A Preference Logic With Four Kinds of Preferences

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Abstract

Decision making with varying kinds of preferences simultaneously is pervasive in realities. In such sights it is essential to know the interaction among the kinds of preferences and the general principles of reasoning with the kinds of preferences. In this paper, we introduce *MKPL*: a many-kinds-preferences logic, to formalize four kinds of preference. Firstly, we consider four kinds of preferences — four interpretations on sets of ordered alternatives. Secondly, we study such properties as reflexive, symmetric, transitive of every kind of preference on partial ordered alternatives, and study the transform of preference kinds considering the changes of subsets of alternatives. Then, we develop *MKPL* by extending proposition logic with four preference connectives. Moreover, we propose a decision procedure of *MKPL*.

Introduction

Preferences are essential for making intelligent choices in complex situations, for mastering large sets of alternatives, and for coordinating a multitude of decisions. In many problems, making choices is to find a solution, where a solution means an assignment of a certain value to each variable in given set of variables such that the value is taken from the finite domain of each domain. Without loss of generality, restrict our discussion to the Boolean case where the values for each variable are *true* or *false*. The number of assignments is exponential in the number of variables. For this reason it is, in general, impossible for a user to describe her preferences by all pairs of the preference relation among assignments. This is where logic comes into play.

From different disciplines, many definitions of preference are given in logical ways. [von Wright 1963, 1972] give definitions of strong, weak, and ceteris paribus preference. [Boutilier 1993, 1994] interprets ceteris paribus by "all other things being normal", whereas [Doyle and Wellman 1994] defines it as "all other things being equal" by mean of contextual equivalence. [Joseph Y. Halpern 1997] extend preference order on worlds to orders on sets of worlds by several approaches. [M. Freund 2004] introduce a logic of rational inferences and preferences. [Otterloo, and Roy 2005] study a whole spectrum of global preferences on $\forall\exists$, $\exists\forall$, $\forall\forall$, and $\exists\exists$ four patterns. E.g., someone many prefer getting a raise (R) over honors (H) can be expressed in a $\forall\exists$ pattern as "Every H-situation of honors can be topped by one of getting a raise". [J. Lang 2004] gives a survey of preferences defined with proposition goals.

A drawback of the present state of the art in the logic of preferences is that proposed logics typically formalize only preferences of one kind. Decision making with varying kinds of preferences simultaneously is a familiar sight in realities. Varying kinds of preferences can be used simultaneously. [S. Kaci and L. van der Torre 2005] first defines four basic preference kinds considering two agent types. Let V be a finite set of propositional variables, W the set of propositional interpretations, $\succeq$ a total pre-order on W, ϕ and ψ propositional logic well-formed formulas. The semantics of preferences formulas defined as follows:

- Locally optimistic: $\phi^M \geq^M \psi$: $\succeq\models\phi^M\geq^M\psi$ iff $\forall w\in\max(\phi\wedge\neg\psi, \succeq)$ and $\forall w'\in\max(\psi\wedge\neg\phi, \succeq)$, $w\succeq w'$.
- Locally pessimistic: $\phi^m\geq^m\psi$: $\succeq\models\phi^m\geq^m\psi$ iff $\forall w\in\min(\phi\wedge\neg\psi, \succeq)$ and $\forall w'\in\min(\psi\wedge\neg\phi, \succeq)$, $w\succeq w'$.
- Opportunistic: $\phi^M\geq^m\psi$: $\succeq\models\phi^M\geq^m\psi$ iff $\forall w\in\max(\phi\wedge\neg\psi, \succeq)$ and $\forall w'\in\min(\psi\wedge\neg\phi, \succeq)$, $w\succeq w'$.
- Careful: $\phi^m\geq^M\psi$: $\succeq\models\phi^m\geq^M\psi$ iff $\forall w\in\min(\phi\wedge\neg\psi, \succeq)$ and $\forall w'\in\max(\psi\wedge\neg\phi, \succeq)$, $w\succeq w'$.

Where $\max(\phi, \succeq)=\{w\in W| w\models\phi, \forall w'\in W: w\models\phi\Rightarrow w\succeq w'\}$, $\min(\phi, \succeq)=\{w\in W| w\models\phi, \forall w'\in W: w\models\phi\Rightarrow w'\succeq w\}$.

Then, they extend them into sixteen strict and non-strict kinds of preferences, and present algorithms to calculate the distinguished preference orders according to a set of preferences formulas.

[G. Brewka 2004] uses ranked knowledge bases together with various preference strategies to represent various kinds of preference, and expresses nested combinations of preference descriptions using various connectives. This gives the user the possibility to represent and reason varying kinds of preferences simultaneously.

Our focus in this paper will be based on qualitative preferences as four basic preference types of [S. Kaci and L. van der Torre 2005]. We hope to give the user the possibility to represent and reason her preferences in a more natural, concise and flexible manner by four approaches:

- Defining four more general kinds of preference based on partial pre-order on W than those based on total pre-order on W in [S. Kaci and L. van der Torre 2005].

