A Relation Graph Formulation for Relationships Among Agents *

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May 30, 1994

Abstract

It seems apparent that there is a variety of empirical structures in multiagent environments which may be represented by graph as for instance: the communication structure, the structure reflecting the degree of cooperation, the structure reflecting influences between agents, the structure reflecting commitments between agents, the authority structure, etc. In this context, graph theory and associated branches of mathematics, and particularly matrix algebra, provide techniques of computation and formulas for calculating certain quantitative and qualitative features of empirical structures.

This paper presents a formal model which can serve as mathematical model of the structural properties of any empirical multiagent system consisting of relationships among pair of agents. Precisely, our work consists of a formal model based on binary relations and graphs. This model has a high computational value since it is essentially based on matrix algebra. In addition, this model takes into account fuzzy relations as for example: none, some, much and a lot. Finally, the proposed model provides a framework for the reasoning about others, the negotiation and communication between agents . We conclude by discussing our initial experiments about a crossroads in which interactions between cars are represented by a cognitive structure.

*This research was supported by the Natural Sciences and Engineering Research Council of Canada (NSERC) and by the Fonds pour la Formation des Chercheurs et l'aide à la Recherche (FCAR) du Québec.

1 Introduction

A central issue in multiagent environment is how to get autonomous agents, each of whom has its goals, actions and plans, to model each other in order to reason about others' activities. Reasoning about others allows agents to coordinate their activities for their mutual benefit [5]. This paper summarizes a cognitive map (or causal graph) [1, 7, 12], that agents can use to model each other in order to make decision in multiagent environments. A cognitive map (CM) is a direct graph in which nodes represent problem constructs and edges or arrows represent dependence or influence between constructs. According to this definition, it seems that in multiagent environments, cognitive mapping (i.e. the process of constructing CM from a given environment):

- enables a local agent to get a sufficient view of its constantly changing distributed environment;
- seems to be a natural way to make and to analyze decisions, taking into account causal relations between agents' goals, actions and utilities.

In fact, cognitive mapping techniques have been used for decision analysis in the fields of international relations [1, 3], administrative sciences [8], management sciences [10], and the distributed group decision support [12]. In this latter context, Zhang and his colleagues provided the notions of NPN (Negative-Positive-Neutral) logic, NPN relations, coupled-type neurons, and coupled-type neural networks. Based on these notions, D-POOL, a cognitive map architecture for the coordination of distributed cooperative agents is constructed. Precisely, D-POOL consists of a collection of distributed nodes, each node is a CM system coupled with a local expert/database system (called agent). To solve a problem, a local node first pools cognitive maps from relevant agents into a NPN relation that retains both negative and positive assertions. Then new cognitive maps are derived and focuses of attentions are generated. With the focuses, a solution is proposed by the local node and passed to the remote systems. Based on their view points, the remote systems respond to the proposal and D-POOL strives for a cooperative or compromised solution through coherent communication and perspective sharing. In summary, D-POOL focuses on CM composition, derivation and not on relationships among agents. In addition, it uses NPN fuzzy relations based on quantitative description, and generally it is not easy to find crisp quantities in real applications.

Our objective in this work is to present a formal model which can serve as mathematical model of the structural properties of any empirical multiagent system consisting of relationships among pair of agents. Precisely, our work consists of a formal theory based on binary relations and graphs. This theory has a high computational value since it is essentially based on matrix algebra. In addition, the fuzzy relations are symbolic in order to make reference to the empirical world. Finally, the proposed model uses the symbolic reasoning, and provides a framework for the reasoning about others, the negotiation and communication between agents.

The remainder of this paper is organized as follows. The next section presents graphs and structures in multiagent environment. We then describe in section 3, our theory for structures based on the calculus of relations. In section 4, we infer the properties of a cognitive structure (or cognitive map) in multiagent environments. Finally, we conclude by discussing our initial experiments about a crossroads in which interactions between cars are represented by a cognitive structure.

2 Graphs and Structures in Multiagent Environments

Our goal in this paper is to present some mathematical theory about the notion of "structure" of a multiagent environment. This theory is concerned with patterns of relationships among pairs of abstract elements. In this context, the graph theory combined with matrix algebra is certainly the convenient mathematical tool for study this type of structure.

Consider, for example, the "communication structure" of a group of agents. We may conceive of each member of the group as an "element" and the fact that a particular member of the group can communicate directly to another as a "relationship". Then, upon coordinating these empirical entities and relationships to the abstract terms of graph theory, we obtain a graph which represents the communication structure of the group. The properties of this graph, with which graph theory is concerned, are at the same time properties of the communication structure.

As another example, consider a structure which reflects the degree of cooperation between many agents. In this case, the elements of the structure are agents' utilities, and the relationships are the fuzzy relations about the cooperation among agents. Notice that the degree of cooperation characterizes the amount of cooperation between agents that can range from fully cooperative (i.e., total cooperation) to antagonistic.

Thus, it seems apparent that there is a variety of empirical structures in multiagent environments which may be represented by graphs as for instance: the structure of a distributed plan, the distribution of roles, the authority structure of an organization, the net of commitments among agents, etc. Or, graph theory and associated branches of mathematics, and particularly matrix algebra, provide techniques of computation and formulas for calculating certain quantitative and qualitative features of empirical structures.

