Memex Music and Gambling Games: EVE's Take on Lucky Number 13

Kevin Burns

The MITRE Corporation
202 Burlington Road
Bedford, MA 01730-1420 USA
kburns@mitre.org

Shlomo Dubnov

University of California in San Diego
9500 Gilman Drive
La Jolla, CA 92093-0326 USA
sdubnov@ucsd.edu

Abstract

Memex is an algorithmic composition, created by a computer program that operates on a database of works by Bach, Mozart and Beethoven. The program steps forward with probability Q and jumps backward with probability P=1-Q along note links in this database – where the P/Q odds are set by a human artist. Coincidentally, the backward/forward odds used in composing Memex are the same as the win/loss odds of a typical slot machine, i.e., P=0.13. Here we use a computational theory of aesthetics to draw a cognitive connection between the "flow" of Memex and the "fun" of gambling. Each aesthetic experience is quantified in terms of marginal entropies, and the computed results are seen to be similar when plotted in the form of "Goldilocks functions" – which show how pleasure varies with probability P. This finding suggests that pleasure may be modeled by information gains in these and perhaps other media experiences.

Introduction

The field of computational aesthetics encompasses many domains (Argamon and Dubnov 2004), yet research studies have typically focused on a single domain. The contribution of this paper is to perform and relate aesthetic analyses across two domains, i.e., music and game play.

In game play, the aesthetic of interest can be characterized as *fun* or *enjoyment*. Historically, things that are fun have not been subject to much study from an aesthetic perspective. More recently, game play has become an important area for aesthetic assessment in digital media (Prensky 2001), but current theories of fun (Koster 2005) are still more colloquial than computational (Burns 2006b).

In music, the aesthetic of interest can be characterized as *flow* (Csikszentmihalyi 1991, Dubnov and Assayag 2005). This sounds more serious than *fun*, but in fact both fun and flow boil down to *enjoyment* – which suggests a cognitive connection between the two. Historically, there have been many analyses of aesthetics in music, for example Meyer's treatment of meaning and emotion (1956) and Narmour's theory of implication-realization (1990).

Recently, mathematical formulas such as Zipf's Law have been used for computing aesthetics in music (Manaris et al. 2005). In this approach, a functional form is assumed (not derived) for the frequencies of features that lead to aesthetic experience – without addressing the mental means by which listeners internalize and anticipate these features. Cancho and Solé (2003) show how Zipf's Law, in human language, can arise as an emergent property of a *game* in which the speaker and hearer interact to minimize a joint cost or *energy* function. In short, these authors use a Bayesian-information approach to derive a functional form like Zipf's Law, rather than assuming it at the outset.

Here we adopt a similar approach to gain a fundamental understanding of aesthetic experience, by deriving (not assuming) a functional form from first principles – and by addressing mental models in the aesthetic analysis.

As one example of our approach, Dubnov et al. (2006) have derived an information-theoretic measure of aesthetics in music. Their measure is a difference in entropies, computed from conditional probabilities based on Markov models. As another example, Burns (2006a, 2006b, 2006c) has derived Bayesian-information measures of aesthetic experience that apply to sketching, game play and other domains.

Because our approach is both Bayesian and information-theoretic, it goes beyond the work of Bense (1965) and others (Staudek 2002), who use information theory to compute the terms of an aesthetic formula first proposed by Birkoff (1933). Later we discuss the limitations of a formula like Birkhoff's, and we note that the same limitations apply to deterministic complexity ratios like those of Machado and Cardoso (1998).

Details of the Bayesian-information theory (EVE') we apply here are discussed elsewhere (Burns 2006a, 2006b, 2006c). Below we provide only a brief summary of the main points as they apply to our analysis of music and game play.

Per EVE', the atomic components of aesthetic experience are *Expectation* (E), *Violation* (V) and *Explanation* (E'). E-V-E' is a cognitive progression that repeats in time as new signals are received and new meanings are assigned and new feelings are evoked. The theory posits two types of pleasure, both involving subjective success in cognitive processing.

