# Contextual reasoning and importing contexts

**Christo Dichev**
Department of Computer Science,
North Carolina A&T State University
Greensboro, NC 27411
e-mail: dichev@ncat.edu

## Abstract

This paper tackles the problem of understanding when and how a reasoning component of a distributed reasoning system is affected by the other components, and how it affects them. A number of relevant concepts which have a precise intuition in terms of cooperative reasoning systems are proposed thus providing a ground for defining a particular class of contexts – importing context. The notion of importing context refers to those contexts of a MCS satisfying the property that only correct with respect to the other contexts formulas are derivable from them. A number of properties of the importing contexts are presented and discussed in a framework of cooperative reasoning systems.

## Introduction

One of the most challenging potential applications of contexts is to provide a knowledge exchange format similar to the network protocols. The knowledge exchange format with respect to the knowledge and inference engines is analogous to the network protocols for data and computers. It can be viewed as a set of rules enabling a standard representation and exchange of knowledge that would facilitate communications between different components of a large knowledge based system by enabling flexible exchange of problems and solutions between them.

However in such complex distributed reasoning systems there are a number of issues that should be taken into account in order to understand their interaction: the way components interact; the constraints imposed on the communicated information and the relations between the communicated and the self-generated information. In the present paper some of these issues are addressed in the framework of the multicontext systems (MCS). The focus is on understanding when and how a reasoning component of a distributed reasoning system is affected by the other components, and how it affects them.

In MCS knowledge and reasoning can be structured in a collection of contexts. Each context is specified by its knowledge base, its inference mechanism and its language. Interrelations among contexts are specified as bridge rules - inference rules with premises and conclusions belonging to different contexts.

More formally a *multicontext system* (MCS) is defined in [7] as a pair $\langle \{c_i\}_{i \in I}, BR \rangle$, where $I$ is a set of indices, $\{c_i\}_{i \in I}$ is a set of contexts and $BR$ is a set of *bridge rules.* A context $c_i$ is a triple $c_i = \langle L_i, A_i, \Delta_i \rangle$, where $L_i$ is the language, $A_i$ is the set of axioms and $\Delta_i$ is the set of inference rules. The later rules specify the "local deduction" in $c_i$, while the bridge rules specify the interaction among contexts. Once a collection of independent contexts is provided with bridge rules it becomes a multicontext system. Bridge rules are inference rules asserting a fact in a context from premises inferred in other contexts. For instance the meaning of the bridge rule written as:

$$\frac{\langle c_i, \varphi \rangle}{\langle c_j, \psi \rangle}\ BR$$

$(i \neq j)$ is that it is possible to derive $\psi$ in context $c_j$, because $\varphi$ has been derived in $c_i$.

The main difference in our approach compared to the other works on formalizing contexts is in the type of bridge rules that are applied. While most of the authors such as [6-9] exploit two types of bridge rules - *reflection up* and *reflection down* bridge rules, in [4,5] we propose a multicontext system framework based on *reflection up* bridge rules. In this paper we continue our study within that framework, i.e. restricted to a class of multicontext systems exploiting bridge rules of the type

$$\frac{\langle c_i, \phi \rangle}{\langle c_j, ist(c_i, \phi) \rangle}$$

$(i \neq j)$ termed by us *concluding ist bridge rules.* We call formula $ist(c_i, \phi)$ [1] *ist*-formula or "switching formula". Despite the analogy in the notation $\langle c_j, ist(c_i, \phi) \rangle$ is a notion different from the McCarthy's lifting axiom [1,10,11]. Intuitively formula $ist(c_i, \phi)$ might be interpreted procedurally as a call from $c_j$ for proving $\phi$ in the context $c_i$.