In the distributed group decision support, a graph should capture the structure of the agent's causal assertions and generate the consequences that follow from this structure. This graph is generally called cognitive map. This map has only two basic types of elements: concepts and causal beliefs. The concepts are treated as variables, and causal beliefs are treated as relationships between variables. A concept variable is something like "a great amount of fuel" or "a small amount of fuel". A cognitive map allows great flexibility in the variables, such as the existence or non-existence of something. But whatever type of concept is represented, it is always regarded as a variable that can take more than one value. Cognitive maps frequently contain concept variables to represent *utility*. Utility means the unspecified best interests of an actor, such as an agent, group or organization.

The second type of basic element in a cognitive map is a causal assertion. Causal assertions are regarded as relating variables to each other, as in the assertion that "the amount of fuel of agent3 promotes its ability to cooperate for task4". Here the causal variable is "the amount of fuel of agent3" and the effect variable is "the ability of agent3 to cooperate for task4". The relationship between these two variables is indicated by the word "promotes". A relationship can be "positive" (R^+), "negative" (R^-), "neutral" (R^0), "neutral or negative" (R_0^-), "neutral or positive"(R_0^+), "nonneutral"(R_-^+), "positive, neutral or negative", that is "universal" (R^u).

In this case, causal links between two concepts v_i and v_j can have one of the following seven values.

Causal Relations	Descriptions
$v_i \xrightarrow{R^+} v_j$	v_i promotes v_j, v_i helps v_j, v_i is benefit to v_j, etc,
$v_i \xrightarrow{R^-} v_j$	v_i retards v_j, v_i hurts v_j, v_i prevents v_j, v_i is harmful to v_j, etc,
$v_i \xrightarrow{R^0} v_j$	v_i has no effect on v_j, v_i does not matter for v_j, etc,
$v_i \xrightarrow{R_0^+} v_j$	v_i does not retard v_j, v_i will not hurt v_j, v_i does not prevent v_j, etc,
$v_i \xrightarrow{R_-^0} v_j$	v_i does not promote v_j, v_i will not help v_j, v_i is of no benefit to v_j, etc,
$v_i \xrightarrow{R_-^+} v_j$	v_i has any effect on v_j, v_i affects in some non-zero way v_j, etc,
$v_i \xrightarrow{R^u} v_j$	R^+, R^- and R^0 can exist between variables v_i and v_j

For the sake of understanding the cognitive map in multiagent environments, here are some definitions that should be remembered.

Definitions:

1. A *cognitive map* is a directed graph which represents an agent's assertions about its beliefs with respect to its environment and more particularly to beliefs about other agents. The components of this graph are the points and the arrows between those points.

2. A *point* represents a concept variable, which may be a plan option or a goal option of any agent, the utility of any agent (or its organization), or any other concept that may take on different values in the context of multiagent reasoning.

3. An *arrow* represents causal relations between concepts. Precisely, an arrow represents a causal assertion of how one concept variable affects another. As stated previously, we have introduced seven causal links between two concept variables v_i and v_j: R^+, R^-, R^0, R_0^+, R_-^0, R_-^+ and R^u.

4. A *path* from variables v_1 to v_n is a collection of distinct points, v_1, v_2, ..., v_n, together with the arrows v_1v_2, v_2v_3 ..., $v_{n-1}v_n$. Notice that a path is *trivial* if it consists of a single point.

5. A *cycle* is a nontrivial path together with a line from the terminal to the initial point of the path. A positive cycle, that is one with an even number of negative (R^-) arrows, is deviation-amplifying because any change in one of its points will eventually be reflected back at that point as an additional indirect effect in the same direction as the original change. Conversely, a negative cycle (one with an odd number of negative arrows) is deviation-counteracting.

6. The *total effect* of v_i on variable v_j is the sum of the indirect effect of all the paths and cycles from v_i to v_j.

7. A *balanced graph* is a graph that has no universal nor non-neutral relation for a total effect of any pair of points.

8. A *acyclic graph* is a graph which has no cycle.

9. The *valency matrix* A of a CM is a square matrix of size n, where n is the number of concepts in the corresponding cognitive map. Each element a_{ij} can take on the values R^+, R^-, R^{+}_{-}, etc, to characterize the relationships between elements i and j of A. The diagonal elements are considered to be R^0.

10. The *degree of a variable* v_i is the ordered pair $[od(v_i), id(v_i)]$. The *outdegree* of concept v_i, $od(v_i)$, precises the number of concepts affected directly by concept v_i. The *indegree* of concept v_i, $id(v_i)$, precises the number of concepts which affect concept v_i directly. The sum of the indegree and indegree for concept v_i gives the total degree (td) of concept v_i: $td(v_i) = id(v_i) + od(v_j)$.

11. The *reachability matrix* R of a CM is a matrix whose entries are denoted r_{ij}, where $r_{ij} = 1$ if v_j is reachable from v_i, and $r_{ij} = 0$ otherwise.

