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Representing Absolute Time Expressions with Vagueness, Indeterminacy, and Different Granularities*

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Abstract

Absolute time expressions are related to the specification of the position of either a time point or an interval on the time axis. The data model we sketch in this paper allows us to represent in a seamless way expressions with indeterminacy ("between 6 and 6:30 p.m., August 10, 1997", "from 5-5:45 to 8-8:15 p.m., January 11, 2000"), with different granularities ("on December 19, 1998", "at 2:30 p.m., September 23, 1998"), with vagueness ("at the end of January 1998"), and with granularities/indeterminacy/vagueness together ("from March to the beginning of May, 1998", "from the end of June 1999 for 34-45 days"). Our data model is mainly based on the integration of different approaches: the framework proposed by Bettini and colleagues for temporal granularities has been suitably merged with the representation of temporal (fuzzy) knowledge by the possibility theory. To deal with uncertainty from temporal relationships, the data model has been based on a multivalued logics.

Introduction

Absolute temporal expressions are those expressions, which refer either to an absolute position of a point (or an interval) on a time axis or to the measurement of a distance between two time points, according to a given metric, e.g., that of the Gregorian Calendar. Temporal expressions can be given in natural language in many different ways: (i) using different time granularities, i.e. time units ("on December 19, 1998", "at 2:30 p.m., September 23, 1998","from January to March, 1998), (ii) with indeterminacy, when the precise temporal location of an event is not known ("between 6 and 6:30 p.m., August 10, 1997", "from 5-5:45 to 8-8:15 p.m., January 11, 2000"), (iii) with vagueness, when also the boundaries delimiting the time interval during which an event happened are not completely known ("at the end of January 1998", "from the beginning of March 1997 to the end of May 1999"), (iv) by mixing expressions of the three previous types ("from March to the beginning of May, 1998", "from the end of June 1999 for 35-45 days").

Research efforts in modeling such kind of expressions and in reasoning on them come from different areas and usually focus separately on one kind of temporal expressions: we cite here temporal databases and temporal reasoning (Bettini *et al.* 1998; Goralwalla *et al.* 1998; Montanari *et al.* 1992), temporal constraints (Dechter, Meiri, & Pearl 1991; Koubarakis & Skiadopoulos 1999), and fuzzy sets (Dubois & Prade 1989; Loganantharaj & Kurkovsky 1997; Godo & Vila 1995).

In this paper we sketch main features of the data model we defined to manage in a seamless way absolute temporal expressions possibly containing vagueness, indeterminacy and different granularities. Our data model is based on the integration of different approaches: the framework proposed by Bettini and colleagues for temporal granularities has been suitably merged with the representation of temporal (fuzzy) knowledge by the possibility theory (Bettini *et al.* 1998; Dubois & Prade 1989). To deal with uncertainty from temporal relationships, the data model has been based on a multivalued logics (Panti 1998). Although the paper is described in a logic-based fashion, it has been designed and implemented inside the framework of temporal object-oriented databases, to support different expressions for valid times of temporal objects (Venuti 1998).

The rest of the paper is organized as follows: we first briefly mention the main research areas related to the data model we are presenting; we, then, describe basic concepts of the data model; the paper ends with some concluding comments.

Related work

Many different approaches and focuses may be distinguished in the literature dealing with temporal infor-

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mation in AI and databases; as regards the problem of representing and reasoning on heterogeneous temporal data, we mention here three different, and usually not integrated, approaches:

- Managing different time granularities. There have been different proposals from AI and database fields for managing different time granularities, i.e. different time units, associated to temporal information (Bettini *et al.* 1998; Goralwalla *et al.* 1998; Montanari *et al.* 1992). The focus here is both on managing (reasoning on, representing and querying) temporal information given at different granularities and on representing and classifying granularity systems, according to properties of their granularities.
- Temporal constraints and indeterminacy. Indeterminacy is present when it is not possible to precisely define the position of a time point or the distance between two time points. Constraint-based reasoning techniques have been defined and applied to classify and solve temporal constraint problems of different kind (Dechter, Meiri, & Pearl 1991; Koubarakis & Skiadopoulos 1999). Temporal constraints expressed by using different time granularities have been considered in (Bettini, Wang, and Jajodia 1998).
- Representing and reasoning on fuzzy temporal knowledge. Several works have been devoted to the problem of representing and managing approximate temporal knowledge, based on Zadeh's possibility theory (Dubois & Prade 1989; Loganantharaj & Kurkovsky 1997; Godo & Vila 1995).

