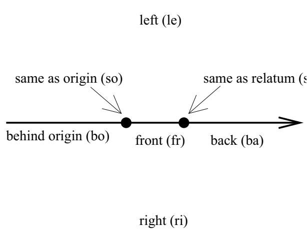

Figure 2: Adding relations for referents on the reference axis

For A, B, and C as origin, relatum, and referent, Figure 3 shows point configurations and their qualitative descriptions, respectively. Isli and Moratz (1999) (Isli and Moratz 1999) introduced two additional configurations in which the origin and the relatum have exactly the same location. In one of the configurations the referent has a different location, this relation is called **dou** (for double point). The configuration with all three points at the same location is called **tri** (for triple point). A system of qualitative relations which describes all the configurations of the domain and does not overlap is called jointly exhaustive and pairwise disjoint (JEPD). Such a calculus was formulated in the scheme of a relation algebra (Düntsch, Wang, and McCloskey 2001) by Scivos and Nebel (Scivos and Nebel 2005).

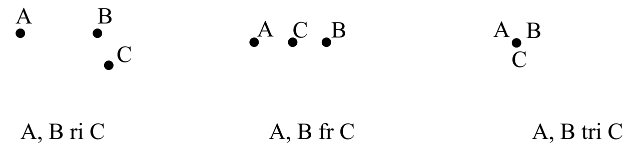

Figure 3: Examples of point configurations and their expressions in the flip-flop calculus. We use an infix notation where the reference system consisting of origin and relatum is in front of the relation symbol and the referent is behind the relation symbol.

The simple flip-flop calculus models “front” and “back” only as linear acceptance regions. Vorwerg et al. (Vorwerg et al. 1997) showed empirically that a cognitively adequate model for projective regions needs acceptance regions for “front” and “back”, which have a similar extent as “left” and “right”. Freksa’s single cross calculus (Freksa 1992b) has this feature (see Figure 4). The front region consists of “left/front” and “right/front”, the left region consists of “left/front” and “left/back”. The intersection of both regions models the left/front relation.

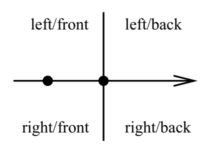

Figure 4: The single cross calculus

For a given calculus one can try to build the closure with respect to a set of operations by iteratively adding the operation results (e.g. potential subsets of the original base relations) to a new set of base relations until a fix point is reached. This construction was performed for Freksa’s single cross calculus by Scivos and Nebel (Scivos and Nebel 2001) for the permutation operations. The resulting calculus is an extension of Freksa’s original Double-Cross calculus (Freksa 1992b). The acceptance regions of the extended Double-Cross calculus are depicted on figure 5.

A first benchmark compares the adequateness of the flip-flop and the Double Cross Calculus with respect to modelling natural language spatial references (see the table on figure 6). The result is that the flip-flop calculus is less adequate for modelling projective predicates than the Double Cross Calculus.

[1]The natural language terms used here are meant to improve the readability of the text. For issues of using QSR representations for modelling natural language expressions please refer to (Moratz and Tenbrink 2006).

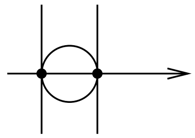

Figure 5: Acceptance regions of the extended Double-Cross calculus

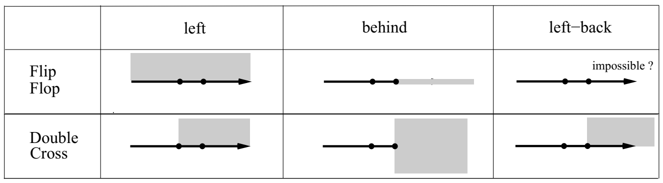

Figure 6: Relative reference by projective predicates for the different calculi

Ambiguous Landmark Problems

In this section we introduce a task which can serve as a benchmark for reasoning with qualitative relative position calculi. In this task ambiguous local landmark observations have to be integrated into survey knowledge. We show that the most prominent relative position calculus, Freksa's Double Cross Calculus can solve a specific instance of this task. The observations can be represented in a constraint network and standard constraint propagation solves the ambiguity problem.

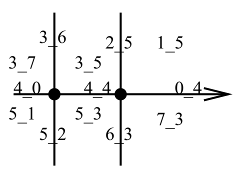

Figure 7: Double Cross reference system/partition

We use for our demonstration the QSR toolbox SparQ (Wallgrün et al. 2007). In this system the original version of the Double Cross Calculus without Thales's circle is used (the relation symbols used in this system can be found on figure 7).