- Ignoring von Wright's expansion principle in the new preference types. The principle means that a preference of ϕ over ψ is interpreted as a preference of $\phi\wedge\neg\psi$ over $\psi\wedge\neg\phi$.
- Regarding preference connectives as general propositional connectives, like $\wedge,\vee,\neg$ etc.
- Studying a whole proof system.

Firstly, we present four general preference types considering four interpretations on sets of partial-ordered alternatives. Secondly, we study such properties as reflexive, symmetric, transitive of every kind of preference on partial ordered alternatives, and study the transform of preference kinds considering the changes of sets of alternatives. Then, we develop *MKPL* by extending proposition logic with four preference connectives. Moreover, we propose a decision procedure with *MKPL*.

Four Kinds of Preferences

Let V be a finite set of propositional variables, W the set of propositional interpretations, ϕ and ψ propositional logic well-formed formulas. If $\succeq$ is a partial order on W, max(ϕ, $\succeq$) and min(ϕ, $\succeq$) may be empty, and are inexplainable. We present four more general preference types as follows:

- Locally optimistic: $\phi\,{}^{\exists}\!\geq^{\forall}\psi$: $\succeq\models\phi\,{}^{\exists}\!\geq^{\forall}\psi$ iff $\exists w\in\{w\in W|\ w\models\phi\}$ and $\forall w'\in\{w\in W|\ w\models\psi\}$, $w\succeq w'$.
- Locally pessimistic: $\phi\,{}^{\forall}\!\geq^{\exists}\psi$: $\succeq\models\phi\,{}^{\forall}\!\geq^{\exists}\psi$ iff $\forall w\in\{w\in W|\ w\models\phi\}$ and $\exists w'\in\{w\in W|\ w\models\psi\}$, $w\succeq w'$.
- Opportunistic: $\phi\,{}^{\exists}\!\geq^{\exists}\psi$: $\succeq\models\phi\,{}^{\exists}\!\geq^{\exists}\psi$ iff $\exists w\in\{w\in W|\ w\models\phi\}$ and $\exists w'\in\{w\in W|\ w\models\psi\}$, $w\succeq w'$.
- Careful: $\phi\,{}^{\forall}\!\geq^{\forall}\psi$: $\succeq\models\phi\,{}^{\forall}\!\geq^{\forall}\psi$ iff $\forall w\in\{w\in W|\ w\models\phi\}$ and $\forall w'\in\{w\in W|\ w\models\psi\}$, $w\succeq w'$.

Evidently, If $\succeq$ is a total pre-order on W, we can get that $(\phi\wedge\neg\psi)\,{}^{\exists}\!\geq^{\forall}(\psi\wedge\neg\phi)\Leftrightarrow\phi\,{}^{M}\!\geq^{M}\psi$, $(\phi\wedge\neg\psi)\,{}^{\forall}\!\geq^{\exists}(\psi\wedge\neg\phi)\Leftrightarrow\phi\,{}^{m}\!\geq^{m}\psi$, $(\phi\wedge\neg\psi)\,{}^{\exists}\!\geq^{\exists}(\psi\wedge\neg\phi)\Leftrightarrow\phi\,{}^{M}\!\geq^{m}\psi$, and $(\phi\wedge\neg\psi)\,{}^{\forall}\!\geq^{\forall}(\psi\wedge\neg\phi)\Leftrightarrow\phi\,{}^{m}\!\geq^{M}\psi$.

Example 1. Let $V=\{p, q\}$, $p\wedge q\succeq p\wedge\neg q\succeq\neg p\wedge q\succeq\neg p\wedge\neg q$. We can see that $\succeq\models p\,{}^{\forall}\!\geq^{\forall}q$, $\succeq\models\neg q\,{}^{\forall}\!\geq^{\forall}\neg p$, $\succeq\models\neg p\,{}^{\exists}\!\geq^{\exists}q$ etc.

Note that our preferences on ∀∃, ∃∀, ∀∀, and ∃∃ four patterns are different from those of [Otterloo, and Roy 2005]. E.g., "someone prefers getting a raise (R) over honors (H)" expressed in our ∀∃ pattern means "For each R-situation of getting a raise, there exist some H-situations of honor that are topped by it".

Properties and Transform

Above four kinds of preference mean four methods of going from an ordering on worlds to one on sets of worlds. With the uniform meaning, methods in [S. Kaci and L. van der Torre 2005] can only go from a total pre-order on worlds to one on sets of worlds, and our methods can go from the more real-life partial order on worlds to one on sets of worlds. A propositional formula ϕ represents a set of worlds $\{w\in W|\ w\models\phi\}$. We can study these preferences from the set theory discipline.