Modeling temporal information

Time domain and granularities

The basic time domain $\mathbf{T}$, called also *time axis*, is isomorphic to the real numbers with the usual ordering relation $\leq$. The set *Gran* of granularity functions is related to the granularities of the Gregorian calendar (years, months, days, hours, seconds). Our granularity mappings consider granularities for both anchored and unanchored time spans (Goralwalla *et al.* 1998): for example, the granularity of months can be used for expressing a certain period in a year (*October, 1999*), as well as for expressing a duration (*for three months*). Granularity functions are based on the framework proposed by Bettini and colleagues in (Bettini *et al.* 1998). Each granularity function G is a temporal type, i.e. a mapping from an index set $\mathbf{I_G}$, isomorphic to a subset of integers $\mathbf{I}$, to $2^{\mathbf{T}}$. More precisely, $Gran = \{Y, mean\_Y, M, mean\_M, D, mean\_D, H, mean\_H, Mi, mean\_Mi, S, mean\_S\}$. The functions Y, M, ... represent the usual granularities of the Gregorian calendar (they manage leap years, months with 28, 29, 30, or 31 days, and so on). The functions $mean\_Y$, $mean\_M$, ... provide regular mappings, that will be used in modeling duration, i.e. unanchored time spans, based on the (astronomical) mean length of a year. Instead of using the notation $\mu(i)$ to identify the ith *tick* (granule) of the mapping (temporal type, or granularity) μ, we will use the symbols $\langle . \rangle$, to denote the granule, and different notations for different index sets, based on that for dates and durations, e.g. `YY/MM/DD/HH/Mi/SS` for the index set $\mathbf{I_S}$ of seconds, `YY/MM/DD` for the index set $\mathbf{I_D}$ of days, n_1 `y` n_2 `m` for the index set $\mathbf{I_{mean\_M}}$ of durations expressed using months and years.

Example 1 The following notations for time granules are equivalent:
$Y(1996) = \langle 1996 \rangle$
$M(1996 * 12 + 4) = \langle 1996/4 \rangle$
$mean\_Y(1) = \langle 1 \text{ y} \rangle$
$mean\_M(1 * 12 + 3) = \langle 1 \text{ y } 3 \text{ m} \rangle$ □

The functions l and u: $2^{\mathbf{T}} \rightarrow \mathbf{T}$, return, respectively, the lower and upper bound of the considered granule. The notation $granule(\mathbf{I_G}, t)$, where $\mathbf{I_G}$ is one of the above time indexes, is a shorthand for referring to the granule $G(i)$ on $\mathbf{T}$ containing the time point t: for example, $granule(\mathbf{I_M}, l(\langle 1997/1/13/0/0/0 \rangle)) \equiv \langle 1997/1 \rangle$.

Time instants and durations

A time instant refers to a point on the time axis, whose location can be given with vagueness, indeterminacy, or different granularities. Instants are represented by the domain **Inst**. The position of an instant on the time axis is given by a function called ***possibility distribution*** π: $\mathbf{Inst} \times \mathbf{T} \rightarrow [0,1] \subset \mathbf{R}$. $\forall t \in \mathbf{T}$, $\pi(i,t)$ is the numerical value estimating the possibility that the instant i is precisely t. When $\pi(i,t) = 0$, it is for sure that i is different from t. It may be that there are several distinct time points t_1, t_2 such that $\pi(i,t_1) = \pi(i,t_2) = 1$. In the following, we will use for the function π the more common notation $\pi_i(t)$, instead of the notation $\pi(i,t)$ (Dubois & Prade 1989). Two other functions are derived from the possibility distribution π, named $poss\_after\_\pi$ and $poss\_before\_\pi$:

$$poss\_after\_\pi_i(t) = sup_{s \leq t} \pi_i(s)$$
$$poss\_before\_\pi_i(t) = sup_{s \geq t} \pi_i(s)$$

Intuitively, these last two functions, given an instant i, provide the numerical values estimating the possibility that time points are after (before) the instant i.
A *duration* is an unanchored time span: it represents the distance between two time points. As for instants, durations can be given with vagueness, indeterminacy, or using different granularities. Formally, durations are represented by the domain **Dur**. For durations, the

possibility distribution π estimates the possibility that a given duration is represented by different distances between time points. The function π: $\mathbf{Dur} \times \mathbf{T} \rightarrow [0,1] \subseteq \mathbf{R}$, returns the numerical values estimating the possibility that different time points on $\mathbf{T}$ represent a given duration.