Now it is important to say how a cognitive map permits to represent relations between agents. Consider for example a multiagent environment where, for three tasks (T_1, T_2, T_3), four agents (A_1, A_2, A_3, A_4) present the capacities summarized in matrix C. In this matrix $c_{ij} = 1$ if A_j has the capacity to achieve the task T_i, and $c_{ij} = 0$ otherwise.

$$C = \begin{array}{c} \\ T_1 \\ T_2 \\ T_3 \end{array} \begin{array}{c} \begin{array}{cccc} A_1 & A_2 & A_3 & A_4 \end{array} \\ \begin{pmatrix} 1 & 1 & 0 & 0 \\ 1 & 0 & 0 & 1 \\ 0 & 1 & 1 & 0 \end{pmatrix} \end{array}$$

Thus, task T_1 for instance can be executed by A_1 or by A_2, and agents have to know how A_1 or A_2 influence them before to allocate T_1. These influences can be reflected in two cognitive maps like those of Figure 1 where a) reflects relations between agents in the case where A_1 will be the actor, and b) the case where A_2 will be the actor.

The valency matrix ($A(1)$) and the reachability matrix ($R(1)$), for the case where A_1 will be the actor, are:

$$A(1) = \begin{array}{c} \\ A_1 \\ A_2 \\ A_3 \\ A_4 \end{array} \begin{array}{c} \begin{array}{cccc} A_1 & A_2 & A_3 & A_4 \end{array} \\ \begin{pmatrix} R^0 & R^- & R^+ & R^+ \\ R^0 & R^0 & R^0 & R^0 \\ R^0 & R^0 & R^0 & R^0 \\ R^0 & R^0 & R^0 & R^0 \end{pmatrix} \end{array} \qquad R(1) = \begin{array}{c} \\ A_1 \\ A_2 \\ A_3 \\ A_4 \end{array} \begin{array}{c} \begin{array}{cccc} A_1 & A_2 & A_3 & A_4 \end{array} \\ \begin{pmatrix} 1 & 1 & 1 & 1 \\ 0 & 1 & 0 & 0 \\ 0 & 0 & 1 & 0 \\ 0 & 0 & 0 & 1 \end{pmatrix} \end{array}$$

In this context, the outdegree and the indegree of agents are summarized in the following matrix.

$$\begin{array}{c} \\ A_1 \\ A_2 \\ A_3 \\ A_4 \end{array} \begin{array}{c} \begin{array}{cc} od & id \end{array} \\ \begin{pmatrix} 3(2+, 1-) & 0 \\ 0 & 1(1-) \\ 0 & 1(1+) \\ 0 & 1(1+) \end{pmatrix} \end{array}$$

Thus, if A_1 will be the actor of task T_1, it will affect positively two agents (A_2 and A_3) and negatively the agent A_3.

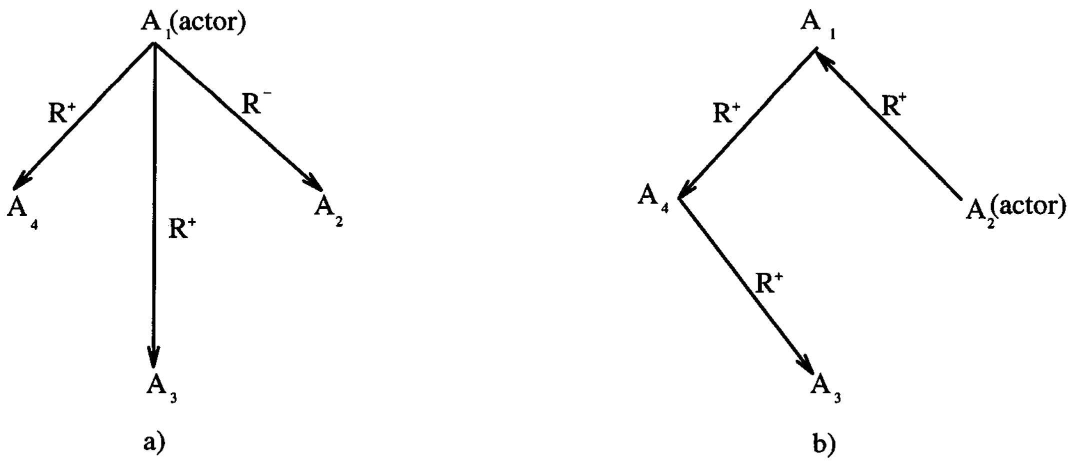

Figure 1: An Example of Influences between Agents

On the other hand, the valency matrix ($A(2)$) and the reachability matrix ($R(2)$, corresponding to the cognitive map of b) (Figure 1), are:

$$A(2) = \begin{array}{c} \\ A_1 \\ A_2 \\ A_3 \\ A_4 \end{array} \begin{array}{c} \begin{array}{cccc} A_1 & A_2 & A_3 & A_4 \end{array} \\ \begin{pmatrix} R^0 & R^0 & R^0 & R^+ \\ R^+ & R^0 & R^0 & R^0 \\ R^0 & R^0 & R^0 & R^0 \\ R^0 & R^0 & R^+ & R^0 \end{pmatrix} \end{array} \qquad R(2) = \begin{array}{c} \\ A_1 \\ A_2 \\ A_3 \\ A_4 \end{array} \begin{array}{c} \begin{array}{cccc} A_1 & A_2 & A_3 & A_4 \end{array} \\ \begin{pmatrix} 1 & 0 & 1 & 1 \\ 1 & 1 & 0 & 1 \\ 0 & 0 & 1 & 0 \\ 0 & 0 & 1 & 1 \end{pmatrix} \end{array}$$

For this case, the outdegree and indegree of the agents are the following:

$$\begin{array}{c} \\ A_1 \\ A_2 \\ A_3 \\ A_4 \end{array} \begin{array}{c} \begin{array}{cc} od & id \end{array} \\ \begin{pmatrix} 1(1+) & 1(1+) \\ 1(1+) & 0 \\ 0 & 1(1+) \\ 1(1+) & 1(1+) \end{pmatrix} \end{array}$$

Thus, if A_2 will be the actor of task T_1, it will affect positively the agent (A_1), which in turn affects positively A_4, which in turn will affect positively A_3.