One type is pleasure p, which arises from success in forming Expectations (E) that indeed pan out. The other type is pleasure-prime p', which arises from success in forming Explanations (E') for Violations (V) of Expectations (E) that did not pan out.

Total pleasure is the sum of p and p', where p is given by EVE's measure of E scaled by a factor G, and p' is given by EVE's measure of E' scaled by a factor G'. Thus, the total pleasure, called *fun* or *flow* (F), is computed as F = G * E + G' * E'.

Gambling Games

A casino slot machine can be modeled by a set of possible payoffs and associated probabilities. Here, to simplify the analysis (also see Burns 2006b), we aggregate the set to model the machine with a single probability P of a single non-zero payoff J. This J is ≈ 1/P, but slightly less because P*J is set ≈ 0.95 so the house makes ≈ 5% profit. P is computed from the number of ways to hit a payoff combination, divided by the number of possible combinations. P for a real slot machine is approximately 0.13, and in fact it is exactly 0.13 for the popular machine analyzed by Scarne (1961).

Since the expected utility is slightly negative (5%), we suggest that slots are best analyzed as an *informational* game, which a player can win, rather than a *financial* game, which a player cannot win (long term). And, since the casino can set P at any value and still make the same average profit, we suggest that P is set at 0.13 because it makes slot play the most fun for slot players (so they play the most games). The question is: Why does P=0.13 make slots most fun?

Below we outline EVE's equations for this game, as derived elsewhere (2006b), with the intent of comparing these equations to similar equations we derive later for music. The equations are based on Bayesian-information measures of Expectations (E) and Explanations (E').

To begin, EVE's expression for E is the negative entropy (Shannon and Weaver 1949) for two possible events with probabilities P and Q=1-P, as follows:

E = [P * log P + Q * log Q]

Here, log P is a measure of Expectation for the event "payoff J", and it is multiplied by the actual frequency (probability) of this event to obtain an average measure of E_J over a set of games. Likewise, Q * log Q is a measure of E_0 for the event "payoff 0". The expression for V is -E, as follows: V = - [P * log P + Q * log Q]. The expression for E' is the same as V, except for weighting factors H^+ and H^- that reflect a player's *sense of humor* in slot play, and factors R_J and R_0 that reflect the degree of *Resolution* ($0 \le R \le 1$) for each Violation. Thus:

E' = - [H^+ * P * log P * R_J + H^- * Q * log Q * R_0]

Here it is assumed (Burns 2006b) that H^+=1.0 and H^-=0.5, which means that a slot player has a good sense of slot humor. It is also assumed that $|R_J|=|R_0|=1$, hence:

E' = - [P * log P - 0.5 * Q * log Q]

Here we have assumed that all Violations will be completely resolved (magnitude 1) with *pleasure* (R_J = +1) when the player has a win, or *displeasure* (R_0 = -1) when the player has a loss. This assumption is reasonable for a simple game of luck like slots, as discussed elsewhere (Burns 2006b). But in general, a more detailed analysis is needed to get R values, as discussed below under "Other Games" and "Memex Music".

Now combining E and E', along with scaling factors G and G', EVE's equation for fun F is as follows:

F = G * [P * log P + Q * log Q]
   - G' * [P * log P - 0.5 * Q * log Q]

When this F is plotted versus probability (P), the first term (re: E) is a bowl-shaped curve and the second term (re: E') is an S-shaped curve. The combination is positive (net fun) only if G'>G. Using a ratio G/G'=1/3, as discussed elsewhere (Burns 2006a, 2006b), the result is an S-shaped curve as shown in Figure 1.

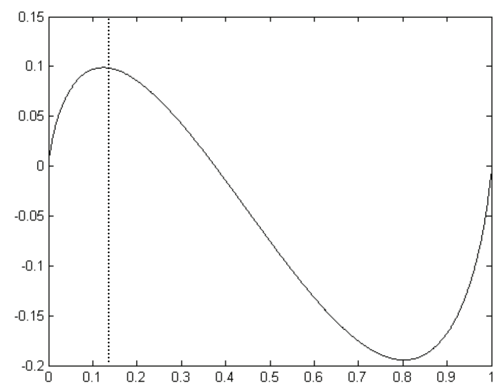

Figure 1. Fun in slot game (F vs. P), by EVE' (curve) and data (line).