Properties of Order on Sets of Worlds

Proposition 1. Let Ω be a finite set of alternatives, $\boldsymbol{R}$ a partial order on Ω, A and B subsets of Ω. We define the following relations on sets of alternatives:

- $\boldsymbol{R}_{\exists\forall}$: $A\boldsymbol{R}_{\exists\forall}B$ iff $A\neq\varnothing$ and $B=\varnothing$, or $\exists a\in A$ and $\forall b\in B$, $a\boldsymbol{R}b$.
- $\boldsymbol{R}_{\forall\exists}$: $A\boldsymbol{R}_{\forall\exists}B$ iff $A\neq\varnothing$ and $B=\varnothing$, or $\forall a\in A$ and $\exists b\in B$, $a\boldsymbol{R}b$.
- $\boldsymbol{R}_{\exists\exists}$: $A\boldsymbol{R}_{\exists\exists}B$ iff $A\neq\varnothing$ and $B=\varnothing$, or $\exists a\in A$ and $\exists b\in B$, $a\boldsymbol{R}b$.
- $\boldsymbol{R}_{\forall\forall}$: $A\boldsymbol{R}_{\forall\forall}B$ iff $A\neq\varnothing$ and $B=\varnothing$, or $\forall a\in A$ and $\forall b\in B$, $a\boldsymbol{R}b$.

then

1) $\boldsymbol{R}_{\exists\exists}$ and $\boldsymbol{R}_{\forall\exists}$ are reflexive on 2^{Ω}-$\varnothing$.

2) $\boldsymbol{R}_{\exists\forall}$, $\boldsymbol{R}_{\forall\exists}$, and $\boldsymbol{R}_{\forall\forall}$ are transitive on 2^{Ω}.

Proof. 1) $\boldsymbol{R}$ is reflexive, then $\forall c\in\Omega$, $c\boldsymbol{R}c$ must hold. For any $A\in 2^{\Omega}$-$\varnothing$, $\forall a\in A\Rightarrow a\in\Omega$, $a\boldsymbol{R}a$ must hold, then $A\boldsymbol{R}_{\exists\exists}A$ and $A\boldsymbol{R}_{\forall\exists}A$.

2) $\boldsymbol{R}$ is transitive, then if $\forall a$, b, $c\in\Omega$, $a\boldsymbol{R}b$ and $b\boldsymbol{R}c$, $a\boldsymbol{R}c$ must hold. Let A, B, $C\in 2^{\Omega}$, then

- when $A\boldsymbol{R}_{\exists\forall}B$ and $B\boldsymbol{R}_{\exists\forall}C$, if $C=\varnothing$ $A\boldsymbol{R}_{\exists\forall}C$ must be true, else if $\exists a\in A$, $\forall b\in B$, $a\boldsymbol{R}b$ and $\exists b\in A$, $\forall c\in B$, $b\boldsymbol{R}c$, then $a\boldsymbol{R}c$ must be true, so $A\boldsymbol{R}_{\exists\forall}C$.
- when $A\boldsymbol{R}_{\forall\exists}B$ and $B\boldsymbol{R}_{\forall\exists}C$, if $C=\varnothing$ $A\boldsymbol{R}_{\forall\exists}C$ must be true, else if $\forall a\in A$ and $\exists b\in B$, $a\boldsymbol{R}b$ and $\forall b\in A$ and $\exists c\in C$, $b\boldsymbol{R}c$, then $a\boldsymbol{R}c$ must be true, so $A\boldsymbol{R}_{\forall\exists}C$.
- when $A\boldsymbol{R}_{\forall\forall}B$ and $B\boldsymbol{R}_{\forall\forall}C$, if $C=\varnothing$ $A\boldsymbol{R}_{\forall\forall}C$ must be true, else if $\forall a\in A$ and $\forall b\in B$, $a\boldsymbol{R}b$ and $\forall b\in A$ and $\forall c\in C$, $b\boldsymbol{R}c$, then $a\boldsymbol{R}c$ must be true, so $A\boldsymbol{R}_{\forall\forall}C$.

Transform of Preference Kinds

As showed in fig.1, preference reasoning with four kinds of preferences is to find complete preference relations $\boldsymbol{R}_{xy}$ (x,y∈{∀, ∃}) on 2^{Ω} that must be consistent with the known part of $\boldsymbol{R}_{xy}$. The part of $\boldsymbol{R}_{xy}$ on single element sets is just $\boldsymbol{R}$.