According to a widely diffused approach in the literature on fuzzy sets, in the following we will use trapezoidal possibility distributions, represented by quadruples $(\underline{a}, a_*, a^*, \overline{a})$ (Dubois & Prade 1989). In the following section, we will introduce different possibility distributions for instants and durations given with different granularities, with vagueness, and indeterminacy.

Granularity, vagueness, and indeterminacy in representing instants and durations Time instants and durations can be given with granularity, indeterminacy, vagueness, or even mixing them in different ways. We identify three main ways of defining an instant or a duration:

1. at a certain granularity (e.g., "on December 15, 1998, at 7:00:00 p.m.", "On September 3, 1998", "for 3 hours, 15 minutes and 23 seconds", "for 3 months and 6 days");
2. with vagueness, using different granularities (e.g., "at the end of May, 1999", "in the middle of 1997", "for about 20 minutes");
3. with indeterminacy, using different granularities and/or vagueness (e.g., "between 2:30:00 p.m. and 7:35:15 p.m. of December 13, 1997", "between September 15, 1997 and November 1997", "between 2 p.m. of November 13, 1997 and the end of December 1997", "for 15 - 25 seconds", "for a time span lasting between 3 minutes and 3 hours", "for a time span lasting between 35 minutes and about one hour").

Constants for instants and durations given with different granularities (case 1) are represented in the standard format for dates and time spans, as, for example, `YY/MM/DD`, `YY/MM/DD/HH`, or `YY/MM/DD/HH/Mi/SS` for instants, and N_1 `yy` N_2 `mm`, N_1 `yy` N_2 `mm` N_3, or N_1 `yy` N_2 `mm` N_3 `dd` N_4 `hh` N_5 `mi` N_6 `ss` for durations, where N_i is the number of considered time units. More formally we can say that such constants correspond to the application of the function $g2i$: $2^{\mathbf{T}} \rightarrow \mathbf{Inst}$; the intervals argument of this function are granules from the calendar-related granularity mappings. For example, the constant `99/03/21` is the result of the function $g2i(\langle 99/03/21 \rangle)$. We call **Gran_I** the set of all the constants made by the previous function. We call $i2g$ the inverse function, which, given an instant belonging to **Gran_I**, returns the corresponding granule.

Constants for instants and durations given with vagueness (case 2) are denoted by applying some qualifier to the previous notations. The qualifiers for instants are `beginning`, `middle`, `end`; the qualifiers for durations are `about`, `almost`. More formally we can say that such constants correspond to the application of suitable functions (corresponding to the previously introduced qualifiers) from $2^{\mathbf{T}}$ to **Inst**. For example, the constant `middle(99/04)` is the result of the function $middle(\langle 99/04 \rangle)$. We call **Vag_I** the set of all the constants made by the previous functions.

The last situation (case 3) happens when the interval of indeterminacy, i.e., the set of contiguous time points with which the considered time point can coincide, is explicitly given. The bounds of the interval of indeterminacy can be vague and/or given with different granularities. Constants for instants and durations given with indeterminacy are represented by two constants belonging to **Gran_I** $\cup$ **Vag_I**, included between the symbols $\prec$, $\succ$. More formally we can say that such constants correspond to the application of a suitable function in_in (for *in*determinate *in*stant): $2^{\mathbf{Gran\_I} \cup \mathbf{Vag\_I}} \rightarrow \mathbf{Inst}$. For example, the constant `≺middle(99/04), 99/05/21≻` corresponds to the result of the function $in\_in(middle(\langle 99/04 \rangle), g2i(\langle 99/05/21 \rangle))$. We call **Indet_I** the set of all the constants made by the previous function.

