A deductive study of the C-CLASSIC$_{\delta\epsilon}$ Description Logic *

Véronique Ventos
LIPN-CNRS URA 1507,
Université Paris-Nord,
93430 Villetaneuse, France,
email: veronique.ventos@ura1507.univ-paris13.fr

Abstract

This paper presents C-CLASSIC$_{\delta\epsilon}$, a Description Logic (DL) which is expressive enough to be practically useful and which can handle default knowledge.
C-CLASSIC$_{\delta\epsilon}$ has been given an intensional semantics (CL$_{\delta\epsilon}$) in which concepts are denoted by a normal form of the set of their properties (rather than the set of their instances as is the case in model-theoric semantics). Therefore, the subsumption algorithm is based on computations and comparisons of elements of CL$_{\delta\epsilon}$, thus giving a good adequacy between the polynomial-time subsumption algorithm and the semantics and allowing the soundness and completeness of the algorithm to be established.

Key-words : Default knowledge, intensional semantics, subsumption algorithm.

1 Introduction

The aim of this paper is to present some theoretical results concerning the deductive aspects[1] of the new DL C-CLASSIC$_{\delta\epsilon}$. This DL (introduced in section 3) is the extension of C-CLASSIC[2] with two "non classical" connectives of $\mathcal{AL}_{\delta\epsilon}$, a toy DL designed by P. Coupey and C. Fouqueré (cf. [5]): the connective δ which describes a concept by default and the connective ϵ which describes an exception to a concept. Using these two new connectives (described in section 2) it is possible to define more concepts and therefore increase the scope of the classifier which works only on defined concepts[3]. Both C-CLASSIC and $\mathcal{AL}_{\delta\epsilon}$ have polynomial, complete and correct subsumption algorithms; however, extending these positive results to C-CLASSIC$_{\delta\epsilon}$ is not straightforward. In order to make a theoretical study of deductive aspects of C-CLASSIC$_{\delta\epsilon}$ (cf. section 4), and especially the subsumption relation, an algebraic approach (similar to [6], for instance, but for a simpler language) is used. In this framework, subsumption is considered from two points of view: descriptive and structural. The descriptive point of view for subsumption consists in comparing *terms (concept descriptions)* of C-CLASSIC$_{\delta\epsilon}$ via an equational system called EQ+. EQ+ fixes the main properties of the C-CLASSIC$_{\delta\epsilon}$ connectives and determines equivalence classes of terms. The structural (and computational) point of view consists in comparing *normal forms* which are computed by applying a homomorphism from the set of terms of C-CLASSIC$_{\delta\epsilon}$ into the set of elements of an intensional semantics (called CL$_{\delta\epsilon}$). The subsumption algorithm reflects this structural computation exactly. The correctness and completeness of the subsumption algorithm is established by proving the equivalence between the descriptive and structural points of view.

*This research is supported as part of the "Creation and Enrichment of a Knowledge-Base: Application to the Supervision of a Telephone Network" project involving CNRS-cognisciences, CNET (National Center for Telecommunication Studies), INRIA, LIPN.

[1] Inductive investigations are described in [9] (we prove that C-CLASSIC$_{\delta\epsilon}$ is PAC-learnable).

[2] C-CLASSIC is equivalent to CLASSIC2 ([2]) without the SAME-AS connective. W.W. Cohen and H. Hirsh described its inductive study in [4].

[3] A defined concept has both necessary and sufficient properties to recognize an instance of this concept.

2 Overview of the connectives δ and ϵ

This section is a description of the connectives δ and ϵ introduced by P. Coupey and C. Fouqueré in the $\mathcal{AL}_{\delta\epsilon}$ DL (cf [5]).

The connective δ intuitively represents the common notion of default.