In summary, cognitive maps with their valency matrices like those of Figure 1 can allow agents to negotiate the allocation of T_1.

3 Towards a new Theory for Structures based on the Calculus of Relations

The mathematical treatment of relations is said to have its origin in Aristote, but its modern study starts with the contributions of G. Boole, A. de Morgan, and C. P. Peirce. Their work was continued in systematic way notably by E. Schröder who around 1890 published his three volumes of *Algebra der Logik.* Half a century later A. Tarski and J. C. C. McKinsey, among others, laid the foundations for the modern calculus of relations.

In ordinary language relation is used to indicate an aspect that *connects* two or more things. The relation "is a father" is defined on a set of persons; the relation "less than" can be defined on the set N of natural numbers. Generally, a relation either does, or does not, hold between two elements; 5 is less than 7, but it is not true that 5 is less than 3. By this alternative, a relation partions the set of pairs of elements of the given set V into those for which the relations hold, and those for which it does not. The relation "is succeeded by" on N corresponds to the subset

$$S := \{(0,1),(1,2),(2,3),...\} \subset N \times N$$

Relations on finite sets can also be represented by tables or by matrix, as shown by the following tables and Boolean matrix about the relation "congruent modulo 2" on the set $1, 2, ..., 7 \subset N$.

	1	2	3	4	5	6	7
1	x			x			x
2		x			x		
3			x			x	
4	x			x			x
5		x			x		
6			x			x	
7	x			x			x

$$\begin{array}{c} \\ 1 \\ 2 \\ 3 \\ 4 \\ 5 \\ 6 \\ 7 \end{array} \begin{array}{c} \begin{array}{ccccccc} 1 & 2 & 3 & 4 & 5 & 6 & 7 \end{array} \\ \begin{pmatrix} 1 & 0 & 0 & 1 & 0 & 0 & 1 \\ 0 & 1 & 0 & 0 & 1 & 0 & 0 \\ 0 & 0 & 1 & 0 & 0 & 1 & 0 \\ 1 & 0 & 0 & 1 & 0 & 0 & 1 \\ 0 & 1 & 0 & 0 & 1 & 0 & 0 \\ 0 & 0 & 1 & 0 & 0 & 1 & 0 \\ 1 & 0 & 0 & 1 & 0 & 0 & 1 \end{pmatrix} \end{array}$$

1	{1,4,7}
2	{2,5}
3	{3,6}
4	{1,4,7}
5	{2,5}
6	{3,6}
7	{1,4,7}

Relations and graph are closely related. Any homogeneous relation [1] can be interpreted as graph and vice versa. One usually represents a relation R on a set V graphically as follows: the elements of V are plotted as points and two points $x, y \in V$ are joined by an arrow having x as its *tail* and y as its *head* precisely when $(x, y) \in R$. A relation depicted in such a way is called a graph and if this graph captures the cognitive structure of an agent it is called a cognitive map. We are working towards a formal model based on binary relations to derive decisions with a cognitive map. This model is now described.

Relations are sets, and consequently we can consider their intersection, union, complement, and inclusion. With respect to these four operations the set of all relations on the set V, namely $\mathbf{2}^{V \times V}$, is a complete Boolean lattice [9]. We are using the symbols $(\vee, \wedge), (\cup, \cap)$, and $(\sqcup, \sqcap)$ for the two Boolean operations on truth values, subsets and relations, respectively. Thus, binary relations allows us to represent firstly R^+, R^- and R^0 by the following matrices:

$$R^+ = \begin{array}{c} \\ + \\ 0 \\ - \end{array} \begin{array}{c} \begin{array}{ccc} + & 0 & - \end{array} \\ \begin{pmatrix} 1 & 0 & 0 \\ 0 & 1 & 0 \\ 0 & 0 & 1 \end{pmatrix} \end{array} \quad R^- = \begin{array}{c} \\ + \\ 0 \\ - \end{array} \begin{array}{c} \begin{array}{ccc} + & 0 & - \end{array} \\ \begin{pmatrix} 0 & 0 & 1 \\ 0 & 1 & 0 \\ 1 & 0 & 0 \end{pmatrix} \end{array} \quad R^0 = \begin{array}{c} \\ + \\ 0 \\ - \end{array} \begin{array}{c} \begin{array}{ccc} + & 0 & - \end{array} \\ \begin{pmatrix} 0 & 1 & 0 \\ 0 & 1 & 0 \\ 0 & 1 & 0 \end{pmatrix} \end{array}$$

[1] Relations between elements of the same set are called homogeneous

Notice that in these matrices, $+$ can mean "increase", "has more chance to happen", etc. Similarly, $-$ can mean "decrease", "has less chance to achieve", etc. Finally, 0 can indicate "no change" or "constant". .