Example 2 Let us consider the following sentences: "On December 13, 1998", "At the end of December, 1998", "between the end of December 1998, and January 15, 1999": the first one is modeled by the constant `1998/12/13` $\in$ **Gran_I**; the second one is modeled by `end(1998/12)` $\in$ **Vag_I**: while the third one is modeled by $\prec$ `end(98/12),99/01/15` $\succ$ $\in$ **Indet_I**. □

We have now to deal with the definition of possibility distributions able to represent instants and durations given with different granularities, degrees of vagueness, and indeterminacy.

In case of instants given at different granularities, the trapezoidal possibility distribution is given as:

$$\forall i \in \mathbf{Gran\_I}\ \pi_i \equiv_{df} \\ (l(i2g(i)) - f(i),\ l(i2g(i)),\ u(i2g(i)),\ u(i2g(i)) + f'(i))$$

Several different definitions can be provided for the functions f and f'; the basic idea is that these two functions are related to the amplitude of the corresponding granule, in order to have a "fuzziness" depending on how coarse is the corresponding granularity. A possible choice could be to define f and f' in the same way as follows:

$$\forall i \in \mathbf{Gran\_I}\ f(i) \equiv_{df} \frac{u(i2g(i)) - l(i2g(i))}{k(i)}$$

Example 3 Let us consider the index set $\mathbf{I_D}$ of days. We can define possibility distributions associated to instants defined on this index set as follows.

$$\pi_{\mathtt{YY/MM/DD}} \equiv_{df} (l(i2g(\mathtt{YY/MM/DD})) - \tfrac{86400}{12},\ l(i2g(\mathtt{YY/MM/DD})), \\ u(i2g(\mathtt{YY/MM/DD})),\ u(i2g(\mathtt{YY/MM/DD})) + \tfrac{86400}{12})$$

where 86400 is the number of seconds in a day granule ($u(i2g(i)) - l(i2g(i)) = 86400$ for each i identified by an element of the index set $\mathbf{I_D}$) and 12 is the value chosen for the constant $k(i)$, for each i.
On the basis of such kind of choices, the user is allowed to suitably define possibility functions. Figure 1 illustrates, for example, the possibility functions associated to the time instants 10:0:0 a.m. of December 15, 1998 (1998/12/15/10/0/0, at the granularity of seconds), 10:0 a.m. of December 15, 1998 (1998/12/15/0/0 at the granularity of minutes), and 10 a.m. of December 15, 1998 (1998/12/15/10 at the granularity of hours). It is worth notice that the possibility function associated to the instant given at the granularity of seconds (the finest one in our granularity system) does not represent any fuzziness: time points within the considered granule have possibility 1, while the time points outside the granule have possibility 0, i.e. they cannot represent the specified instant. □

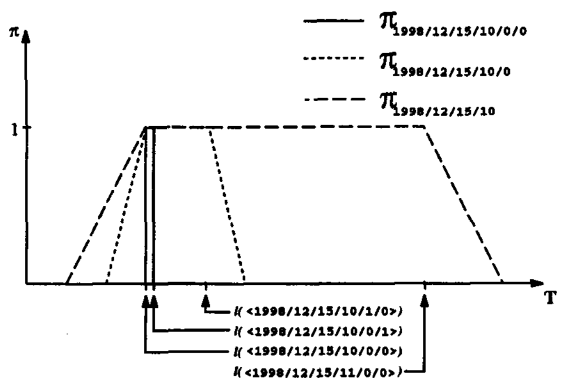

Figure 1: An example of possibility functions related to time instants given at different granularities.

Let us now consider the second case, i.e. instants given with vagueness. A simple choice is to adopt functions, allowing one to perform a fuzzy partition of the considered granule: a fuzzy partition for a granule X is composed by possibility functions π_i such that $\forall t \in X\ \sum_i \pi_i(t) = 1$, where i is ranging on all the constants, whose possibility functions compose the partition.