Ex[4]: *Mammal* $\equiv$ *Animal* $\sqcap$ δ*Viviparous* $\sqcap$ *Vertebrate* defines the Mammal concept as a Vertebrate Animal which is usually Viviparous. Instances of *Mammal* necessarily have the properties *Animal* and *Vertebrate* but they may not have the property *Viviparous*. Unfortunately, using this definition of *Mammal* we can infer that - for instance - a *Duck* (*Duck* $\equiv$ *Animal* $\sqcap$ *Oviparous* $\sqcap$ *Vertebrate* $\sqcap$ *With-beak* $\sqcap$ *Quack* $\sqcap$ *Palmiped* $\sqcap$ δ *Fly*) is a *Mammal* (as it is a Vertebrate Animal). Thus, as R.J. Brachman claimed in [3], automatic classification with default knowledge seems impossible since a default property is not necessary. To solve this problem, P. Coupey and C. Fouqueré introduced in [5] the connective ϵ which represents an exception to a concept. They defined ***a definitional point of view for default knowledge*** and express the following constitutive property: ***an object is an instance of a concept C iff it satisfies the strict definitional knowledge of C, and satisfies or is explicitly "exceptional" w.r.t. the default knowledge of C.*** With the constitutive property we can no longer infer that a Duck is a Mammal as it is neither Viviparous nor exceptional w.r.t. Viviparous. On the other hand, an *Ornithorynchus*[5] (*Ornithorynchus* $\equiv$ *Animal* $\sqcap$ *Vertebrate* $\sqcap$ *Oviparous* $\sqcap$ *With-beak* $\sqcap$ *Viviparous*$^{\epsilon}$) will be classified under the *Mammal* concept since it is exceptional w.r.t. *Viviparous*.

In the framework described here, the classification process is monotonic despite the presence of default knowledge and, at this level, the exceptions are not applied (defaults are not inhibited by exceptions). The nonmonotonicity of defaults is recovered during the inheritance process, e.g. *Mammal* inherits the properties *Animal*, *Viviparous* and *Vertebrate*. *Ornithorynchus* is subsumed by *Mammal* but it does not inherit the property *Viviparous* since an exception to this property inhibits it.

P. Coupey and C. Fouquere showed in [5] that the introduction of the connectives δ and ϵ considerably improves the capabilities of classification processes since few concepts are definable with only strict knowledge. In the Supervision of Telephone Network application [1], default knowledge was integrated so as to be able to give a full definition for many concepts. However, since $\mathcal{AL}_{\delta\epsilon}$ is too restricted to be used in practical applications, it was necessary to design C-CLASSIC$_{\delta\epsilon}$.

3 The C-CLASSIC$_{\delta\epsilon}$ DL

The set of connectives of C-CLASSIC$_{\delta\epsilon}$ is the union of the set of connectives of C-CLASSIC [4] and of $\mathcal{AL}_{\delta\epsilon}$ [5].

[4]The formal notation (see the syntactic rule, section 3) is used for descriptions, not the CLASSIC one.

[5]Ornithoryncus = duck-billed platypus.

C-CLASSIC$_{\delta\epsilon}$ is defined using a set **R** of primitive roles, a set **P** of primitive concepts, the constants $\top$ and $\bot$, a set **I** of individuals (called *classic-individuals*), and the following syntactic rule (C and D are concepts, P is a primitive concept, R is a primitive role, u is a real, n is an integer and I_i are classic-individuals):

$C, D \rightarrow$	$\top$	the most general concept
	$\mid \bot$	the most specific concept
	$\mid P$	primitive concept
	$\mid$ *ONE-OF* $\{I_1 \ldots I_n\}$	concept in extension
	$\mid$ *MINu*	u is a real number
	$\mid$ *MAXu*	u is a real number
	$\mid C \sqcap D$	concept conjunction
	$\mid \forall R : C$	value restriction
	$\mid$ *R FILLS* $\{I_1 \ldots I_n\}$	subset of values for R
	$\mid$ *R AT-LEAST n*	cardinality for R (minimum)
	$\mid$ *R AT-MOST n*	cardinality for R (maximum)
	$\mid \delta C$	default concept
	$\mid C^{\epsilon}$	exception to the concept C

For example, *Switch* $\sqcap$ δ(*Ineffective-beam AT-LEAST 2*) $\sqcap$ $\forall$*Alarm-level:MIN 3* $\sqcap$ *Linked-to FILLS* {*CT-Paris1 CT-Paris3*} $\sqcap$ $\forall$*Broken-beam: ONE-OF*{*Lyon1 Lyon3*} describes all the switches which usually have at least two ineffective beams, at least level-3 alarms, are linked to at least CT-Paris1 and CT-Paris3, and all the broken beams of which are Lyon1 or Lyon3.

Defining a concept[6] means giving a name A to a *term* C of the C-CLASSIC$_{\delta\epsilon}$ language using the expression $A \equiv C$.