This curve (Figure 1) is called a *Goldilocks function*, because it shows the sweet spot of pleasure where P is not too small and not too large – but just right for peak fun. Here we see peak fun when P is around 10%. In the limit as G/G' goes to zero, fun is given by E' alone, which peaks when P is around 0.15. These values compare well to the P=0.13 of a real slot machine, noted above.

This match to the P of real slot machines shows how fun can be modeled and measured by signed and weighted entropies. In short, when EVE's F is plotted versus P, the *fun in slots* can be seen as arising largely from a difference in *marginal entropy* between good outcomes and bad outcomes in the E' of EVE'.

Other Games

In a similar analysis (Burns 2006b), EVE' was also applied to games of skill, which can be modeled by a win probability P that reflects a player's level of skill. The main difference between slot games and skill games lies in the Bayesian modeling of *Resolutions* (R) for Violations (V), which are quite different and in fact opposite for games of skill than for games of luck.

In the analysis of skill games (Burns 2006b), EVE' was used to compute a measure of fun, which compared well to data on players' judgments of enjoyment (versus margin of victory) in a simple video boxing game called "Punch Out" (Piselli 2006). Further details are discussed elsewhere (Burns 2006b), as the main points to be made here are merely as follows: (1) EVE's equations can also be applied to other games. (2) The ratio G/G'=1/3 (see above) was also assumed for Punch Out, hence the match of EVE' theory to actual data for Punch Out offers further support for this same ratio in gambling games (above) and Memex music (below).

Memex Music

"Memex" (Dubnov 2006) is an algorithmic composition. The name refers to a fictional machine (Bush 1945) that was, at the time, a futuristic device for creating and recalling associations in the form of memory trails. "Memex" (the music) was composed by a computer program that was designed to create music from structure without explicitly specifying this structure in the form of deterministic knowledge – but rather by implicitly associating snapshots of music with other snapshots of music along probabilistic trails.

The program works by taking a constrained-random walk through an indexed database of works by Bach, Mozart and Beethoven. The database (Dubnov et al. 2003) is designed such that each note adds to a string of preceding notes called a "suffix". The database is indexed such that, at each note, there is a pointer forward to the next note in the "old" music. There is also a pointer backward to a previous note, namely the previous note whose suffix is the longest in common with the current note's suffix. This allows "new" music to be composed in a constrained-random walk, where each note in the new sequence is either a step *forward* or a jump *backward* (before advancing forward) along suffix links.

Like a series of payoffs (games) in slot play, the sequence of events (notes) in Memex is based on a note-wise probability P for jump backward and Q=1-P for step forward. The jump backward is similar to the P of a "jackpot" payoff event in a slot machine. The step forward is similar to the Q of a "zero" payoff event in a slot machine.

The value of P used for Memex was set by the human artist who wrote the program, by trial-and-error "tuning", until the composed music sounded best to him. This occurred at P=0.13. At the time the artist did not know that the P for a slot machine was also 0.13.

This composer's optimization of the Memex song at P=0.13 is coincidentally identical to the casino's optimization of a slot machine at P=0.13. Below we explore a common basis for this coincidence, using EVE's equations to analyze Memex music much like we did for the gambling game.