Example 4 Let us consider the following constants, which identify an instant at the beginning, in the middle, and at the end of November, 1997: `beginning(1997/11)`, `middle(1997/11)`, `end(1997/11)`. The following possibility functions can be defined for these instants:

$$\pi_{\mathtt{beginning(1997/11)}} \equiv_{def} (l(\langle 1997/11 \rangle),\ l(\langle 1997/11 \rangle),\ l(\langle 1997/11 \rangle) + a,\ l(\langle 1997/11 \rangle) + 2a)$$

$$\pi_{\mathtt{middle(1997/11)}} \equiv_{def} (l(\langle 1997/11 \rangle) + a,\ l(\langle 1997/11 \rangle) + 2a,\ l(\langle 1997/11 \rangle) + 3a,\ l(\langle 1997/11 \rangle) + 4a)$$

$$\pi_{\mathtt{end(1997/11)}} \equiv_{def} (l(\langle 1997/11 \rangle) + 3a,\ l(\langle 1997/11 \rangle) + 4a,\ u(\langle 1997/11 \rangle),\ u(\langle 1997/11 \rangle))$$

where a is equal to $\frac{u(i2g(1997/11)) - l(i2g(1997/11))}{5}$, i.e., $6 \cdot 24 \cdot 60 \cdot 60$ seconds, according to an equi-partition of the granule, i.e. November 1997, among the three considered possibility functions. Figure 2 graphically represents the three possibility functions. □

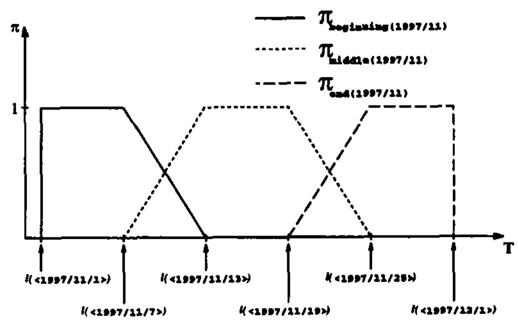

Figure 2: An example of possibility functions related to time instants given with vagueness.

More generally, we can define the following possibility functions for instants belonging to **Vag_I**, considering an equi-partition of the considered granule for the `beginning`, `middle`, and `end` qualifiers.

$$\pi_{beginning(i)} \equiv_{df} (L, L, L + \tfrac{U-L}{5},\ L + 2\tfrac{U-L}{5})$$

$$\pi_{middle(i)} \equiv_{df} (L + \tfrac{U-L}{5}, L + 2\tfrac{U-L}{5},\ L + 3\tfrac{U-L}{5},\ L + 4\tfrac{U-L}{5})$$

$$\pi_{end(i)} \equiv_{df} (L + 3\tfrac{U-L}{5}, L + 4\tfrac{U-L}{5},\ U,\ U)$$

where $L = l(i2g(i))$ and $U = u(i2g(i))$

Let us finally consider the possibility distribution related to the third case, i.e. instants given with indeterminacy. In this case, the possibility functions can be obtained by combining those functions related to the instants used to express instants with indeterminacy. More precisely, we use the classical intersection between possibility functions (Dubois & Prade 1989):

$$\forall i, j \in \mathbf{Gran\_I} \cup \mathbf{Vag\_I}$$
$$\pi_{in\_in(i,j)} \equiv_{df} possibly\_after\_\pi_i \cap possibly\_before\_\pi_j$$

Example 5 Let us consider the instant `≺end(1998/11), 1999/01/15≻`, representing a time point located between the end of November, 1998 and January 15, 1999. The related possibility function can be obtained as intersection of the functions $\pi_{\mathtt{end(1998/12)}}$ and $\pi_{\mathtt{1999/1/15}}$, as shown in Figure 3.

$\pi_{in\_in(\mathtt{end(98/11),99/01/15})} \equiv_{df}$
$possibly\_after\_\pi_{\mathtt{end(98/11)}} \cap possibly\_before\_\pi_{\mathtt{99/01/15}} \equiv$
$(l(i2g(\mathtt{1998/11/19})),\ l(i2g(\mathtt{1998/11/25})),$
$l(i2g(\mathtt{1999/1/16})),\ l(i2g(\mathtt{1999/1/16/2})))$ $\square$

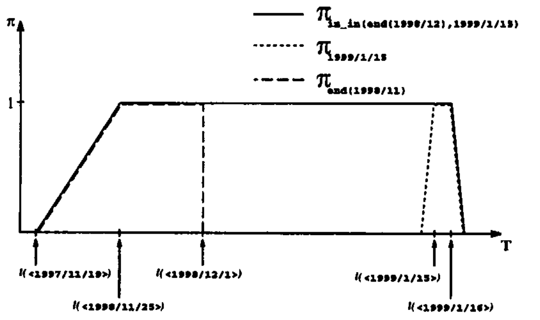

Figure 3: An example of possibility functions related to time instants given with indeterminacy.