4 Deductive study of C-CLASSIC$_{\delta\epsilon}$

This section is divided as follows: section 4.1 focuses on the *descriptive* point of view, beginning with the equational system EQ+ and following by a formal characterization of *descriptive susbsumption*. Section 4.2 focuses on the *structural* point of view. C-CLASSIC$_{\delta\epsilon}$ is given an intensional semantics which is used to formally define *structural subsumption*, the basis of the subsumption algorithm. Finally, it is shown that *structural subsumption* computation in C-CLASSIC$_{\delta\epsilon}$ is polynomial and that the subsumption algorithm is correct and complete.

4.1 Descriptive point of view

Equational system

In order to formalize the subsumption relation in C-CLASSIC$_{\delta\epsilon}$, the equational system EQ+ is defined (see below). This system fixes the main properties of the connectives (e.g. axiom 2 expresses the commutativity of concepts conjunction), and is used to define an equivalence relation between *terms*. Moreover, EQ+ serves as the basis for the definition of an intensional semantics[7].

$\forall A, B, C \in$ C-CLASSIC$_{\delta\epsilon}$, $I_j \in I$, $Ei \in 2^I$:
01. $(A \sqcap B) \sqcap C = A \sqcap (B \sqcap C)$
02. $A \sqcap B = B \sqcap A$
03. $A \sqcap A = A$
04. $\top \sqcap A = A$

[6]Note that cyclic concept definitions are not allowed.

[7]The presence of individuals in the description language can lead to intractable reasoning. To avoid this problem, A. Borgida and P.F. Patel-Schneider's point of view is adopted (see [2]), where individuals are regarded as disjoint sets of objects rather than as distinct objects.

05. $\bot \sqcap A = \bot$
06. $\forall R : (A \sqcap B) = (\forall R : A) \sqcap (\forall R : B)$
07. $\forall R : \top = \top$
08. *ONE-OF* $E1 \sqcap$ *ONE-OF* $E2$ = *ONE-OF*$(E1 \cap E2)$
09. *MIN* $m \sqcap$ *MIN* n = *MIN* $maxi(m, n)$
10. *MAX* $m \sqcap$ *MAX* n = *MAX* $mini(m, n)$
11. *R FILLS* $E1 \sqcap$ *R FILLS* $E2$ = *R FILLS*$(E1 \cup E2)$
12. *R FILLS* $\emptyset = \top$
13. *R AT-LEAST* $m \sqcap$*R AT-LEAST* n = *R AT-LEAST* $maxi(m, n)$
14. *R AT-LEAST* $0 = \top$
15. *R AT-MOST* $m \sqcap$*R AT-MOST* n = *R AT-MOST* $mini(m, n)$
16. *R AT-MOST* $0 = \forall R{:}\bot$
17. *R FILLS* $\{I_1 \ldots I_n\}$ = *R FILLS* $\{I_1 \ldots I_n\} \sqcap$ *R AT-LEAST* n
18. $\forall R$: *ONE-OF* $\{I_1 \ldots I_n\} = \forall R$: *ONE-OF* $\{I_1 \ldots I_n\} \sqcap$ *R AT-MOST* n
19. *R AT-LEAST* $n \sqcap \forall R$: *ONE-OF* $\{I_1 \ldots I_n\}$ = *R AT-LEAST* n $\sqcap \forall R$: *ONE-OF* $\{I_1 \ldots I_n\}$ $\sqcap$ *R FILLS* $\{I_1 \ldots I_n\}$
20. *R AT-MOST* $n \sqcap$*R FILLS* $\{I_1 \ldots I_n\}$ = *R AT-MOST* n $\sqcap$*R FILLS* $\{I_1 \ldots I_n\}$ $\sqcap \forall R$: *ONE-OF* $\{I_1 \ldots I_n\}$
21. $(\delta A)^\epsilon = A^\epsilon$
22. $\delta(A \sqcap B) = (\delta A) \sqcap (\delta B)$
23. $A \sqcap \delta A = A$
24. $A^\epsilon \sqcap \delta A = A^\epsilon$
25. $\delta\delta A = \delta A$

The axioms 1 to 20 are explained and justified in [10]. They formally describe the C-CLASSIC connective properties which are informally expressed in [2][8]. The axioms concerning δ and ϵ (axioms 21 to 25) are defined in [5]. Put simply, (21) presupposes that an exception has a meaning only if it concerns a *default concept.* (25) allows redundant chains of δ to be removed, and (22) is a distributivity property. The *definitional point of view* of default knowledge described section 2 expresses a subsumption relation between A and δA (A is subsumed by δA) and between A^ϵ and δA (A^ϵ is subsumed by δA). These subsumption relations are expressed by the axioms (23) and (24)[9]. From an extensional point of view, the set of δA's instances is seen as a superset of A's instances and a superset of the instances which are exceptions to A, i.e. to be an A (resp. an A^ϵ) is more specific than to be a δA.