Secondly, we use the operation $\sqcup$ on matrices R^+, R^- and R^0 to determine R_0^+, R_-^0 and R_-^+. Generally, let $R, S \subset V \times V$ be relations; their union is given by:

$$R \sqcup S \stackrel{\text{def}}{=} \{(x,y) \mid (x,y) \in R \vee (x,y) \in S\}$$

This definition allows us to state: $R_0^+ = R^+ \sqcup R_0$, $R_-^0 = R^0 \sqcup R_-$, $R_-^+ = R^+ \sqcup R_-$ and $R^u = R^0 \sqcup R^+ \sqcup R^-$. The matrices relative to these statements are:

$$R_0^+ = \begin{array}{c} \\ + \\ 0 \\ - \end{array} \begin{array}{c} \begin{array}{ccc} + & 0 & - \end{array} \\ \begin{pmatrix} 1 & 1 & 0 \\ 0 & 1 & 0 \\ 0 & 1 & 1 \end{pmatrix} \end{array} \qquad R_-^0 = \begin{array}{c} \\ + \\ 0 \\ - \end{array} \begin{array}{c} \begin{array}{ccc} + & 0 & - \end{array} \\ \begin{pmatrix} 0 & 1 & 1 \\ 0 & 1 & 0 \\ 1 & 1 & 0 \end{pmatrix} \end{array}$$

$$R_-^+ = \begin{array}{c} \\ + \\ 0 \\ - \end{array} \begin{array}{c} \begin{array}{ccc} + & 0 & - \end{array} \\ \begin{pmatrix} 1 & 0 & 1 \\ 0 & 1 & 0 \\ 1 & 0 & 1 \end{pmatrix} \end{array} \qquad R^u = \begin{array}{c} \\ + \\ 0 \\ - \end{array} \begin{array}{c} \begin{array}{ccc} + & 0 & - \end{array} \\ \begin{pmatrix} 1 & 1 & 1 \\ 0 & 1 & 0 \\ 1 & 1 & 1 \end{pmatrix} \end{array}$$

Once the relationships between all the variables are determined, the cognitive map can be drawn. Relationships that are in sequence form paths, and paths transmit indirect effects. The operation of combining direct effects of relationships that are in sequence into indirect effects of path is called composition or multiplication. More generally, let $R, S \subset V \times V$ be relations, their composition or multiplication is given by:

$$R \circ S \stackrel{\text{def}}{=} \{(x,z) \in V \times V \mid \exists y \in V : (x,y) \in R \wedge (y,z) \in S\}$$

Notice that the product $R \circ S$ is also written as RS and therefore $R \circ R$ is written as R^2. We will simply write RS for the product and $R^2, R^3...$ for the powers of R. Notice also that finite relation can conveniently be represented by a matrix. In this case the product consists to form the conjunction element by element, of the x-row of R with the z-column of S, and then form the disjunction of all these results with respect to y:

$$(R_{xx} \wedge S_{xz}) \vee (R_{xy} \wedge S_{yz}) \vee (R_{xz} \wedge S_{zz}) = \exists i : R_{xi} \wedge S_{iz} = (R \circ S)_{xz}$$

In addition, important properties of $\circ$ are: 1) $\circ$ distributes over $\sqcup$ (for example $R^- R_0^+ = (R^-)(R^- \sqcup R^0) = (R^- R^-) \sqcup (R^- R^0) = R^+ \sqcup R^0 = R_0^+$) and, 2) $\circ$ is symmetric (for example, $R_-^0 R^- = R^- R_-^0$).

The rules of composition or multiplication that could be used for combining direct effects are given by the following matrix:

$$
\circ : \quad \begin{array}{c c}
 & \begin{matrix} R^0 & R^+ & R^- & R^0_- & R^+_0 & R^+_- & R^u \end{matrix} \\
\begin{matrix} R^0 \\ R^+ \\ R^- \\ R^0_- \\ R^+_0 \\ R^+_- \\ R^u \end{matrix} &
\begin{pmatrix}
R^0 & R^0 & R^0 & R^0 & R^0 & R^0 & R^0 \\
R^0 & R^+ & R^- & R^0_- & R^+_0 & R^+_- & R^u \\
R^0 & R^- & R^+ & R^+_0 & R^0_- & R^+_- & R^u \\
R^0 & R^0_- & R^+_0 & R^+_0 & R^0_- & R^u & R^u \\
R^0 & R^+_0 & R^0_- & R^0_- & R^+_0 & R^u & R^u \\
R^0 & R^+_- & R^+_- & R^u & R^u & R^+_- & R^u \\
R^0 & R^u & R^u & R^u & R^u & R^u & R^u
\end{pmatrix}
\end{array}
$$