[1] The formula $ist(c, \phi)$, meaning that $\phi$ is true in $c$, was first proposed by McCarthy [11]

One motivation for addressing multicontext systems limited to concluding *ist* bridge rules is grounded on our observation that the multicontext systems extended with a *premising ist* type bridge rule such as

$$\frac{\langle c_j, ist(c_i, \phi)\rangle}{\langle c_i, \phi\rangle}$$

are in a sense more vulnerable to inconsistencies compared to the former class. In MCS provided with *premising ist* bridge rules (*Pist*) any inconsistency derivable in one context is propagated to the other contexts of the system:

$$\frac{\dfrac{\langle c_i, \bot\rangle}{\langle c_i, ist(c_j, \bot)\rangle}}{\langle c_j, \bot\rangle}\ Pist$$

One problem arising from the contexts limited to concluding *ist* interactions is that it is possible from a given multicontext system $\mathcal{C}$ to derive facts such as $\mathcal{C} \vdash \langle c_i, ist(c_j, \varphi)\rangle$, while $\langle c_j, \varphi\rangle$ is not derivable in $\mathcal{C}$. Thus it is possible a context $c_i$ to assert that a formula $\varphi$ holds in some other context $c_j$, while it does not. Such *ist*-assertions make $c_i$ *incorrect for* $c_j$ and in some cases they must be eliminated. Observations such as this one provided a motivation for defining the notion of *importing context* denoting a particular family of contexts in which only correct *ist*-formulas are derivable. Two problems can be mentioned in relation to the importing contexts.

- Is it possible to define some criteria for distinguishing importing contexts from not importing ones?
- Is it possible to transform a context into an importing one preserving the essential properties of the original?

The above questions are addressed in the present paper, which is a continuation of our previous works on multicontext systems restricted to concluding *ist* bridge rules [2,3,4,5]. We propose a number of relevant concepts which have a precise intuition in terms of cooperative reasoning systems and can be used to understand their interaction. The introduced basic notions and definitions are twofold: first they enable us to make a distinction between the *internal* and the *imported part* of a context and next provide a background for defining the notion of *importing context*. The later notion refers to those contexts of a MCS satisfying the property that only *ist*-formulas that are correct with respect to the other contexts are derivable from them. In the sections following the definition of importing contexts we examine and discuss some properties of those contexts in a multicontext system framework.

## Contexts as interacting theories

In this section we present the idea of importing contexts. We first introduce some basic notions that allow us to make a distinction between the facts derivable independently from the other contexts and the facts whose derivation is supported from some other contexts. Then we define the notion of importing context and discuss some of the properties characterizing the class of importing contexts.

### Basic definitions and assumptions

As the bridge rules enable interactions between contexts a distinction between the facts "internal" with respect to a given context from the "external" ones can be important. For instance an inconsistency in a context $c_i$ might be either a "local" property, or it might be a consequence from the facts imported by the bridge rules. Thus one problem related to the properties of a multicontext system is whether inconsistency can be imported into a context from the other ones. Another problem is how to define contexts making no incorrect assertions for the other contexts. We therefore introduce some adequate notions in order to distinguish between the *internal* and the *imported part* of a context $c_i$.

**Definition 1.** A *primitive context* $c_i$ is defined as a triple $c_i = \langle L_i, A_i, \Delta_i\rangle$, where the language $L_i$ of $c_i$ is a set of $L_i$-wff of the type $\langle c_i, \varphi\rangle$, $A_i \subseteq L_i$ is the set of axioms of $c_i$ and $\Delta_i$ is the set of inference rules.

In fact the notion of primitive context denotes a separate context, which can be viewed as a trivial multicontext system with a single context.

**Definition 2.** The set of formulas derivable from a multicontext system $\mathcal{C}$ is called *multicontext theory* denoted by $Mcth(\mathcal{C}) = \{\langle c, \varphi\rangle |\ \mathcal{C} \vdash \langle c, \varphi\rangle\}$.

**Definition 3.** Let $c_i$ be a context of the multicontext system $\mathcal{C} = \langle \{c_i\}_{i\in I}, BR\rangle$. The set of formulas $Ker(c_i) = \{\langle c_i, \varphi\rangle |\ \mathcal{C}_0 \vdash \langle c_i, \varphi\rangle\}$ derivable from the multicontext system $\mathcal{C}_0 = \langle \{c_i\}_{i\in I}, \emptyset\rangle$ is called *internal part* or *kernel* of $c_i$.