Note that EQ+ is the result of choices linked to the application. Clearly, other choices could have been made and, for instance, in other applications defaults could well be non distributive.

Descriptive subsumption

Let $=_{EQ+}$ denote the equality (modulo EQ+ axioms) between two *terms* of C-CLASSIC$_{\delta\epsilon}$. $=_{EQ+}$ defines equivalence classes of *terms* (e.g. $A \sqcap \delta\delta A =_{EQ+} A$ thanks to axioms 25 and 23). *Descriptive subsumption* $\sqsubseteq_d$ is then as follows:

Let C,D be elements of C-CLASSIC$_{\delta\epsilon}$, $C \sqsubseteq_d D$, i.e. D *descriptively* subsumes C, iff $C \sqcap D =_{EQ+} C$.

Example: *Ornithoryncus* $\sqcap$ *Mammal* = *Animal* $\sqcap$ *Vertebrate* $\sqcap$ *Oviparous* $\sqcap$ *With-beak* $\sqcap$ *Viviparous*$^\epsilon$ $\sqcap$ *Animal* $\sqcap$ δ*Viviparous* $\sqcap$ *Vertebrate* = *Animal* $\sqcap$ *Vertebrate* $\sqcap$ *Oviparous* $\sqcap$ *With-beak* $\sqcap$ *Viviparous*$^\epsilon$ $\sqcap$ δ*Viviparous* (applying (2) and (3)) = *Animal* $\sqcap$ *Vertebrate* $\sqcap$ *Oviparous* $\sqcap$ *With-beak* $\sqcap$ *Viviparous*$^\epsilon$ (applying (24)). As *Ornithoryncus* $\sqcap$ *Mammal* $=_{EQ+}$ *Ornithoryncus*, *Mammal* subsumes *Ornithoryncus*. It turns out that *terms* are not a suitable representation to compute subsumption and so another equivalent representation must be defined. This is the purpose of the following section.

4.2 Structural point of view

Intensional semantics

Following Birkhoff's theorem (cf. [7; 8] for a presentation of sets of equations and universal algebras), we show in [10] that our equational system induces a class of CL$_{\delta\epsilon}$-algebras. From this class we propose a structural algebra, which provides C-CLASSIC$_{\delta\epsilon}$ with an intensional semantics we called CL$_{\delta\epsilon}$.

The elements of CL$_{\delta\epsilon}$ are structures the definition of which is given in appendix. Intuitively, these structures are "normalized structural representations" of C-CLASSIC$_{\delta\epsilon}$ *terms* (i.e. a *normal form* of their set of properties)[10]. To obtain a normal form, implicit information is added[11]. This normalization strategy is a kind of *partial saturation* which has been adopted to ease the computation of the Least Common Subsumption algorithm (cf. [9]).

To define CL$_{\delta\epsilon}$, a homomorphism from the set of terms of C-CLASSIC$_{\delta\epsilon}$ into the set of elements of CL$_{\delta}\epsilon$ had to be defined. This homomorphism which is described fully in [10] and briefly in appendix, takes into account the axioms of EQ+ and the normalization strategy used. From a practical point of view, it consists in associating to each connective and constant of C-CLASSIC$_{\delta\epsilon}$ its interpretation in CL$_{\delta\epsilon}$.

Structural subsumption and the subsumption algorithm

Two terms C and D of C-CLASSIC$_{\delta\epsilon}$ are structurally equivalent iff the *normal form* of C is equal to the *normal form* of D. This equality is noted as $C =_{CL_{\delta\epsilon}} D$. The formal definition of *structural subsumption* is then defined as follows:

Let C, D be elements of C-CLASSIC$_{\delta\epsilon}$, $C \sqsubseteq_s D$, i.e. D *structurally* subsumes C, iff $C \sqcap D =_{CL_{\delta\epsilon}} C$.