Applying EVE' to Memex, at each event in the music there is a measure of *Expectation* (E). EVE's measure is E_B = log P for a step backward, and E_F = log Q for a step forward. Like the case of a slot machine, these measures assume that the listener has a mental model of the machine's P (and Q=1-P) in his head as a basis for forming Expectations (but see "Learning" below). Now, multiplying these measures of E by their frequency of occurrence, the average measures for E_B and E_F are P * log P and Q * log Q, so the total measure of *Expectation* (E) is as follows:

$$E = [P * \log P + Q * \log Q]$$

This is the same as the equation for E in slots (see above). The measures of *Violation* (V) are $V_B = -E_B$ and $V_F = -E_F$, and these will give rise to measures of *Explanation* (E') based on the sign and size of *Resolution* (R) for the Violations. In the case of Memex (unlike slot games), it is reasonable to assume that all resolutions will give rise to pleasure – i.e., that any resolution *causes pleasure*. A lack of resolution will *prevent pleasure*, but unlike game play we assume that for music there is no resolution that causes displeasure like the bad feeling of a gambling loss. [This may be different if the listener does not like the classical composers in Memex's database.]

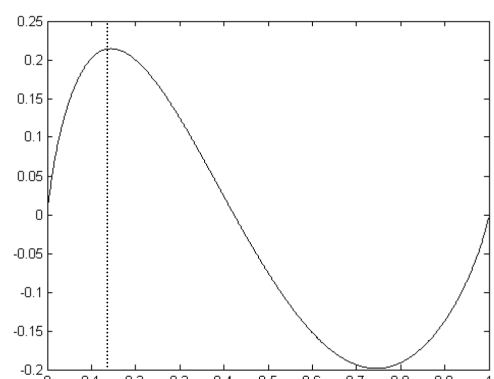

Figure 2. Flow from Memex (F vs. P), by EVE' (curve) and data (line).

In short, unlike game play, for music we assume that all Resolution (R) will bring pleasure – but not all Violations will be completely resolved. As a first order approximation, assuming that the listener is relatively familiar with the classical composers in the database, a simple measure of R is the non-linear function Q^2.

This function is admittedly speculative and is proposed only as a gross and average measure of R. Nevertheless, the function is reasonable because, if Q is large, then there is a high probability that the music will continue in step-forward fashion – and hence a high probability that a given jump backward or step forward will be "recognized" as fitting a model (motif). Moreover, it is reasonable to expect that both R and the rate of increase in R will increase with Q, hence $R \sim Q^2$ rather than $R \sim Q$. Finally, as discussed further under S-shapes (below), the final result (F) of EVE's calculation will be similar for any function R that decreases with P (increases with Q).

Thus, assuming $R_B = R_F = Q^2$, the total measure of pleasure from E and E' is as follows:

$$F = G * [P * \log P + Q * \log Q] - G' * Q^2 * [P * \log P + Q * \log Q]$$

Compared to the equation for fun in slots (see above), we see two major differences in the term for E'. One difference is the additional factor Q^2 to account for incomplete Resolution. The other difference is the + sign to account for pleasure from any Resolution.

The above equation for F is plotted in Figure 2, using the same scaling ratio G/G'=1/3 used in previous analyses (see above). Note that the peak F is near P=0.13, much like it was for fun in slots (Figure 1). Clearly the factors driving this mode are different in the two media, and in that sense it is a coincidence that the peak F is at P=0.13 for both music and game play. But beneath this coincidental result there are computational reasons for the common mode, as discussed further below.

Of course the above approach is extremely simplified, for both gambling and music. In particular, the analysis assumed that the temporal aesthetics of atomic events – like payoffs in gambling and note links in music – can be reduced to a time-averaged function F of a single parameter P (and Q=1-P). Nevertheless, this simple model does provide some insight into the common mode for F at P=0.13 that arises in S-shaped Goldilocks functions (Figures 1 and 2). The question now is: Why S-shapes?

S-Shapes

To see how the S-shapes arise, as well as to gain further insight into EVE's tradeoff between E and E', it is useful to re-write the function F as a product of two terms rather than as a sum of two terms. Here we focus on the case of Memex and re-write the expression for F as follows:

$$F = [-G * (P * \log P + Q * \log Q)] * [G'/G * Q^2 - 1]$$

In this form we see two terms in brackets. The first term is a G-weighted *entropy*, which is a measure of unpredictability or *surprise*. One might liken this to the perceived "complexity" of the song. The second term is a *conversion* factor, which multiplies entropy by a factor that measures the degree to which surprise is, on average, *resolved*. One might liken this to the perceived "order" of the song. With these two terms, EVE's product of *surprise * resolve* (see above) is much like the aesthetic measure (M) of Birkhoff (1933), defined as M = O/C. Birkhoff's O is "order" like EVE's *resolve*, i.e., the conversion factor, and Birkhoff's C is "complexity" like EVE's *surprise*, i.e., the entropy factor. But, unlike EVE's *product*, Birkhoff's measure is a *quotient*.