According to this definition the kernel of a context is the set of formulas derivable in that context assuming no effects from the other contexts.

**Definition 4.** Let $\mathcal{C}$ be a multicontext system, such that $c_i$ is one of its contexts. The set of formulas $Exter(c_i) = \{\langle c_j, \varphi\rangle |\ \mathcal{C} \vdash \langle c_j, \varphi\rangle, j \neq i\}$ is called *external part* of $c_i, i \in I$.

**Definition 5.** Let $\mathcal{C}$ be a multicontext system, such that $c_i$ is one of its contexts. The set of *ist*-formulas

$$Imp(c_i) = \{\langle c_i, ist(c_j, \varphi)\rangle |\ \mathcal{C} \vdash \langle c_i, ist(c_j, \varphi)\rangle,\ \mathcal{C} \vdash \langle c_j, \varphi\rangle\}$$

derivable from $c_i$ as a result of applying concluding *ist* bridge rules to derived formulas, is called *imported part* of $c_i, i \in I$.

The imported part of a context is the set of all formulas derivable in that context by importing them from the other contexts.

**Definition 6.** Let $\mathcal{C}$ be a multicontext system and $\mathcal{C}_{-c_i} = Mcth(\mathcal{C}(i|\langle L_i, \emptyset, \Delta_i\rangle))$ be the multicontext theory obtained from $\mathcal{C}$ by substituting $c_i$ with a context containing no axioms. The set of formulas $Pextr(c_i) = \{\langle c_j, \varphi\rangle | \ \mathcal{C}_{-c_i} \vdash \langle c_j, \varphi\rangle, j \neq i\}$ is called *pure external part* of $c_i, i \in I$.

**Definition 7.** Let $c_i, i \in I$ be a context of the multicontext system $\mathcal{C}$ and $\mathcal{C}_{-c_i} = Mcth(\mathcal{C}(i|\langle L_i, \emptyset, \Delta_i\rangle))$ be the multicontext theory obtained from $\mathcal{C}$ by substituting $c_i$ with a context containing no axioms. The set of *ist*-formulas $Pimp(c_i) = \{\langle c_i, ist(c_j, \varphi)\rangle | \ \mathcal{C}_{-c_i} \vdash \langle c_i, ist(c_j, \varphi)\rangle\}$ is called *pure imported part* of $c_i$.

Obviously $Pexter(c_i) \subseteq Exter(c_i)$ and as we will see later in this section $Pimp(c_i) \subseteq Imp(c_i)$, which enable us to introduce the following definition.

**Definition 8.** Let $Imp(c_i)$ be the imported part of $c_i$ and $Pimp(c_i)$ be the pure imported part of $c_i$. The set of formulas $Echo(c_i) = Imp(c_i) \setminus Pimp(c_i)$ is called *echo* of $c_i, i \in I$.

Notice that the kernel of a context $c_i$ refers to the corresponding theory assuming that the context $c_i$ has been "extracted" from the multicontext system, it is a component of and closed for communications with the other components. So the notion of kernel of a context reflects the behaviour of the context assuming that it has been disconnected from the system. The imported part of a context corresponds to those set of facts that are generated from external sources and made distinct from the facts belonging to the internal part of that context. Finally the notion of pure imported part (pure external part) of a context enable us to distinguish external facts that are supported by its kernel from the external ones derived assuming that the kernel is substituted with an empty theory.

## Properties of interacting contexts

The conceptual separation of a context into a kernel and imported part makes it possible the task of examining context to be reduced to examining its parts.

**Theorem 1.** (*Locality of inconsistency*). For any context $c_i, i \in I$ holds:

$$Imp(c_i) \nvdash \bot$$

According to the above theorem inconsistency can not be "imported" into any context. It is a property of the context itself.

**Theorem 2.** Let $\mathcal{C} = \langle \{c_i\}_{i \in I}, BR\rangle$ be a multicontext system. The imported part of the context $c_i$ is correct for all contexts $c_j, j \in I, j \neq i$ of the multicontext system $\mathcal{C}$.