We can use the Double Cross Calculus to represent our local observation based underdetermined spatial knowledge of the robotics example depicted in figure 8. The robot's observation at time point 1 (the red landmarks are close and can be distinguished, the green ones are to far away to be distinguished):

$$R1, R2 \quad (2\_5, 3\_6) \quad G1 \tag{1}$$
$$R1, R2 \quad (2\_5, 3\_6) \quad G2 \tag{2}$$

The robot's observation at time point 2 (the green landmarks are close and can be distinguished, the red ones are to far away to be distinguished):

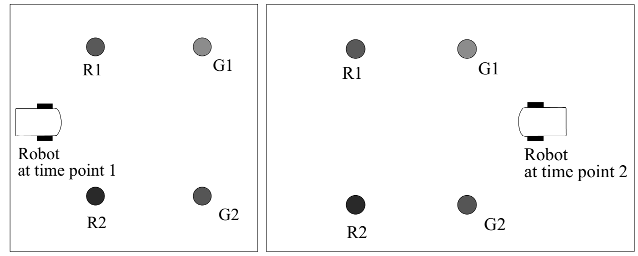

Figure 8: Two observation resulting in ambiguous spatial knowlede

$$G1, G2 \quad (5\_2, 6\_3) \quad R1 \tag{3}$$
$$G1, G2 \quad (5\_2, 6\_3) \quad R2 \tag{4}$$

The observation corresponding to equation (4) can be reformulated:

$$G1, R2 \quad (3\_5, 3\_6) \quad G2 \tag{5}$$

It follows:

$$R2, G1 \quad \textsc{Inv}\,(3\_5, 3\_6) \quad G2 \tag{6}$$
$$R1, R2 \quad (2\_5, 3\_6) \diamond \textsc{Inv}\,(3\_5, 3\_6) \quad G2 \tag{7}$$
$$R1, R2 \quad (3\_5, 2\_5, 1\_5) \quad G2 \tag{8}$$

The conjunction (intersection) of equation (2) and equation (8) yields:

$$R1, R2 \quad 2\_5 \quad G2 \tag{9}$$

This manual deduction shows how the ambiguity is resolved in this landmark configuration. In general the observations can be represented in a constraint network and standard constraint propagation solves the ambiguity problem.

However, since the Double Cross calculus is coarse only special configurations of landmarks can be solved with this formalism. In the configuration which we used for our demonstration the landmarks are arranged as corner points of a rectangle. This rectangular shape corresponds to the structure of the double cross. Landmark configurations which do not follow this structure cannot be disambiguated based on constraint-propagation reasoning with the Double Cross Calculus.

More fine grained calculi like the $\textsc{Gpcc}_m$ calculi described in the next section are capable of solving much more general problems. This approach is ongoing work, first results are promising.

Generalizing ternary point configuration calculi

Applications exist in which finer qualitative acceptance areas are helpful. The possibility to use finer qualitative distinctions can be viewed as a stepwise transition to quantitative knowledge. The idea of using context dependant direction and distance intervals for the representation of spatial knowledge can be traced back to Clementini, di Felice, and Hernandez (Clementini, Di Felice, and Hernandez

1997). However, only special cases of reasoning were considered in their work. Here, we will propose a calculus that makes direct use of general purpose constraint propagation. Thereby robot applications including reasoning about ambiguous perceptions like in our proposed benchmark task will be made possible. In 2-dimensional space, two points *A* and *B* can be used to "localise" a third point *C*; this is relative localisation, which means that no absolute reference system, such as in (Frank 1991), is used: (1) *A* is the origin (which may be, for instance, the speaker's location); (2) *B* is the relatum; and (3) *C* is the reference object. The localisation of *C* relative to *A* and *B* consists then of describing *C* relative to the reference system determined by *A* and *B*. We shall be considering two kinds of relative localisation:

1. Relative distance: how far is *C* from *B* compared to *A*? In other words, how does the distance from *C* to *B* compare with the distance from *A* to *B*?
2. Relative direction: what is the direction of *C* from *B* for an observer placed at *A*? In other words, what is the angle determined by the directed straight lines (BA) and (BC)?

These two relative localisations will then be combined to lead to relative position.

The newly proposed calculus is called granular point configuration calculus GPCC. In this calculus two points are the basis for a reference system. The reference system can be interpreted as a partition of the plane into acceptance regions for a third point. All options for places of the third point which are in the same part of the partition are considered to be in an aquivalence class and are treated in the same way in categorization and reasoning tasks by subsequent modules. One variant of the GPCC calculus and its partition on the plane is shown in figure 9.