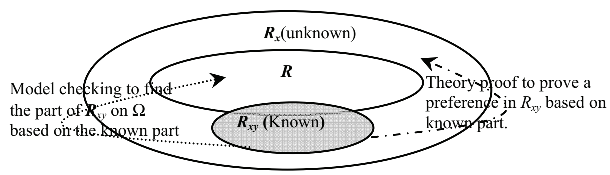

Fig.1. Preference Reasoning

We study preference reasoning in the following patterns:

(1) ⊆ patterns

- $C\subseteq A$, $A\boldsymbol{R}_{xy}B\Rightarrow C\boldsymbol{R}_{x'y'}B$
- $C\subseteq B$, $A\boldsymbol{R}_{xy}B\Rightarrow A\boldsymbol{R}_{x'y'}C$
- $A\subseteq C$, $A\boldsymbol{R}_{xy}B\Rightarrow C\boldsymbol{R}_{x'y'}B$
- $B\subseteq C$, $A\boldsymbol{R}_{xy}B\Rightarrow A\boldsymbol{R}_{x'y'}C$

(2) ∩ patterns

- $(A\cap C)\boldsymbol{R}_{xy}B\Rightarrow C\boldsymbol{R}_{x'y'}B$, $A\boldsymbol{R}_{x'y'}B$
- $A\boldsymbol{R}_{xy}(B\cap C)\Rightarrow A\boldsymbol{R}_{x'y'}B$, $A\boldsymbol{R}_{x'y'}C$
- $A\boldsymbol{R}_{xy}B\Rightarrow(A\cap C)\boldsymbol{R}_{x'y'}B$, $(A\cap C^{c})\boldsymbol{R}_{x''y''}B$
- $A\boldsymbol{R}_{xy}B\Rightarrow A\boldsymbol{R}_{x'y'}(B\cap C)$, $A\boldsymbol{R}_{x''y''}(B\cap C^{c})$

(3) ∪ patterns

- $(A\cup C)\boldsymbol{R}_{xy}B\Rightarrow C\,\boldsymbol{R}_{x'y'}B$, $A\boldsymbol{R}_{x''y''}B$
- $A\boldsymbol{R}_{xy}(B\cup C)\Rightarrow A\boldsymbol{R}_{x'y'}B$, $A\boldsymbol{R}_{x''y''}C$

- $A\boldsymbol{R}_{xy}B \Rightarrow (A\cup C)\boldsymbol{R}_{x'y'}B$, $(A\cup C^c)\boldsymbol{R}_{x''y''}B$
- $A\boldsymbol{R}_{xy}B \Rightarrow A\boldsymbol{R}_{x'y'}(B\cup C)$, $A\boldsymbol{R}_{x''y''}(B\cup C^c)$

By predigesting iterative results, we conclude the following proposition.

Proposition 2. Let Ω be a finite set of alternatives, $\boldsymbol{R}$ a partial order on Ω, A, B and C nonempty subsets of Ω. It can be concluded that

(1) for $\boldsymbol{R}_{\exists\forall}$

- $A\boldsymbol{R}_{\exists\forall}B, A\subseteq C\Rightarrow C\boldsymbol{R}_{\exists\forall}B$
- $A\boldsymbol{R}_{\exists\forall}B\Rightarrow(A\cap C)\boldsymbol{R}_{\exists\forall}B$, or $(A\cap C^c)\boldsymbol{R}_{\exists\forall}B$
- $A\boldsymbol{R}_{\exists\forall}B, B\subseteq C\Rightarrow A\boldsymbol{R}_{\exists\exists}C$
- $A\boldsymbol{R}_{\exists\forall}B, C\subseteq B\Rightarrow A\boldsymbol{R}_{\exists\forall}C$

(2) for $\boldsymbol{R}_{\exists\exists}$

- $A\boldsymbol{R}_{\exists\exists}B, A\subseteq C\Rightarrow C\boldsymbol{R}_{\exists\exists}B$
- $A\boldsymbol{R}_{\exists\exists}B\Rightarrow(A\cap C)\boldsymbol{R}_{\exists\exists}B$, or $(A\cap C^c)\boldsymbol{R}_{\exists\exists}B$
- $A\boldsymbol{R}_{\exists\exists}B, B\subseteq C\Rightarrow A\boldsymbol{R}_{\exists\exists}C$
- $A\boldsymbol{R}_{\exists\exists}B\Rightarrow A\boldsymbol{R}_{\exists\exists}(B\cap C)$, or $A\boldsymbol{R}_{\exists\exists}(B\cap C^c)$

(3) for $\boldsymbol{R}_{\forall\forall}$

- $A\boldsymbol{R}_{\forall\forall}B, C\subseteq A\Rightarrow C\boldsymbol{R}_{\forall\forall}B$
- $A\boldsymbol{R}_{\forall\forall}B, A\subseteq C\Rightarrow C\boldsymbol{R}_{\exists\forall}B$
- $A\boldsymbol{R}_{\forall\forall}B, B\subseteq C\Rightarrow A\boldsymbol{R}_{\forall\exists}C$
- $A\boldsymbol{R}_{\forall\forall}B, C\subseteq B\Rightarrow A\boldsymbol{R}_{\forall\forall}C$

(4) for $\boldsymbol{R}_{\forall\exists}$

- $A\boldsymbol{R}_{\forall\exists}B, A\subseteq C\Rightarrow C\boldsymbol{R}_{\exists\exists}B$
- $A\boldsymbol{R}_{\forall\exists}B, C\subseteq A\Rightarrow C\boldsymbol{R}_{\forall\exists}B$
- $A\boldsymbol{R}_{\forall\exists}B, B\subseteq C\Rightarrow A\boldsymbol{R}_{\forall\exists}C$
- $A\boldsymbol{R}_{\forall\exists}B\Rightarrow A\boldsymbol{R}_{\forall\exists}(B\cap C)$, or $A\boldsymbol{R}_{\forall\exists}(B\cap C^c)$

Proof. A, B, and C are nonempty.