Per EVE', F goes to zero when resolve goes to zero (total confusion) or when surprise goes to zero (total boredom). Birkhoff's M goes to zero when order O goes to zero, but M goes to infinity when complexity C goes to zero. This suggests that total boredom is complete beauty, which misses the common sense of "no pain, no gain".

Besides a more intuitive treatment of this limiting condition, EVE's F improves on other aspects of Birkhoff's measure and others (Bense 1965, Machado and Cardoso 1998, Staudek 2002) that use similar ratios. EVE' does so through mathematical modeling of the tradeoff between *surprise* and *resolve*, in a Bayesian formulation that considers both temporal and contextual factors (Burns 2006c).

To gain further insight into this tradeoff between surprise and resolve, EVE's measure of F (fun or flow) can be likened to a measure of *energy*. That is, the product of surprise * resolve can be seen as measure of how much *entropy* has been *converted* to energy. In these terms, an information gain in the listener's mind is much like an energy "felt" as fun or flow – where the S-shape of F arises from the product of *entropy* (surprise) and *conversion* (resolve).

Figure 3 plots the entropy factor versus P, where entropy is measured as -[P * log P + Q * log Q]. Figure 4 plots the conversion factor versus P, where conversion is measured by $[G'/G * Q^2 - 1]$. The product (Figure 2) is an S-shape.

Thus, to generalize, we might expect an S-shape for any product of an *entropy* factor (hump-shape) and a *conversion* factor when the latter decreases with P. And this suggests that the S-shape applies more generally to any aesthetic experience where people are dealing with entropy (surprise) and their conversion (resolve) ability is a decreasing function of a parameter P that underlies entropy.

Applying this same logic to game play, we can re-write EVE's equation for slot fun as the product of an *entropy* factor and a *conversion* factor, as follows:

$$F = [-G * (P * \log P + Q * \log Q)] * [\{(G'/G-1) * P * \log P - (0.5*G'/G+1) * Q * \log Q\} / (P * \log P + Q * \log Q)]$$

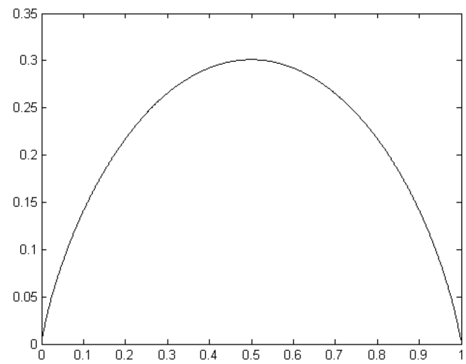

Figure 3. Plot of entropy versus P for Memex. The total pleasure F is entropy * conversion.

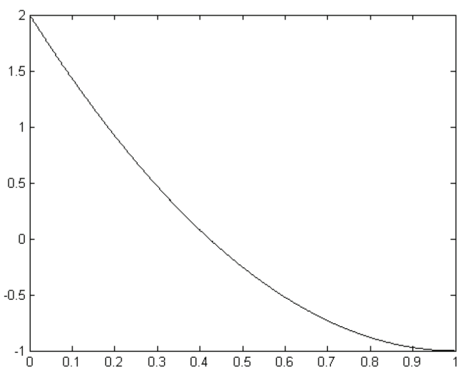

Figure 4. Plot of conversion versus P for Memex. The total pleasure F is entropy * conversion.