There are obviously some constraints in defining instants with indeterminacy: intuitively, it is not possible to define an interval of indeterminacy having the first bound following the second one. More precisely, in our case, the two possibility functions related to the bounds of an interval of indeterminacy must satisfy the following condition:

$$inf(t : \pi_i(t) = 1) < inf(u : \pi_j(u) = 1)$$

By this condition, we are sure that the resulting possibility function is a usual trapezoidal function.

Time intervals

A time interval refers to a set of contiguous time points. A starting instant, an ending instant, and a duration identify an interval. **Itvl** is the domain of variables and constants for intervals. For variables of interval type the functions $from(\cdot)$, $to(\cdot)$ return values belonging to the domain **Inst**; $dur(\cdot)$ returns a value belonging to the domain **Dur**. Different functions allow us to define interval constants on the basis of granules, instants, and durations. These functions, we call *interval constructors*, are left implicit in the following of the paper, using the symbol $\langle ... \rangle$ for expressing intervals in different ways. We will use the following notations, according to the different natural language expressions used to define an interval.

- Notation 1. $\langle$`YY`$\rangle$, or $\langle$`YY/MM`$\rangle$, or $\langle$`YY/MM/DD`$\rangle$, and so on, when the interval is a granule of the Calendar, e.g. $\langle$`1994/10`$\rangle$; this notation is used to model intervals given by sentences like "the year 1994", "January '89". By these different notations we refer to intervals given as granules at one of the granularities of the Gregorian Calendar. The interval constructor corresponding to this kind of notation is the function *gran_int*: $2^{\mathbf{T}} \rightarrow \mathbf{Itvl}$.
- Notation 2. $\langle from(x),\ to(x) \rangle$ when starting and ending instants are given, e.g. $\langle$`end(94/10/10),95/3/2/15`$\rangle$; this notation is used to model intervals given by "from ... to ..." sentences. This is the usual way to express intervals in temporal databases (allowing only one granularity without any vagueness). The interval constructor corresponding to this kind of notation is the function *from_to*: $2^{\mathbf{Inst}} \rightarrow \mathbf{Itvl}$.
- Notation 3. $\langle from(x),\ dur(x) \rangle$ when starting instant and duration are given, e.g. $\langle$`middle(94/10/10),` $\prec$`3 h, 3 h 30 mi`$\succ\rangle$; this notation is used to model intervals given by "from ... for ..." sentences. The interval constructor corresponding to this kind of notation is the function *from_for*: $\mathbf{Inst} \times \mathbf{Dur} \rightarrow \mathbf{Itvl}$.
- Notation 4. $\langle dur(x),\ to(x) \rangle$ when ending instant and duration are given, e.g. $\langle$`33 h,96/10`$\rangle$; this notation is used to model intervals given by "for ... to ..." sentences. The interval constructor corresponding to this kind of notation is the function *for_to*: $\mathbf{Dur} \times \mathbf{Inst} \rightarrow \mathbf{Itvl}$.
- Notation 5. $\langle in,\ dur(x) \rangle$, where in is a granule; this notation allows one to express intervals given by "in ... for ..." , e.g., $\langle\langle$`94/10`$\rangle$`,6mi3s`$\rangle$. The interval constructor corresponding to this kind of notation is the function *in_for*: $2^{\mathbf{T}} \times \mathbf{Dur} \rightarrow \mathbf{Itvl}$.
- Notation 6. $\langle from(x),\ dur(x),\ to(x) \rangle$ when both starting instant, duration, and ending instant are given, e.g. $\langle$`96/9/10,` $\prec$`4 h 6 mi 3 s, 24 h 5 mi 2 s`$\succ$`, 96/9/11`$\rangle$; this is the more general notation, allowing to express also all the intervals expressible by the previous notations. The interval constructor corresponding to this kind of notation is the function *from_for_to*: $2^{\mathbf{Inst}} \times \mathbf{Dur} \rightarrow \mathbf{Itvl}$.