(1) $A\boldsymbol{R}_{\exists\forall}B\Leftrightarrow\exists a\in A, \forall b\in B, a\boldsymbol{R}b$,

- $A\subseteq C\Rightarrow a\in C$, then $\exists c\in C$, $c\boldsymbol{R}b$. That is $C\boldsymbol{R}_{\exists\forall}B$.
- it must be $a\in A\cap C$ or $a\in A\cap C^c$, then $\exists c\in A\cap C, \forall b\in B$, $a\boldsymbol{R}b$ or $\exists c\in A\cap C^c, \forall b\in B$, $c\boldsymbol{R}b$. That is $(A\cap C)\boldsymbol{R}_{\exists\forall}B$, or $(A\cap C^c)\boldsymbol{R}_{\exists\forall}B$.
- $B\subseteq C\Rightarrow b\in C$, then $\exists c\in C$, $a\boldsymbol{R}c$. That is $A\boldsymbol{R}_{\exists\exists}C$.
- if $C\subseteq B$, then $\forall c\in C\Rightarrow c\in B$, and $a\boldsymbol{R}c$. That is $A\boldsymbol{R}_{\exists\forall}C$.

(2) $A\boldsymbol{R}_{\exists\exists}B\Leftrightarrow\exists a\in A, \exists b\in B, a\boldsymbol{R}b$,

- $A\subseteq C\Rightarrow a\in C$, then $\exists c\in C, \exists b\in B$, $c\boldsymbol{R}b$. That is $C\boldsymbol{R}_{\exists\exists}B$.
- it must be $a\in A\cap C$ or $a\in A\cap C^c$, then $\exists c\in A\cap C, \forall b\in B$, $a\boldsymbol{R}b$ or $\exists c\in A\cap C^c, \forall b\in B$, $c\boldsymbol{R}b$. That is $(A\cap C)\boldsymbol{R}_{\exists\forall}B$, or $(A\cap C^c)\boldsymbol{R}_{\exists\forall}B$.
- $B\subseteq C\Rightarrow b\in C$, then $\exists a\in A$, $b\in C$, $a\boldsymbol{R}b$. That is $A\boldsymbol{R}_{\exists\exists}C$.
- it must be $b\in B\cap C$ or $b\in B\cap C^c$, then $\exists a\in A$, $\exists c\in B\cap C$ or $\exists c\in B\cap C^c$, $a\boldsymbol{R}c$. That is $A\boldsymbol{R}_{\exists\exists}(B\cap C)$, or $A\boldsymbol{R}_{\exists\exists}(B\cap C^c)$.

(3) $A\boldsymbol{R}_{\forall\forall}B\Leftrightarrow\forall a\in A, \forall b\in B, a\boldsymbol{R}b$,

- $C\subseteq A$ then $\forall c\in C\Rightarrow c\in A$, and $c\boldsymbol{R}b$. That is $C\boldsymbol{R}_{\forall\forall}B$.
- $A\subseteq C\Rightarrow a\in C$, then $\exists c\in C$, $c\boldsymbol{R}b$. That is $C\boldsymbol{R}_{\exists\forall}B$.
- $B\subseteq C\Rightarrow b\in C$, then $\exists c\in C$, $a\boldsymbol{R}c$. That is $A\boldsymbol{R}_{\forall\exists}C$.
- $C\subseteq B$, then $\forall c\in C\Rightarrow c\in B$, and $a\boldsymbol{R}c$. That is $A\boldsymbol{R}_{\forall\forall}C$.

(4) $A\boldsymbol{R}_{\forall\exists}B\Leftrightarrow\forall a\in A, \exists b\in B, a\boldsymbol{R}b$,

- $A\subseteq C\Rightarrow a\in C$, then $\exists c\in C$, $c\boldsymbol{R}b$. That is $C\boldsymbol{R}_{\exists\exists}B$.
- $C\subseteq A$ then $\forall c\in C\Rightarrow c\in A$, and $c\boldsymbol{R}b$. That is $C\boldsymbol{R}_{\forall\exists}B$.
- $B\subseteq C\Rightarrow b\in C$, then $\exists c\in C$, $a\boldsymbol{R}c$. That is $A\boldsymbol{R}_{\forall\exists}C$.
- it must be $b\in B\cap C$ or $b\in B\cap C^c$, then $\exists c\in B\cap C$ or $\exists c\in B\cap C^c$, $a\boldsymbol{R}c$. That is $A\boldsymbol{R}_{\forall\exists}(B\cap C)$, or $A\boldsymbol{R}_{\forall\exists}(B\cap C^c)$.