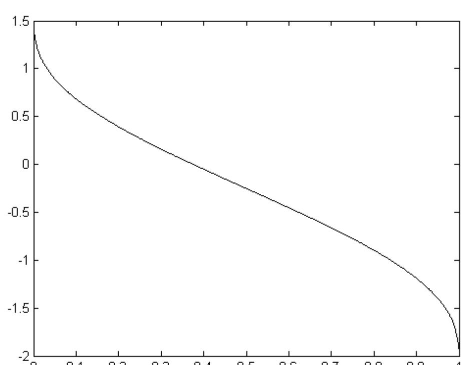

Figure 5. Plot of conversion versus P for slot game. The total pleasure F is entropy * conversion.

The entropy factor (first term) is the same as that shown in Figure 3. The conversion factor (second term) is plotted in Figure 5. As expected, the conversion factor decreases with P.

Thus, to summarize: In a psychological analysis, the average fun or flow of aesthetic experience can be expressed as a sum, F = G*E + G'*E', where the first term gives average pleasure from subjective success in forming Expectations (E), and the second term gives average pleasure-prime from subjective success in forming Explanations (E'). In a physical analogy, the same sum can be expressed as a product F = *surprise* * *resolve*, where the first term is an entropy factor and the second term is a conversion factor.

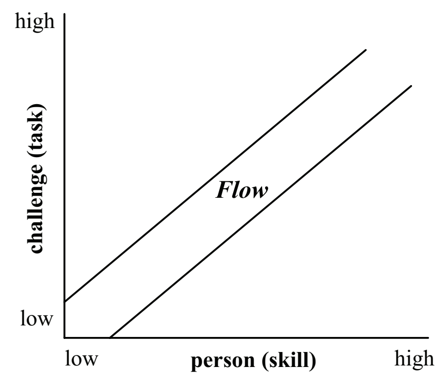

Figure 6. Plot of flow. Adapted from Csikszentmihalyi (1991).

A similar idea is found in the theory of "flow" popularized by Csikszentmihalyi (1991), who draws the "channel" of flow (see Figure 6) as a tradeoff between difficulty (y-axis) and ability (x-axis) – where flow occurs when the task matches one's skill. This is similar to EVE's tradeoff between E and E', which can be expressed as a product of surprise and resolve. But it is not clear exactly how one would go about computing the magnitude of difficulty (y-axis) or ability (x-axis) in Figure 6.

EVE's advantage is that the theory provides both a psychological basis and a mathematical framework for modeling and measuring aesthetics computationally.

Learning

A key assumption in the above analysis is that the perceiver's mental models are accurate. That is, the log P and log Q terms, which reflect mental beliefs, were multiplied by P and Q, which reflect actual frequencies. And this raises the question: What if the mental models for probabilities P and Q do not match the actual frequencies for P and Q?

Here we use P_m and Q_m to denote the mental probabilities in a listener's head, and we use P_s and Q_s to denote the actual frequencies in a song or slot game. With this distinction, the entropy term -[P * log P + Q * log Q] becomes $-[P_s * \log P_m + Q_s * \log Q_m]$, where the latter is always greater than the former by an amount known as the Kullback-Leibler (KL) divergence. Thus, when a listener's models for P and Q do not match the actual models, then the entropy factor in EVE's function F will be higher. This is interesting because it suggests that fun or flow may actually be enhanced when the perceiver (of a song) or player (of a game) does not have an accurate mental model.

However, EVE's function F is more than just entropy; it is a product of both an entropy factor and a conversion factor, so the potential for more fun or flow depends on both factors. Figures 7 and 8 plot the product (F) for slot play and Memex. In each case, the dotted lines assume that the mental Q is 0.75 times the actual Q, and the dashed lines assume that the mental P is 0.75 times the actual P. All results for F are plotted versus the actual P.

In other words, the *dotted* lines show F when the mental P is *optimistic* (higher) than actual, and the *dashed* lines show F when the mental P is *pessimistic* (lower) than actual.