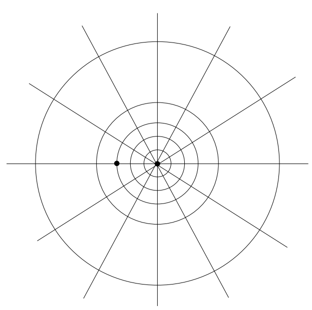

Figure 9: The partition of the GPCC_3-Calculus

To give a precise, geometric definition of the GPCC-relations we describe the corresponding geometric configurations in an analogue way to the TPCC calculus (Moratz and Ragni 2008) on the basis of a Cartesian coordinate system represented by $\mathbb{R}^2$. First we define the special cases for $A = (x_A, y_A)$, $B = (x_B, y_B)$ and $C = (x_C, y_C)$.

$$A, B \text{ dou } C \quad := \quad x_A = x_B \wedge y_A = y_B \wedge (x_C \neq x_A \vee y_C \neq y_A)$$
$$A, B \text{ tri } C \quad := \quad x_A = x_B = x_C \wedge y_A = y_B = y_C$$

For the cases with $A \neq B$ we define a relative radius $r_{A,B,C}$

$$r_{A,B,C} \quad := \quad \frac{\sqrt{(x_C - x_B)^2 + (y_C - y_B)^2}}{\sqrt{(x_B - x_A)^2 + (y_B - y_A)^2}}$$

$$A, B \text{ sam } C \quad := \quad r_{A,B,C} = 0$$

and for $A \neq B \neq C$ a relative angle $\phi_{A,B,C}$:

$$\phi_{A,B,C} \quad := \quad \tan^{-1} \frac{y_C - y_B}{x_C - x_B} - \tan^{-1} \frac{y_B - y_A}{x_B - x_A}$$

The further base relations have an acceptance area depending on the granularity of the calculus to be applied. The calculus shown in figure 9, GPCC_3, has a level of granularity of 3 and 267 relations. A calculus of the granularity level m, described below as GPCC_m, has $(4m-1)(8m)+3$ base relations. The base relations of GPCC_3 are thus defined:

$$
\begin{aligned}
A, B\ {}_3\!\perp^1_0\ C &:= 0 < r_{A,B,C} \leq 1/3 \wedge \phi_{A,B,C} = 0 \\
A, B\ {}_3\!\perp^1_1\ C &:= 0 < r_{A,B,C} \leq 1/3 \wedge 0 \leq \phi_{A,B,C} \leq 1/6\pi \\
A, B\ {}_3\!\perp^1_2\ C &:= 0 < r_{A,B,C} \leq 1/3 \wedge \phi_{A,B,C} = 1/6\pi \\
A, B\ {}_3\!\perp^1_3\ C &:= 0 < r_{A,B,C} \leq 1/3 \wedge 1/6\pi \leq \phi_{A,B,C} \leq 2/6\pi \\
&\vdots \\
A, B\ {}_3\!\perp^1_{23}\ C &:= 0 < r_{A,B,C} \leq 1/3 \wedge \\
&\quad\ 11/6\pi \leq \phi_{A,B,C} \leq 12/6\pi \\
A, B\ {}_3\!\perp^2_0\ C &:= r_{A,B,C} = 1/3 \wedge \phi_{A,B,C} = 0 \\
&\vdots \\
A, B\ {}_3\!\perp^3_0\ C &:= 1/3 \leq r_{A,B,C} \leq 2/3 \wedge \phi_{A,B,C} = 0 \\
&\vdots \\
A, B\ {}_3\!\perp^9_0\ C &:= 3/2 \leq r_{A,B,C} \leq 3/1 \wedge \phi_{A,B,C} = 0 \\
&\vdots \\
A, B\ {}_3\!\perp^{11}_{23}\ C &:= 3/1 \leq r_{A,B,C} \wedge 11/6\pi \leq \phi_{A,B,C} \leq 12/6\pi
\end{aligned}
$$

This schema can be transferred and applied to arbitrary granularity m of a calculus GPCC_m. The general segments $A, B\ {}_m\!\perp^i_j\ C$ are then so defined:

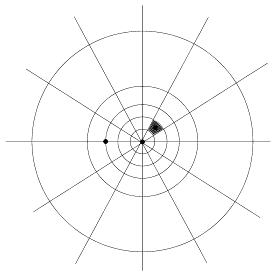

Figure 10: An example configuration of three points A, B, C. The depicted configuration corresponds to $A, B\ {}_3\bot^3_3$

$$
\begin{aligned}
0 \le j \le 8m-2 \wedge j \bmod 2 = 0 &\rightarrow \phi_{A,B,C} = \frac{j}{4m}\pi \\
1 \le j \le 8m-1 \wedge j \bmod 2 = 1 &\rightarrow \frac{j-1}{4m}\pi < \phi_{A,B,C} < \frac{j+1}{4m}\pi \\
1 \le i \le 2m-1 \wedge i \bmod 2 = 1 &\rightarrow \frac{i-1}{2m} < r_{A,B,C} < \frac{i+1}{2m} \\
2 \le i \le 2m \wedge i \bmod 2 = 0 &\rightarrow r_{A,B,C} = \frac{i}{2m} \\
2m+1 \le i \le 4m-3 \wedge \\
i \bmod 2 = 1 &\rightarrow \frac{m}{2m - \frac{i-1}{2}} < r_{A,B,C} < \frac{m}{2m - \frac{i+1}{2}} \\
2m+2 \le i \le 4m-2 \wedge \\
i \bmod 2 = 0 &\rightarrow r_{A,B,C} = \frac{m}{2m - \frac{i}{2}} \\
i = 4m-1 &\rightarrow m < r_{A,B,C}
\end{aligned}
$$

Because we have three arguments, we have 3! = 6 possible ways of arranging the arguments for a transformation. Following Zimmermann and Freksa (Zimmermann and Freksa 1996) we use the following terminology and symbols to refer to these permutations of the arguments (a,b : c):

term	symbol	arguments
identical	ID	a,b : c
inversion	INV	b,a : c
short cut	SC	a,c : b
inverse short cut	SCI	c,a : b
homing	HM	b,c : a
inverse homing	HMI	c,b : a

With ternary relations, one can think of different ways of composing them. However there are only a few ways to compose them in a way such that we can use it for enforcing local consistency (Scivos and Nebel 2001). In trying to generalize the path-consistency algorithm (Montanari 1974), we would like to enforce 4-consistency (Isli and Cohn 2000). We then had to use the following (strong) composition operation:

$$\forall A, B, D:\ A, B\,(r_1 \diamond r_2)\,D \leftrightarrow \exists C:\ A, B\,(r_1)\,C \wedge B, C\,(r_2)\,D$$

Unfortunately, the GPCC$_m$ calculi are not closed under strong composition. For that reason we can not directly enforce 4-consistency. But we can define a weak composition operation $r_1 \diamondsuit r_2$ of two relations r_1 and r_2. It is the most specific relation such that:

$$\forall A, B, D:\ A, B\,(r_1 \diamondsuit r_2)\,D \leftarrow \exists C:\ A, B\,(r_1)\,C \wedge B, C\,(r_2)\,D$$

While using the weak composition we can not enforce 4-consistency we still get usefull inferences.

The problem is calculating the permutation and composition results for such structures by machine. The operation tables can be approximated with the aid of a composition of distance orientation intervals (DOI) (Moratz and Wallgrün 2003). Thereby areal segments and their borders are summarized. Thus one obtains thereby a quasi-partition in which only linear overlappings occur.

The calculi are, with respect to the transformation HMI, closed:

$$\mathrm{HMI}\left({}_m\bot^i_j\right) = {}_m\bot^{4m-i}_{8m-1-j}$$

In robotic applications the relevant areal base relations with their borders are summarized into general relations. Out of this, one obtains a closed region in a plane (with the exception of its exterior segments which continue infinitely) as acceptance area for the third point of a ternary relational proposition. The bounded line segment acceptance areas belong to both neighboring segments and border points typically belong to four segments. All inner segments contain the point which corresponds to the relation sam.

The areal measure of these ambiguous acceptance areas is however 0. In the event that a corresponding border point triple is to be represented qualitatively, a disjunction of all bordering base relations must be used. As a result one obtains then a fine grained quasi-partition for the representation of the relative position of a point with respect to a reference system build by two points.

Obviously, the calculi GPCC$_3$, GPCC$_4$, and GPCC$_5$ can solve more natural instances of the ambiguous landmark problem than the Double Cross Calculus. Which granularity is needed to solve reasonably designed random instances of the ambiguous landmark benchmark is subject to future investigations.

Conclusion

We showed a robotics problem about the disambiguation of landmarks. This disambiguation of landmarks can be achived by constraint-propagation only, since the underdetermined spatial knowledge about the landmark position can be expressed as constraint networks. The Double Cross Calculus is capable to solve a simple instance of this problem. For more general tasks one needs a finer granularity of the position calculus. We presented a first draft of such a calculus which in principle can solve general instances of the landmark disambiguation problem.

With the ambiguous landmark benchmark we have a test case which puts an emphasis on a *qualitative decision* as output of qualitative spatial reasoning based on observed data. From my point of view this is a more natural task than abstract constraint satisfaction problems which try to find spatial instances based on purely abstract input constraints.

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