Normal forms are the fundamental data handled by our *subsumption algorithm.* So, given two terms C and D

[8] A careful reader will have noted the lack of certain axioms linked to inconsistency (e.g. ONE-OF $\emptyset = \bot$). In fact this is not the case but the technical explanations required to highlight this point are too long to be given here (cf. [10]). Intuitively, note that in this framework the absorption property of $\bot$ is undesirable. With respect to subsumption, the equational system can detect subsumptions between concepts that are equivalent to $\bot$ from an extensional point of view (i.e. $\emptyset$). In other words, it is possible to detect intensional subsumptions which are not detected from the extensional point of view (i.e. a triangle which has four sides is intensionally different from a square circle even if their extension is equal to the empty set (cf. [11] for more details)).

[9] Note that subsumption is defined from equality: A is subsumed by B iff A $\sqcap$ B = A.

[10] Henceforth, elements of CL$_{\delta\epsilon}$ are called normal forms.

[11] For instance, according to axiom 17, the property R FILLS {a, b, c} leads us to add the property R AT-LEAST 3.

of C-CLASSIC$_{\delta\epsilon}$, answering the question "Does D subsume C?" means performing the following procedure:
The *normal forms* of C and "$C \sqcap D$" are computed using the homomorphism described in [10]. If these two *normal forms* are equal then the algorithm returns "yes" otherwise it returns "no".
We showed in [10] that: i) the computation of the *normal form* is polynomial in time, ii) the comparison of two *normal forms* is straightforward, iii) the size of *normal forms* is polynomial with respect to the size of *terms*. Consequently, it is true to say that the subsumption algorithm is polynomial. In order to show that it is correct and complete, it is sufficient to prove the equivalence between *descriptive* and *structural subsumption*.

Theorem 1 *Let C and D be terms of C-CLASSIC$_{\delta\epsilon}$, $C \sqsubseteq_s D$ iff $C \sqsubseteq_d D$.*

The complete proof of this theorem can be found in [10]. The "only if" part (i.e. completeness of the subsumption algorithm) consists in proving that each axiom of EQ+ is valid in CL$_{\delta\epsilon}$. To prove the "if" part (i.e. correctness of the subsumption algorithm), a *descriptive normal form* is defined in C-CLASSIC$_{\delta\epsilon}$, and then both its uniqueness and the fact that the equality in CL$_{\delta\epsilon}$ implies the equality of the *descriptive normal forms* are proved.

5 Conclusion

This paper presents the C-CLASSIC$_{\delta\epsilon}$ DL which is an extension of C-CLASSIC so as to handle default knowledge. C-CLASSIC$_{\delta\epsilon}$ has been given an intensional semantics based on concept algebras, and then it has been proved that subsumption in C-CLASSIC$_{\delta\epsilon}$ (i.e. the main reasoning operation) is polynomial, correct and complete. C-CLASSIC$_{\delta\epsilon}$ has been implemented in C^{++} and is being used in an industrial application.

References

[1] B. Biébow, P. Coupey, C. Fouqueré, J. Nobecourt, S. Szulman, and V. Ventos. Final report: creation and enrichment of a knowledge base, application to the supervision of the telephone network. Technical report, LIPN, Villetaneuse, France, 1996.

[2] A. Borgida and P. F. Patel-Schneider. Complete algorithm for subsumption in the CLASSIC description logic. *Artificial Intelligence Research*, 1:278–308, 1994.

[3] R.J. Brachman. I lied about the trees. *AI Mag.*, 6(3):80–93, 1985.

[4] W. W. Cohen and H. Hirsh. Learning the CLASSIC description logic: Theoretical and experimental results. In *International Conference on Knowledge Representation and Reasoning*, pages 121–133. 1994.

[5] P. Coupey and C. Fouqueré. Extending conceptual definitions with default knowledge (accepted paper). *Computational Intelligence*, 1996.

[6] R. Dionne, E. Mays, and F.J. Oles. The equivalence of model-theoretic and structural subsumption in description logics. In *13th International Joint Conference on Artificial Intelligence*, pages 710–716, Chambéry, France, 1993.

[7] G. Gratzer. *Universal Algebra*. Van Nostrand Company, 1968.

[8] N. Jacobson. *Basic Algebra II*. W.H. Freeman and Company, 1989.