Comparing the dashed and dotted lines to the solid lines (KL=0), we see an interesting difference between Figures 7 and 8. In Figure 7, for slot play, pleasure (fun) is always *lower when* $P_m > P_s$ (dotted line), and pleasure (fun) is always higher when $P_m < P_s$ (dashed line). This is interesting because it suggests that "wishful thinking" (i.e., $P_m > P_s$) in slot play is not the source of fun – and in fact, to the contrary, wishful thinking actually reduces the computed measure fun.

In short, our results suggest that a player with an inaccurate mental model will find slots more fun when his mental model is biased pessimistically (dashed line) than when it is accurate or biased optimistically (dotted line).

On the other hand, the Memex plot in Figure 8 shows that peak pleasure (flow) is increased when there is *any inaccuracy* in the listener's mental model – either $P_m>P_s$ or $P_m<P_s$ – but the increase in peak flow is higher for the latter case (dashed line) than the former. This is interesting because it suggests that one reason why people may become bored of the same song after they have heard it many times is that KL goes to zero as they "learn" the song, so they will get less pleasure from the peak P at which the song was composed.

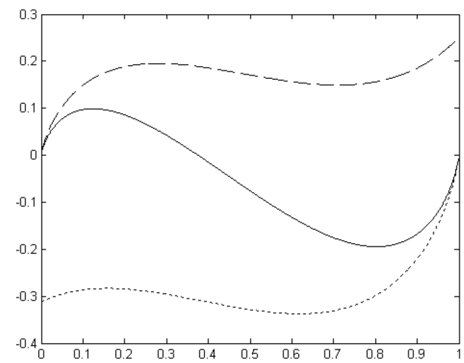

Figure 7. Plot of fun function F versus P for slot game, showing the base case with $P_m=P_s$ as well as two cases with $P_m \neq P_s$. The dashed line is for $P_m<P_s$. The dotted line is for $P_m>P_s$. The plot shows that, compared to the base case, the latter case of "wishful thinking" actually reduces fun in slot play.

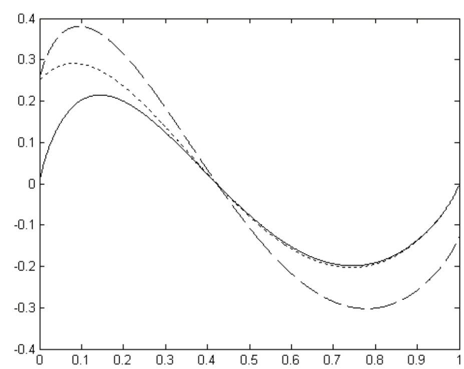

Figure 8. Plot of flow function F versus P for Memex, showing the base case with $P_m=P_s$ as well as two cases with $P_m \neq P_s$. The dashed line is for $P_m<P_s$. The dotted line is for $P_m>P_s$. The plot shows that, compared to the base case, peak F is higher for either $P_m<P$ or $P_m>P_s$.

The same insight may explain why people enjoy listening to the same music or hearing the same stories over and over again – i.e., if they "fool" themselves and "pretend" not to know the structure (model) of the music or story at some level akin to P and Q. Since the KL divergence is driven by subjective beliefs, it is possible that people can pretend in order to increase their pleasure. And perhaps this pretending is a good way to increase one's enjoyment of media experiences.

Conclusion

This study was motivated by a common mode of peak pleasure at P=0.13, which was found in both gambling games and Memex music. Using a Bayesian-information theory (EVE'), we analyzed the underlying basis for this coincidence and computed "Goldilocks functions" that plot pleasure F (fun or flow) versus probability (P).

The resulting S-shaped curves show how the same (or similar) mode arises at P=0.13, even though the underlying models and contextual details of the media experiences (game play versus music) are different. The theoretical formulation was given a physical interpretation – in which pleasure (fun or flow) is the product of an *entropy* (surprise) factor and a *conversion* (resolve) factor. In these terms, enjoyment can be seen as *energy* that one gets from entropy conversion.

This energy interpretation is encouraging, because it suggests that the psychology of pleasure is rooted in fundamental regularities of our physical embodiment and surrounding environment. It also suggests that more sensual (less cognitive) aesthetic experiences may also be amenable to analysis by EVE' in similar entropy-conversion modeling. This possibility, as well as extensions of EVE' to other domains of cognitive aesthetics, will be explored in future work.