Some constraints and relations exist between the possibility distributions of instants and durations of an interval: for example, given the possibility distributions for the starting instant and for the duration of an interval, the possibility distribution of the ending instant can be computed by the fuzzy arithmetic addition of the two given distributions (Dubois & Prade 1989). In general, we have to define constraints which allow us to have some kind of "consistency" between the possibility distributions of starting and ending instants, and duration. A simple constraint consists in disallowing overlapping distributions for starting and ending instants (Dubois & Prade 1989);nevertheless, in this way we cannot express, for example, intervals having starting and ending instants in the same granule. A detailed analysis of more general constraints between possibility distributions for instants and duration of an interval is out of the scope of this paper, and will be the focus of future works.

Temporal Relations

Formulas and predicates in our model return truth values of the set MV_logic $= [0,1] \subset \mathbf{R}$. The meaning of the formulas $\neg A$, $A \wedge B$, $A \vee B$, $A \rightarrow B$ is given as follows:

$$\neg A \equiv_{df} 1 - A$$

$$A \wedge B \equiv_{df} min(A, B)$$

$$A \vee B \equiv_{df} max(A, B)$$

$$A \rightarrow B \equiv_{df} min(1 - A + B, 1).$$

Relations between instants, durations, and intervals are predicates in the previously introduced multivalued logics. The basic idea in defining different relations is to use possibility theory as the basis for the definition of multivalued predicates: fuzzy sets allow us to represent approximate temporal information and to evaluate predicates, which return (crisp) truth values that range from "complete truth" to "complete falsity" in a seamless way (Panti 1998). In the following, for space restrictions, we briefly sketch the definitions of main relationships between instants.

Definition 1 *Let be i a generic (fuzzy) instant or duration; we define* measure *of π_i as* $|\pi_i| \equiv_{df} \int_{-\infty}^{+\infty} \pi_i(t)dt$

On the basis of this definition, we are able to define the following relations between instants (relations between durations are defined similarly).

Definition 2 *Let be i and j two elements of domain **Inst**; the relation $before(i, j)$: **Inst** $\times$ **Inst** $\mapsto$ MV_logic, is defined as*

$$before(i,j) \equiv_{df} min\left(\frac{|poss\_after\_\pi_i \cap poss\_after\_\pi_j|}{|\pi_i|+|\pi_j|}, 1\right)$$

Definition 3 *Let be i and j two elements of domain **Inst**; the relation $after(i, j)$: **Inst** $\times$ **Inst** $\mapsto$ MV_logic, is defined as*

$$after(i,j) \equiv_{df} 1 - before(i,j)$$

Definition 4 *Let be i and j two elements of domain **Inst**; the relation $equal(i, j)$: **Inst** $\times$ **Inst** $\mapsto$ MV_logic, is defined as*

$$equal(i,j) \equiv_{df} \frac{1 - max(before(i,j) - after(i,j), after(i,j) - before(i,j))}{1 + abs\left(\frac{|\pi_i| - |\pi_j|}{|\pi_i| + |\pi_j|}\right)}$$

Further relations can be defined to perform comparisons at different levels of granularity. Relations between instants and durations can suitably composed to define the usual interval relations, i.e. equal, meets, finishes, overlap, and so on.

Conclusions

In this paper we have briefly described main features of a data model, allowing one to represent absolute time expressions involving different granularities, indeterminacy and vagueness. The novelty of our work is related to the seamless representation of such kind of temporal information; our approach is methodologically based on the integration of granularity systems, which provide a tool for defining granularities and their relationships, and possibility theory, which allow to represent approximate temporal knowledge. Relations between instants/durations/intervals are evaluated according to a multivalued logics.
This is only a first step towards providing temporal databases with the capability of expressing and querying data with heterogeneous valid times; further work is needed, for example, to define suitable query languages and to design and implement efficient algorithms for evaluating formulas and predicates on temporal data.

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