In fact the above theorem establishes that a premising *ist* bridge rule such as

$$\frac{\langle c_i, ist(c_j, \varphi)\rangle}{\langle c_j, \varphi\rangle}$$

$(i \neq j)$ applied only to the imported part of any context $c_i$ produces no effect on the other contexts $c_j, j \in I$ of the multicontext system $\mathcal{C}$. Therefore the effect (if any) of the premising *ist* bridge rules to a multicontext theory results from the facts outside the imported part of the contexts i.e. from their kernels. In other words the imported part of a context might be interpreted as the one in which the premising *ist* bridge rule is a derivable property.

## Importing contexts

One problem arising in contexts limited to concluding *ist* interaction is that it is possible to derive facts such as $\mathcal{C} \vdash \langle c_i, ist(c_j, \varphi)\rangle$, while $\langle c_j, \varphi\rangle$ is not derivable, as it is shown in the following example. Imagine a multicontext system $\mathcal{C}$ containing contexts $c_1$ and $c_2$ defined as follows:

$$\langle c_1, \ \phi \rightarrow ist(c_2, \varphi)\rangle$$
$$\langle c_1, \phi\rangle$$

$$\langle c_2, \psi\rangle$$

Assuming that

$$\frac{\langle c_2, \phi\rangle}{\langle c_1, ist(c_2, \phi)\rangle}$$

is the only bridge rule defined in the multicontext system, then

$$\mathcal{C} \vdash \langle c_1, ist(c_2, \varphi)\rangle,$$

while $\langle c_2, \varphi\rangle$ is not derivable from $\mathcal{C}$. Situation such as this one are acceptable and even useful when modeling an agent beliefs. An agent can believe that another agent knows some facts, wile he does not. However there are domains where assertions such as $\mathcal{C} \vdash \langle c_i, ist(c_j, \varphi)\rangle$ that make $c_i$ *incorrect for* $c_j$ must be detected and then eliminated. These observations provide a motivations for defining the notion of *importing context*.

**Definition 9.** A context $c_i = \langle L_i, A_i, \Delta_i\rangle$ of a multicontext system $\mathcal{C} = \langle \{c_j\}_{j \in I}, BR\rangle$ with concluding *ist* bridge rules is an *importing context* if and only if $\forall j \neq i$ there is no any formulae $\langle c_j, \varphi\rangle$ such that

$$\mathcal{C} \vdash \langle c_i, ist(c_j, \varphi)\rangle \ \text{ and } \ \mathcal{C} \nvdash \langle c_j, \varphi\rangle$$

In other words, $c_i$ is an importing context if whenever is derivable formula $\langle c_i, ist(c_j, \varphi)\rangle$, then formula $\langle c_j, \varphi\rangle$ is derivable too.

An intuitive illustration of importing contexts can be found by switching to a Horn-clause language extended accordingly with *ist*-formula. A context $c_i$ in this particular case is an importing context if in each its clause

$$A \leftarrow B_1, ..., ist(c_j, B), ..., B_n$$

*ist*-formula can occur only in its body.

In the following we restrict our consideration to multicontext systems $\mathcal{C} = \langle \{c_j\}_{j\in I}, BR\rangle$, where all bridge rules are assumed to be of the form

$$\frac{\langle c_i, \phi\rangle}{\langle c_j, ist(c_i, \phi)\rangle} \in BR$$

## Properties of importing contexts

The notion of importing context refers to those contexts of a MCS satisfying the property that only correct with respect to the other contexts formulas are derivable from them. In this section we characterize further the class of importing contexts.

**Theorem 3.** Let $c_i$ be an importing context of the multicontext system $\mathcal{C}$. Then for any formula $\langle c_i, ist(c_j, \varphi)\rangle$ holds $\mathcal{C} \vdash \langle c_i, ist(c_j, \varphi)\rangle$ if and only if $\langle c_i, ist(c_j, \varphi)\rangle \in Imp(c_i)$.