When two or more paths start from some variable x and all end at another variable y, their effects can be added into a total effect of x on y. The operation on relations in this case is $\sqcup$ (corresponding to the sum operation introduced in definition 6 of section 2). The rules governing addition of effects of paths are given by the following truth matrix:

$$
\sqcup : \quad \begin{array}{c c}
 & \begin{matrix} R^0 & R^+ & R^- & R^0_- & R^+_0 & R^+_- & R^u \end{matrix} \\
\begin{matrix} R^0 \\ R^+ \\ R^- \\ R^0_- \\ R^+_0 \\ R^+_- \\ R^u \end{matrix} &
\begin{pmatrix}
R^0 & R^+_0 & R^0_- & R^0_- & R^+_0 & R^u & R^u \\
R^+ & R^+_- & R^+_- & R^u & R^+_0 & R^+_- & R^u \\
R^0_- & R^+_- & R^- & R^0_- & R^u & R^+_- & R^u \\
R^0_- & R^u & R^0_- & R^0_- & R^u & R^u & R^u \\
R^+_0 & R^+_0 & R^u & R^u & R^+_0 & R^u & R^u \\
R^u & R^+_- & R^+_- & R^u & R^u & R^+_- & R^u \\
R^u & R^u & R^u & R^u & R^u & R^u & R^u
\end{pmatrix}
\end{array}
$$

Until now, we have considered a model based on NPN crisp logic. This model is necessary to solve and analyze the distributed decision-making problem. However, it might be not sufficient since causality and influence between agents are generally fuzzy and admits many degrees, and vague degrees such as: none, very little, some, sometimes, much, a lot, usually, more or less, etc.

In this paper, we restrict the fuzzy scale to none, some, much and a lot, in order to give an indication on how our model, based on binary relations, takes into account fuzzy relations. Precisely, *none, some, much* and *alot* are given by the following matrices:

$$
none = \begin{pmatrix} 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 1 \end{pmatrix} \qquad some = \begin{pmatrix} 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 \\ 0 & 0 & 1 & 0 \\ 0 & 0 & 0 & 1 \end{pmatrix}
$$

$$
much = \begin{pmatrix} 0 & 0 & 0 & 0 \\ 0 & 1 & 0 & 0 \\ 0 & 0 & 1 & 0 \\ 0 & 0 & 0 & 1 \end{pmatrix} \qquad alot = \begin{pmatrix} 1 & 0 & 0 & 0 \\ 0 & 1 & 0 & 0 \\ 0 & 0 & 1 & 0 \\ 0 & 0 & 0 & 1 \end{pmatrix}
$$

With these definitions, we also introduce the operator $\subseteq$ (inclusion) between two relations R, S.

$$R \subseteq S \stackrel{\text{def}}{=} \forall x, y : [(x, y) \in R \rightarrow (x, y) \in S].$$

Obviously, if $R \subseteq S$ then $S \supseteq R$. In addition, the previous definition allows us to state:

$$none \subseteq some \subseteq much \subseteq alot$$

Now let $P = \{none, some, much, alot\}$ and $R, S \in P$ be relations. Furthermore, let $\otimes$ the operation that is used for combining direct fuzzy effects, and the operation $\oplus$ that is used for combining total fuzzy effect. In fact, $\otimes$ and $\oplus$ may be any respective T-norm (T) (i.e. triangular norm) and T-conorm (S) extended from interval $[0, 1]$ to P (see [2] for definitions and properties of T and S). Yager [11] proves the following maximality/minimality property of *min* and *max* for fuzzy logic:

$$T \subseteq min \subseteq max \subseteq S$$

Thus, T-norms other than *min* are never more causally lenient than *min*, and S-conorms other than *max* are never more causally stringent than *max*. This justifies selecting *min* and *max*, as default when little is known about relations. In our model, the $min(R, S)$ operation can be obtained by the $\circ$ operation between R and S. Similarly, the *max* can obtained by the $\sqcup$ operation. This justifies selecting $\circ$ and $\sqcup$ for $\otimes$ and $\oplus$. In summary, we have:

$$\forall R, S \in P, \qquad R \otimes S = min(R, S) = R \circ S$$

$$\forall R, S \in P, \qquad R \oplus S = max(R, S) = R \sqcup S$$

Thus for instance, $much \circ some = some$, $some \circ none = none$ and $much \sqcup some = much$, $some \sqcup none = some$ etc.

In summary, if causality is not fuzzy, we use NPN crisp logic to determine indirect total effect and total effect. In this case causality between concepts has the following form:

$$concept_i \xrightarrow{R^+} concept_j \tag{1}$$

If however the causality is fuzzy, relation between concepts is given by

$$concept_i \xrightarrow{R^+:s} concept_j \tag{2}$$

Or by

$$concept_i \xrightarrow{R^+_-:s,m} concept_j \tag{3}$$

Or by

$$concept_i \xrightarrow{R^u:s,m,n} concept_j \tag{4}$$

Notice that (1) means that $concept_i$ promotes, helps, etc, $concept_j$ and that this influence is not fuzzy. This total effect can result from the addition of several products of indirect effects. A product is obtained by the $\circ$ operation on crisp terms R^+, R_-, R^0, etc, and the addition is obtained by the $\sqcup$ operation (on crisp terms).

A representation like (2) means that $concept_i$ promotes, helps, etc, $concept_j$ and that the influence between these two concepts is fuzzy and is characterized by s (i.e. some). A such representation can result from the addition of several products of indirect effects. In this case however, we distinguish two terms: one is crisp and is obtained by the addition of some products on crisp terms, and the second is fuzzy and is obtained by the addition of some products on fuzzy terms.

Similarly, a representation like (3) means that $concept_i$ has any effect on $concept_j$ and that the negative and positive effects are respectively characterized by s (i.e. some) and by m (i.e. much). This total effect is obtained by the combination of $\sqcup$ and $\circ$ on crisp terms to determine R^+_- and on fuzzy terms to determine s, m.