The above principles give us a clear picture of four kinds of preference from set theory discipline.

MKPL Logic

We develop *MKPL* by extending proposition logic with four preference connectives ${}^{\exists}{\geq}^{\forall}$, ${}^{\forall}{\geq}^{\exists}$, ${}^{\exists}{\geq}^{\exists}$, and ${}^{\forall}{\geq}^{\forall}$.

Definition 1 (Language). Given a set $V=\{p_1, \ldots, p_n\}$ be a finite set of propositional variables, we define the set L_{MKPL} of *MKPL* formulas as follows.

- $p_i\in L_{MKPL}$
- $\phi,\psi\in L_{MKPL}\Rightarrow\neg\phi\in L_{\mathrm{PL}}, \phi\to\psi\in L_{MKPL}, \phi\,{}^{x}{\geq}^{y}\,\psi\in L_{MKPL}$, $x,y\in\{\forall, \exists\}$
- $\leftrightarrow, \vee, \wedge, \top, \bot$, (,) are defined as usually, ${}^{x}{\geq}^{y}$ has the same priority as that of $\vee$.

Definition 2 (Semantics). Given a preference model $M=(W, \boldsymbol{R})$, W the set of propositional interpretations 2^V, $\boldsymbol{R}$ a partial order on W, $w\in W$, we write w a set of variable, $p\in w$ iff p is true. The semantics of *MKPL* is defined as follows.

- $M, w\models p$ iff $p\in w$
- $M, w\models\neg\phi$ iff $M, w\not\models\phi$
- $M, w\models\phi\to\psi$ iff $M, w\models\neg\phi$, or $M, w\models\psi$
- $M, w\models\phi\,{}^{\forall}{\geq}^{\forall}\,\psi$ iff $M, w\models\phi$ and $\forall w'\in W(M, w'\not\models\psi)$, or $\forall w_1\in W(M, w_1\models\phi)$ and $\forall w_2\in W(M, w_2\models\psi)$, $w_1\boldsymbol{R}w_2$
- $M, w\models\phi\,{}^{\exists}{\geq}^{\forall}\,\psi$ iff $M, w\models\phi$ and $\forall w'\in W(M, w'\not\models\psi)$, or $\exists w_1\in W(M, w_1\models\phi)$ and $\forall w_2\in W(M, w_2\models\psi)$, $w_1\boldsymbol{R}w_2$
- $M, w\models\phi\,{}^{\exists}{\geq}^{\exists}\,\psi$ iff $M, w\models\phi$ and $\forall w'\in W(M, w'\not\models\psi)$, or $\exists w_1\in W(M, w_1\models\phi)$ and $\exists w_2\in W(M, w_2\models\psi)$, $w_1\boldsymbol{R}w_2$
- $M, w\models\phi\,{}^{\forall}{\geq}^{\exists}\,\psi$ iff $M, w\models\phi$ and $\forall w'\in W(M, w'\not\models\psi)$, or $\forall w_1\in W(M, w_1\models\phi)$ and $\exists w_2\in W(M, w_2\models\psi)$, $w_1\boldsymbol{R}w_2$

Lemma 1. Given a *MKPL* formulas ϕ, if $\Diamond\phi$ expresses that ϕ is satisfiable, then $\Diamond\phi\leftrightarrow(\phi\,{}^{\exists}{\geq}^{\exists}\,\phi)$. If $\blacklozenge\phi$ expresses that ϕ is a tautology, then $\blacklozenge\phi\leftrightarrow\neg(\neg\phi\,{}^{\exists}{\geq}^{\exists}\,\neg\phi)$.

Proof. Let $M=(W, \boldsymbol{R})$, w be a preference model,

(1) $M, w\models\Diamond\phi$ iff $\exists w'\in W$, $M, w'\models\phi$. $M, w\models\phi\,{}^{\exists}{\geq}^{\exists}\,\phi$ iff $\exists w_1\in W(M, w_1\models\phi)$ and $\exists w_2\in W(M, w_2\models\psi)$, $w_1\boldsymbol{R}w_2$, that is $\exists w'\in W$, $M, w'\models\phi$.

(2) $\blacklozenge\phi\leftrightarrow\neg\Diamond\neg\phi\leftrightarrow\neg(\neg\phi\,{}^{\exists}{\geq}^{\exists}\,\neg\phi)$.

Definition 3 (Maximal Propositional Conjunction). A maximal propositional conjunction is an ordered conjunction of atoms or negated atoms such that every atom in V is mentioned exactly once.