In the following by $c \vdash_i$ we will denote the derivability relation within the context $c_i$, that is, the derivability relation assuming no applications of any bridge rules.

**Lemma 1.** $Imp(c_i) \vdash_i \langle c_i, ist(c_j, \varphi)\rangle$ if and only if $\langle c_i, ist(c_j, \varphi)\rangle \in Imp(c_i)$.

**Lemma 2.** $\mathcal{C} \vdash \langle c_i, \varphi\rangle$ iff $(Ker(c_i) \cup Imp(c_i)) \vdash_i \langle c_i, \varphi\rangle$.

According to the later lemma the theory $\mathbf{c}_i$ associated with the context $c_i = \langle L_i, A_i, \Delta_i\rangle$ of the multicontext system $\mathcal{C} = \langle \{c_i\}_{i\in I}, BR\rangle$ is to be understood as the transitive closure of the union of its kernel $Ker(c_i)$ and its imported part $Imp(c_i)$ with respect to the derivability relation $c \vdash_i$, i.e.

$$\mathbf{c}_i = \{\langle c_i, \varphi\rangle |\ (Ker(c_i) \cup Imp(c_i)) \vdash_i \langle c_i, \varphi\rangle\}$$

Therefore the set of formulas derivable from a given context can be split into three components: its kernel, its imported part and a collection of formulas not belonging to the former components but derivable from them.

**Lemma 3.** If $\langle c_i, ist(c_j, \varphi)\rangle \in Pimp(c_i)$, then $\mathcal{C} \vdash \langle c_j, \varphi\rangle$.

**Corollary.** $Pimp(c_i) \subseteq Imp(c_i)$.

The basic property of the importing contexts is that they are correct for the other contexts of the multicontext system. An important consequence of this fact is that they do not affect the other contexts even when premising *ist* bridge rules are applied to them. More precisely, assume that $c_i$ is an importing context of the multicontext system $\mathcal{C} = \langle \{c_j\}_{j\in I}, BR\rangle$ and $BR_D$ is an arbitrary set of premising *ist* bridge rules. If we add to $\mathcal{C}$ the set of bridge rules $BR_D$, then in the resulting multicontext system $\mathcal{C}_\mathcal{D} = \langle \{c_j\}_{j\in I}, BR \cup BR_D\rangle$ all theories $\mathbf{c}_i$ associated with contexts $c_i$ in $\mathcal{C}_\mathcal{D}$ remain unchanged with respect to the corresponding theories associated with the originals $c_i$ in $\mathcal{C}$ for any $i \in I$.

**Theorem 4.** Let $\mathcal{C} = \langle \{c_j\}_{j\in I}, BR\rangle$ be a multicontext system, where $c_j, j \in I$ are importing contexts and let $\mathcal{C}_\mathcal{D} = \langle \{c_j\}_{j\in I}, BR \cup BR_D\rangle$ be a multicontext system obtained from $\mathcal{C}$ by adding to the concluding *ist* bridge rules $BR$ a set of premising *ist* bridge rules $BR_D$. Then for any formula $\langle c_i, \varphi\rangle$ and any $i \in I$

$$\mathcal{C} \vdash \langle c_i, \varphi\rangle \quad \text{iff} \quad \mathcal{C}_\mathcal{D} \vdash \langle c_i, \varphi\rangle$$

that is $Mcth(\mathcal{C}) = Mcth(\mathcal{C}_\mathcal{D})$.

**Proof.** This property of importing contexts can be proved by simply substituting in any proof in $c_i' \in \mathcal{C}_\mathcal{D}$ each application of a premising *ist* type bridge rule with the proof of $\mathcal{C} \vdash \langle c_i, \phi\rangle$. Such a proof exists, because $c_i$ is an importing context and it is assumed that the assertion $\langle c_i', ist(c_i, \phi)\rangle$ holds. In the other direction the property is obvious since any proof in $c_i$ is also a proof in $c_i'$ $\Box$.