Finally, (4) means that R^-, R^0 and R^+ can exist between $concept_i$ and $concept_j$ and the force of these influences are respectively s, m, n (i.e. some, much and none). This total effect between $concept_i$ and $concept_j$ is obtained by the combination of $\sqcup$ and $\circ$ on crisp terms to determine R^u and on fuzzy terms to determine s, m, n.

4 Inferring the Properties of a Cognitive Map in Multiagent Environments

Now it is important to say how a cognitive map can contribute to reasoning about others in a multiagent environment. In fact, concepts in cognitive map such as "decision variables", "goals", "actions", "utility", etc. may be the concepts of the decision-maker, the concepts of another agent on which the decision maker has to reason in order to make its decision, or the concepts of a group or an organization on which the decision maker has to decide.

It is also important to say how to solve the decision-making problem in a multiagent environment. Generally, given a cognitive map with one or more decision variables and a *utility* variable, which decision should be chosen and which should be rejected? To achieve this, the concerned agent should calculate the total effect of each decision on the utility variable. Those decisions that have a positive total effect on utility should be chosen, and decisions that have a negative total effect should be rejected. If many decisions have a positive total effect, those that have the sub-set fuzzy terms should be chosen. Decisions with a nonnegative total effect should not be rejected, decisions with a nonpositive total effect should not be accepted. All this depend on fuzzy terms and on adopted strategies on how to choose.

To solve the decision-making problem, an agent generally needs to analyze its cognitive map or the cognitive map of its organization if this map is available. In fact, the cognitive map relevant to reasoning in multi-agent environment being analyzed is converted to the form of a *valency matrix* V. This matrix is a square matrix of size n, where n is the number of concepts in the corresponding cognitive map (see definition 9 of section 2). Each element v_{ij}, consisting of *crisp term:fuzzy terms*, characterizes the relationships between elements i and j. The valency matrix has a number of useful properties [6]. With the valency matrix, we can calculate indirect paths of length 2, 3, 4, etc. Matrices relative to these indirect paths are: V^2 (i.e. $V \circ V$), V^3, V^4, etc. Finally, the total effect matrix T for an acyclic cognitive map can be calculated by:

$$T = \bigsqcup_{i=1}^{n-1} V^i$$

This total effects matrix, can be used for generating advice based on the total effect of each plan, action or strategy on the utility variable. More generally, the elements of T may be used to guide a dynamic decision process until one goal is reached. Based on the calculation of T, we have the possibility to solve:

1. problems of a given cognitive map like this one: "could $concept_i$ be strengthened if $concept_j$ is strengthened?", "could $concept_i$ be weakened if $concept_j$ is strengthened?", etc.;

2. the problem of changes which can be formulated by: if certain crisp and/or fuzzy relations change, what will happen to the considered cognitive map?

3. the explanation problem which consists to find consistent explanations with the observed changes.

In multiagent environment, the emerging potential for cooperation can be also study by analyzing agents' cognitive maps. This suggests that cognitive map *analysis* can be used to explore the following: 1) How do the agent's characteristics (e.g. its rationality, its sincerity, its benevolent, etc.) affect its cognitive map? 2) How the conflicts of interest between agents are affected by the alteration of perceptions of specific causal links? 3) How do efficiently focus on the patterns of similarities and differences between agents with respect to both their causal beliefs and their positions? How do separately analyze the cognitive maps in terms of complexity, density, imbalance and inconsistencies relative to salient goals?

The Objective behind this analysis is to locate groupings of goals that could be result in a *coalition of agents* on the issue of establishment of a cooperation.

5 Example

In order to keep the discussion on a concrete level, consider the use of the cognitive map in Figure 3 concerning a crossroads scenario (Figure 2). The implementation and experiments about this scenario are discussed in [4]. Here, we only explain how our relation graph formulation takes into account interactions between cars considered as intelligent agents. To do this, we assume that in our scenario of figure 2, there is no policeman and that traffic lights are out. Consequently, agents are in unfamiliar situation in which each participant has to reason about others in order to coordinate its behaviors with others. Figure 3 is a part of this scenario. Precisely, this is part of an actual agent's `C5` cognitive map which is assumed to be the decision maker which reasons about other agents: `C7`, `C2` and `C3`. This cognitive map is constructed according to the following arriving order: `C2`, `C7`, `C5`, `C3`. With this cognitive map, `C5` determines the relation between `C2` and itself (the dashed arrow). Since this influence is positive, agent `C5` evaluates two plans: `Plan1` and `Plan2`. The first plan suggests that `C5` passes at the same time as `C2` (instead of passing after `C7`) by asking `C7` its permission. The second plan suggests that it should wait its turn. To make its decision, `C5` considers `Plan1` and `Plan2` as policy concepts and calculates its total effect matrix T.

This matrix allows it to know effects of `Plan1` and `Plan2` on its utility and on others' utilities. In our example, `Plan1` has the effect $+:m$ on `C5`'s utility and has no effects on others' utilities; and `Plan2` has the effect $+:s$ on `C5`'s utility and has no effects on others' utilities. Consequently, `C5` decides to pass at the same time as `C2` (instead of passing after `C7`) by asking `C7` his permission.