According to a usual procedure of transforming a propositional formula into complete normal propositional disjunction, it is easy to decide whether ϕ can be expressed by a maximal propositional conjunction. We write $\spadesuit\phi$ if ϕ can be expressed by a maximal propositional conjunction.

Evidently, no matter what $\boldsymbol{R}$ is, there is exactly only $w\in W$, we have $M, w\models\phi$. Therefore, we can treat a maximal propositional conjunction as a world of 2^{V} in the following.

Definition 4 (Proof System S_{MKPL}).

Axiom

1. all propositional tautologies
2. $\phi^{x}\geq^{y}\psi\rightarrow\Diamond\phi$
3. $\spadesuit\phi\wedge\spadesuit\psi\rightarrow((\phi^{x}\geq^{y}\psi)\leftrightarrow(\phi^{x'}\geq^{y'}\psi))$
4. $\Diamond\psi\wedge(((\phi^{x}\geq^{y}\psi)\wedge\gamma)^{x'}\geq^{y'}\chi)\leftrightarrow\Diamond\psi\wedge(\phi^{x}\geq^{y}\psi)\wedge(\gamma^{x'}\geq^{y'}\chi)$
5. $\Diamond\psi\wedge((\neg(\phi^{x}\geq^{y}\psi)\wedge\gamma)^{x'}\geq^{y'}\chi)\leftrightarrow\Diamond\psi\wedge\neg(\phi^{x}\geq^{y}\psi)\wedge(\gamma^{x'}\geq^{y'}\chi)$
6. $\Diamond\psi\wedge(\chi^{x'}\geq^{y'}((\phi^{x}\geq^{y}\psi)\wedge\gamma))\leftrightarrow\Diamond\psi\wedge(\phi^{x}\geq^{y}\psi)\wedge(\chi^{x'}\geq^{y'}\gamma)$
7. $\Diamond\psi\wedge(\chi^{x'}\geq^{y'}(\neg(\phi^{x}\geq^{y}\psi)\wedge\gamma))\leftrightarrow\Diamond\psi\wedge\neg(\phi^{x}\geq^{y}\psi)\wedge(\chi^{x'}\geq^{y'}\gamma)$
8. Order axiom:
   - **R∃∃:** $\Diamond\phi\rightarrow\phi^{\exists}\geq^{\exists}\phi$
   - **R∀∃:** $\Diamond\phi\rightarrow\phi^{\forall}\geq^{\exists}\phi$
   - **T∀∀**: $(\phi^{\forall}\geq^{\forall}\psi)\wedge(\psi^{\forall}\geq^{\forall}\gamma)\rightarrow\phi^{\forall}\geq^{\forall}\gamma$
   - **T∀∃**: $(\phi^{\forall}\geq^{\exists}\psi)\wedge(\psi^{\forall}\geq^{\exists}\gamma)\rightarrow\phi^{\forall}\geq^{\exists}\gamma$
   - **T∃∀:** $(\phi^{\exists}\geq^{\forall}\psi)\wedge(\psi^{\exists}\geq^{\forall}\gamma)\rightarrow\phi^{\exists}\geq^{\forall}\gamma$
9. Transform axiom:
   - **∃∀R1:** $(\Diamond\phi\wedge\Diamond\psi\wedge\Diamond\gamma)\wedge\blacklozenge(\phi\rightarrow\gamma)\wedge(\phi^{\exists}\geq^{\forall}\psi)\rightarrow\gamma^{\exists}\geq^{\forall}\psi$
   - **∃∀R2:** $(\Diamond\phi\wedge\Diamond\psi\wedge\Diamond\gamma)\wedge(\phi^{\exists}\geq^{\forall}\psi)\rightarrow((\phi\wedge\gamma)^{\exists}\geq^{\forall}\psi)\vee((\phi\wedge\neg\gamma)^{\exists}\geq^{\forall}\psi)$
   - **∃∀R3:** $(\Diamond\phi\wedge\Diamond\psi\wedge\Diamond\gamma)\wedge(\phi^{\exists}\geq^{\forall}\psi)\wedge\blacklozenge(\psi\rightarrow\gamma)\rightarrow\phi^{\exists}\geq^{\exists}\gamma$
   - **∃∀R4:** $(\Diamond\phi\wedge\Diamond\psi\wedge\Diamond\gamma)\wedge(\phi^{\exists}\geq^{\forall}\psi)\wedge\blacklozenge(\gamma\rightarrow\psi)\rightarrow\phi^{\exists}\geq^{\forall}\gamma$
   - **∃∃R1:** $(\Diamond\phi\wedge\Diamond\psi\wedge\Diamond\gamma)\wedge\blacklozenge(\phi\rightarrow\gamma)\wedge(\phi^{\exists}\geq^{\exists}\psi)\rightarrow\gamma^{\exists}\geq^{\exists}\psi$