The next corollary states that in a multicontext system with premising-*ist* bridge rules, containing only importing contexts premising *ist* bridge rules can be eliminated without any effects on the resulting multicontext theory.

**Corollary.** If $\mathcal{C}_\mathcal{D} = \langle \{c_j\}_{j\in I}, BR \cup BR_D\rangle$ is a multicontext system, such that each $c_j$ is an importing context then the multicontext theory $Mcth(\mathcal{C}_\mathcal{D})$ is invariant with respect to the set of the premising *ist* bridge rules $BR_D$.

In the following we define some sufficient conditions for importing contexts. The first one reflects our attempt to find a sufficient condition for identifying importing contexts independently of the multicontext system in which a context takes part.

The next example illustrates the intuition supporting the following theorem. Consider the context $c_i$

..........

$\langle c_i,\ \psi_1 \wedge ist(c_k, \psi_2) \wedge \psi_3 \rightarrow ist(c_j, \varphi)\rangle$
$\langle c_i,\ \psi_1 \wedge \psi_3\rangle$

..........

Assuming that $\langle c_i, ist(c_k, \psi_2)\rangle \in Imp(c_i)$, we can derive $\langle c_i, ist(c_j, \varphi)\rangle$, and if $\varphi$ does not hold in $c_j$, then $c_i$ is incorrect for $c_j$. Contexts such as $c_i$ are prospective members of the class of not importing contexts and might be ignored.

**Theorem 5.** A sufficient condition for the context $c_i = \langle L_i, A_i, \Delta_i\rangle$ of the multicontext system $\mathcal{C} = \langle \{c_i\}_{i\in I}, BR\rangle$ to be an importing context is that for any subset of its imported part $S \subseteq Imp(c_i)$ and for any formula $\langle c_i, ist(c_j, \varphi)\rangle \notin Imp(c_i)$ holds

$$S \cup Ker(c_i) \nvdash_i \langle c_i, ist(c_j, \varphi)\rangle$$

The next theorem presents another sufficient condition for importing contexts based on the notion "pure imported part of a context".

**Theorem 6.** If $c_i$ is a context of the multicontext system $\mathcal{C} = \langle \{c_i\}_{i \in I}, BR \rangle$, such that for any formula $\langle c_i, ist(c_j, \varphi) \rangle$: $\mathcal{C} \vdash \langle c_i, ist(c_j, \varphi) \rangle$ implies $\langle c_i, ist(c_j, \varphi) \rangle \in Pimp(c_i)$, then $c_i$ is an importing context in $\mathcal{C}$.

The following theorem shows that any Horn-clause context can be replaced by an equivalent importing context.

**Theorem 7.** Let $c_i$ be a Horn-clause context of the multicontext system $\mathcal{C}$ and $\langle c_i, ist(c_j, A) \leftarrow B_1, B_2, ..., B_n \rangle$ be a clause of $c_i$. Let $\mathcal{C}'$ be a multicontext system obtained from $\mathcal{C}$ by removing the clause $\langle c_i, ist(c_j, A) \leftarrow B_1, B_2, ..., B_n \rangle$ from $c_i$ and adding to the context $c_j$ a new clause $\langle c_j, A \leftarrow ist(c_i, B_1), ist(c_i, B_2), ..., ist(c_i, B_n) \rangle$, then for any formulae $\varphi$: $\mathcal{C} \vdash \varphi$ implies $\mathcal{C}' \vdash \varphi$.

**Corollary.** Any Horn-clause context $c_i$ can be transformed into an importing one by multiple applications of that theorem until no formulae of the form $\langle c_i, ist(c_j, A) \leftarrow B_1, B_2, ..., B_n \rangle$ is left.

The above theorem can be generalized to handle not only Horn-clause theories.