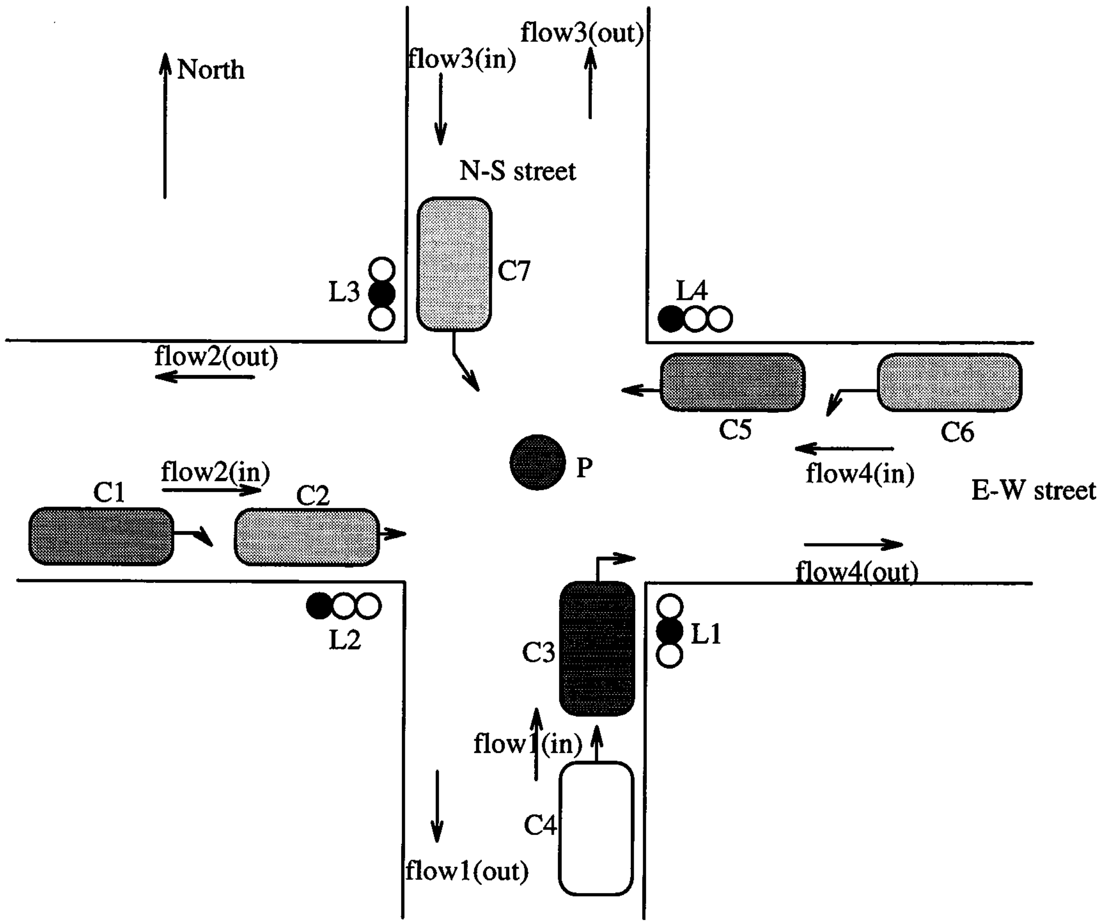

Figure 2: A Multiagent Scenario in Urban Traffic: The Crossroads. `C denotes a car, L traffic-lights, and P the place of a policemen`

6 Conclusion

We have presented a relation graph formulation about the notion of "structure" in a multiagent environment. In this context, a formal model based on binary relations and graphs, and taking into account fuzzy relations was proposed. This model allows agents 1) to make a decision in a multiagent situation, 2) to negotiate tasks and resources allocations, 3) to reason about others in order to cooperate or to coordinate its activities, etc. Preliminary overview of our experiments about crossroads were presented.

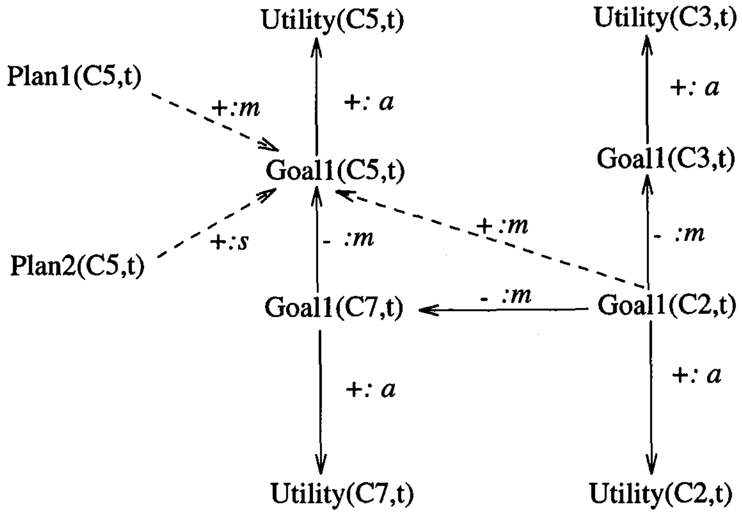

Figure 3: Example of a Cognitive Map in a Crossroads Scenario

This research marks an important step in the use of a mathematical theory, such as graph and relations, to real-world DAI environments, because it seems that those environments need a more rigorous treatment of the features of their empirical structures.

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