References

Argamon, S., and Dubnov, S. 2004. Style and Meaning in Language, Art, Music and Design. *Proceedings of the 2004 AAAI Symposium*. Arlington, VA. Technical Report FS-04-07. http://music.ucsd.edu/~sdubnov/style2004.htm

Bense, M. 1965. *Aesthetica: Einführung in die neue Aesthetik*. Baden-Baden: Agis-Verlag.

Birkhoff, G. 1933. *Aesthetic Measure*. Cambridge, MA: Harvard University Press.

Burns, K. 2006a. Atoms of EVE': A Bayesian Basis for Aesthetic Analysis of Style in Sketching. *Artificial Intelligence for Engineering Design, Analysis and Manufacturing* 20:185-199.

Burns, K. 2006b. Fun in Slots. *Proceedings of the IEEE Conference on Computational Intelligence in Games*. Reno, NV.

Burns, K. 2006c. Bayesian Beauty: On the Art of EVE' and the Act of Enjoyment. *Proceedings of the AAAI'06 Workshop on Computational Aesthetics*. Boston, MA.

Bush, V. 1945. As We May Think. *The Atlantic Monthly*. July, 1945.

Cancho, R., and Solé, R. 2003. Least Effort and the Origins of Scaling in Human Language. *Proceedings of the National Academy of Sciences of the United States of America* 100(3):788-791.

Csikszentmihalyi, M. 1991. *Flow: The Psychology of Optimal Experience*. New York, NY: Harper Collins.

Dubnov, S. 2006. Memex (music) and Thoughts about Memex. http://music.ucsd.edu/~sdubnov

Dubnov, S., McAdams, S., and Reynolds, R. 2006. Structural and Affective Aspects of Music from Statistical Audio Signal Analysis. *Journal of the American Society for Information Science and Technology, Special Issue on Style*. Forthcoming.

Dubnov, S., and Assayag, G. 2005. Improvisation Planning and Jam Session Design Using Concepts of Sequence Variation and Flow Experience. *Proceedings of Sound and Music Computing*. Salerno, Italy.

Dubnov, S., Assayag, G., Lartillot, O., and Bejerano, G. 2003. Using Machine-Learning Methods for Musical Style Modeling. *IEEE Computers* 36(10):73-80.

Koster, R. 2005. *A Theory of Fun for Game Design*. Scottsdale, AZ: Paraglyph Press.

Machado, P., and Cardoso, A. 1998. Computing Aesthetics. *Proceedings of the Brazilian Symposium on Artificial Intelligence, SBIA'98*, pp. 219-229. New York, NY: Springer-Verlag.

Manaris, B., Romero, J., Machado, P., Krehbiel, D., Hirzel, T., Pharr, W., Davis, R. 2005. Zipf's Law, Music Classification and Aesthetics. *Computer Music Journal* 29(1):55-69.

Meyer, L. 1956. *Emotion and Meaning in Music*. Chicago, IL: University of Chicago Press.

Narmour, E. 1990. *The Analysis and Cognition of Basic Melodic Structures: The Implication-Realization Model*. Chicago, IL: University of Chicago Press.

Piselli, P. 2006. Relating Cognitive Models of Computer Games to User Evaluations of Entertainment. M.S. Thesis, Department of Computer Science, Worcester Polytechnic Institute.

Prensky, M. 2001. Fun, Play and Games: What Makes Games Engaging. In Prensky, M. *Digital Game-Based Learning*, Chapter 5. McGraw-Hill.

Scarne, J. 1961. *Scarne's New Complete Guide to Gambling*, pp. 445-448. New York, NY: Simon & Schuster.

Shannon, C., and Weaver, W. 1949. *The Mathematical Theory of Communication*. Urbana, IL: University of Illinois Press.

Staudek, T. 2002. Exact Aesthetics: Object and Scene to Message. Ph.D. Thesis, Faculty of Informatics, Masaryk University.