**Theorem 8.** Let $c_i$ be a context of the multicontext system $\mathcal{C}$ and $\langle c_i, ist(c_j, \varphi) \leftarrow \psi \rangle$ be a formula of $c_i$. Let $\mathcal{C}'$ be a multicontext system obtained from $\mathcal{C}$ by removing the clause $\langle c_i, ist(c_j, \varphi) \leftarrow \psi \rangle$ from $c_i$ and adding to the context $c_j$ a new clause $\langle c_j, \varphi \leftarrow ist(c_i, \psi) \rangle$, then for any formulae $\phi$ $\mathcal{C} \vdash \phi$ implies $\mathcal{C}' \vdash \phi$.

Finally, we will prove the undecidability of the problem of demonstrating whether a context is an importing one or not, by reducing it to the problem of undecidability of the predicate calculus. We observe first that all logical predicate laws are valid in any multicontext system defined in this section and also that, if a formula including no *ist*-subformula is valid in any such multicontext system, then it is a tautology. Consider now a two-context system such that in the first context $c_1$ are valid only the logical tautologies, i.e. the axioms and the theorems following from them, while in addition to those facts in the second context $c_2$ is also valid the formula $\langle c_2, ist(c_1, \alpha) \rangle$, where $\alpha$ is any formula with no *ist*-modality in it. Then the proof of the fact that $c_2$ is an importing context or not importing one is equivalent to the proof of derivability or not derivability of the formula $\alpha$, which in general, for arbitrary $\alpha$ is impossible.

## Conclusion

In this paper we tackle the problem of understanding when and how a reasoning component of a distributed reasoning system is affected by the other reasoning components, and how it affects them. We introduce first a number of concepts such as imported part, exported part and kernel of a context which have a precise intuition in terms of cooperative reasoning and can be used to understand their interaction. Based on this ground we introduced also a particular class of multicontext systems with a single bridge rule. Within these MCS we have defined a particular class of contexts - importing contexts. The importing contexts model situations where the interacting agents can import facts proven to be true in the outside world, but are not allowed to support believes incompatible with the outside world. The properties of the importing contexts are presented and discussed in the light of contextual reasoning including some properties related to the Horn importing contexts.


## Acknowledgments

I wish to express my thanks to Trendafil Madarov for its valuable comments on the draft of this paper.


## References


1 Buvač S. 'Quantificational Logic of Context'. In Proceedings of the Thirteenth National Conference on Artificial Intelligence. Menlo Park, Calif.: American Association for Artificial Intelligence.

2. Dichev Ch. A framework for knowledge structuring. *Proc. of the 6-th Conference: Artificial Intelligence: Methodology, Systems, Applications*, World Scientific Publishing, 1994, pp. 55-65.

3. Dichev Ch. Context formation in logic programming setting. *Proceedings of TAINN'95*, Turkey, June 1995, pp. 207-218.

4. Dichev Ch. Contexts as communicating theories. *AI Communications - The European Journal on Artificial Intelligence*, No 10(3), 1997.

5. Dichev C., Multicontext systems and importing contexts, Guinchiglia F. (Ed.) Proceedings of the 8th International Conference on Artificial Intelligence: Methodology, Systems, Applications (AIMSA'98), LNAI, Springer-Verlag, 1998, pp. 209-220.

6. Giunchiglia, E., Traverso.P., Giunchiglia, F. Multicontext systems as a specification framework for complex reasoning systems. *IRST Technical Report 9206-22, IRST, Trento, Italy*, 1992.

7. Giunchiglia, F. Contextual reasoning. *Proc. of the IJCAI'93 Workshop on "Using knowledge in context"*, pp. 39-50.

8. Giunchiglia, F., L. Serafini. Multilanguage hierarchical logics or: how we can do without modal logics. *Artificial Intelligence*, 65:29-70, 1994.

9. Giunchiglia, F., L. Serafini, Giunchiglia, E, Frixione M. Non-omnicent belief as context-based reasoning. *Proc. of the IJCAI'93*, Morgan Caufmann, 1993, pp. 548-554.

10. Guha,R.V. Contexts: a formalizations and some applications, *PhD Thesis, Stanford*, 1991.

11. McCarthy J. Notes on formalizing context. In *Proc. of Thirteen International Joint Conference on Artificial Intelligence (IJCAI'93)*, France, 1993, pp. 555-560.