[9] V. Ventos, P. Brézellec, P. Coupey, and H. Soldano. C-classic$_{\delta\epsilon}$: un langage de descriptions pac-learnable. In *Actes Journées Acquisition Validation Apprentissage, JAVA95*, pages 27–40, Grenoble, France, 1995.

[10] V. Ventos, P. Brézellec, P. Coupey, and H. Soldano. Aspects déductifs et inductifs d'un langage de descriptions incluant les connecteurs défaut et exception. Technical report, LIPN, université Paris Nord, Villetaneuse, France, 1996.

[11] W.A. Woods. Understanding subsumption and taxonomy: a framework for progress. In Morgan-Kaufmann, editor, *Principles of Semantic Networks*, pages 45–94. J. Sowa, 1991.

APPENDIX

Definition: A *structure* S of CL$_{\delta\epsilon}$ corresponding to a *term* T of C-CLASSIC$_{\delta\epsilon}$ is a pair $< S_\sigma, S_\delta >$ where S_σ contains the strict properties of T and S_δ its default properties. S_σ and S_δ are tuples defined as follows: (**dom**, **min**, **max**, π, **r**, ϵ) where **dom** is either a set of individuals if the definition of T contains a *ONE-OF* property, or the special symbol *UNIV* otherwise; **min** (resp. **max**) is either a real if T contains a *MIN* property (resp. *MAX*), or the special symbol *MIN-R* (resp. *MAX-R*) otherwise; π is the set of primitive concepts belonging to T; **r** is a set of elements defined as follows: $< R, fillers, least, most, c >$ where R is a name role; *fillers* is either a set of individuals if T contains a R *FILLS* property in its definition, or $\emptyset$ otherwise; *least* is an integer representing an R *AT-LEAST* property; *most* is either an integer representing an R *AT-MOST* property, or the special symbol *NOLIMIT* otherwise; c is a *structure* if T contains $\forall R : C$ in its definition; ϵ is a set of tuples (*dom*, *min*, *max*, π, *r*, ϵ).

Extract of the homomorphism from C-CLASSIC$_{\delta\epsilon}$ into CL$_{\delta\epsilon}$:

C-CLASSIC$_{\delta\epsilon}$	CL$_{\delta\epsilon}$
$\top$	< (*UNIV*, *MIN-R*, *MAX-R*, $\emptyset$, $\emptyset$, $\emptyset$), (*UNIV*, *MIN-R*, *MAX-R*, $\emptyset$, $\emptyset$, $\emptyset$) >
P	< (*UNIV*, *MIN-R*, *MAX-R*, {**P**}, $\emptyset$, $\emptyset$), (*UNIV*, *MIN-R*, *MAX-R*, {**P**}, $\emptyset$, $\emptyset$) >
MIN **u**	< (*UNIV*, **u**, *MAX-R*, $\emptyset$, $\emptyset$, $\emptyset$), (*UNIV*, **u**, *MAX-R*, $\emptyset$, $\emptyset$, $\emptyset$) >

$C \sqcap D \qquad c \otimes d$

Let us examine this extract in more detail:

- The interpretation of the primitive concept **P** consists in filling the field π of the strict tuple (i.e. $S_\sigma = (dom, min, max, \pi, r, \epsilon)$) with the name of this concept and, using the normalization strategy (adding implicit information) and axiom 23, doing the same thing for the default part (i.e. S_δ).
- c and d represent respectively the *normal forms* of concepts C and D. The internal operation $\otimes$ of $CL_{\delta\epsilon}$ is defined as follows: $c = < c\sigma, c\delta >$ and $d = < d\sigma, d\delta >$ are two elements of $CL_{\delta\epsilon}$, where $c\sigma, d\sigma, c\delta, d\delta$ are (strict or default) tuples. $c \otimes d = < c\sigma \oplus d\sigma, c\delta \oplus d\delta >$. The "tuples union operation" $\oplus$ is fully defined in [10]. Put simply, to define $\oplus$ requires defining the result of "union" on each field of tuples. Thus, for instance, "union" of fields *min* is equivalent to the maximum of the two *min* (cf. axiom 9); "union" of fields *prim* is equivalent to the standard set union.

Example: The structure (and therefore the *normal form*) corresponding to the *Mammal* concept is:
$\ll$ (UNIV,MIN-R,MAX-R, {**Animal, Vertebrate**}, ∅, ∅), (UNIV,MIN-R,MAX-R, {**Animal, Vertebrate, Viviparous**}, ∅, ∅) $\gg$