   - **∃∃R2:** $(\Diamond\phi\wedge\Diamond\psi\wedge\Diamond\gamma)\wedge\phi^{\exists}\geq^{\exists}\psi\rightarrow((\phi\wedge\gamma)^{\exists}\geq^{\exists}\psi)\vee((\phi\wedge\neg\gamma)^{\exists}\geq^{\exists}\psi)$
   - **∃∃R3:** $(\Diamond\phi\wedge\Diamond\psi\wedge\Diamond\gamma)\wedge(\phi^{\exists}\geq^{\exists}\psi)\wedge\blacklozenge(\psi\rightarrow\gamma)\rightarrow\phi^{\exists}\geq^{\exists}\gamma$
   - **∃∃R4:** $(\Diamond\phi\wedge\Diamond\psi\wedge\Diamond\gamma)\wedge\phi^{\exists}\geq^{\exists}\psi\rightarrow(\phi^{\exists}\geq^{\exists}(\psi\wedge\gamma))\vee(\phi^{\exists}\geq^{\exists}(\psi\wedge\neg\gamma))$
   - **∀∀R1:** $(\Diamond\phi\wedge\Diamond\psi\wedge\Diamond\gamma)\wedge\blacklozenge(\gamma\rightarrow\phi)\wedge(\phi^{\forall}\geq^{\forall}\psi)\rightarrow\gamma^{\forall}\geq^{\forall}\psi$
   - **∀∀R2:** $(\Diamond\phi\wedge\Diamond\psi\wedge\Diamond\gamma)\wedge\blacklozenge(\phi\rightarrow\gamma)\wedge(\phi^{\forall}\geq^{\forall}\psi)\rightarrow\gamma^{\forall}\geq^{\forall}\psi$
   - **∀∀R3:** $(\Diamond\phi\wedge\Diamond\psi\wedge\Diamond\gamma)\wedge(\phi^{\forall}\geq^{\forall}\psi)\wedge\blacklozenge(\psi\rightarrow\gamma)\rightarrow\phi^{\forall}\geq^{\exists}\gamma$
   - **∀∀R4:** $(\Diamond\phi\wedge\Diamond\psi\wedge\Diamond\gamma)\wedge(\phi^{\forall}\geq^{\forall}\psi)\wedge\blacklozenge(\gamma\rightarrow\psi)\rightarrow\phi^{\forall}\geq^{\forall}\gamma$
   - **∀∃R1:** $(\Diamond\phi\wedge\Diamond\psi\wedge\Diamond\gamma)\wedge\blacklozenge(\phi\rightarrow\gamma)\wedge(\phi^{\forall}\geq^{\exists}\psi)\rightarrow\gamma^{\exists}\geq^{\exists}\psi$
   - **∀∃R2:** $(\Diamond\phi\wedge\Diamond\psi\wedge\Diamond\gamma)\wedge\blacklozenge(\gamma\rightarrow\phi)\wedge(\phi^{\forall}\geq^{\exists}\psi)\rightarrow\gamma^{\forall}\geq^{\exists}\psi$
   - **∀∃R3:** $(\Diamond\phi\wedge\Diamond\psi\wedge\Diamond\gamma)\wedge(\phi^{\forall}\geq^{\exists}\psi)\wedge\blacklozenge(\psi\rightarrow\gamma)\rightarrow\phi^{\forall}\geq^{\exists}\gamma$
   - **∀∃R4:** $(\Diamond\phi\wedge\Diamond\psi\wedge\Diamond\gamma)\wedge\phi^{\forall}\geq^{\exists}\psi\rightarrow(\phi)^{\forall}\geq^{\exists}(\psi\wedge\gamma))\vee(\phi^{\forall}\geq^{\exists}(\psi\wedge\neg\gamma))$
10. Modus Ponens: $\phi, \phi\rightarrow\psi\Rightarrow\psi$

Where $x, y, x', y'\in\{\exists,\forall\}$, Order axiom and Transform axiom are rewrite of proposition 1 and proposition 2 in L_{MKPL}.

Proposition 3. S_{MKPL} is sound and complete.

Proof. (1) Soundness. The validity proofs for each axiom are given below.

- Axiom 2: Suppose that $M, w\models\phi^{x}\geq^{y}\psi$. We have that $M, w\models\phi$ and $\forall w'\in W$, $M, w'\not\models\psi$, or $xw_1\in W(M, w_1\models\phi)$ and $yw_2\in W(M, w_2\models\psi)$, $w_1\boldsymbol{R}w_2$, where $x, y\in\{\exists,\forall\}$. This means that there exist(s) models of ϕ, that is $M, w\models\Diamond\phi$.
- Axiom 3: Suppose that $M, w\models\spadesuit\phi\wedge\spadesuit\psi$. There is exactly only one w'', $M, w''\models\phi$, and There is exactly only one w', $M, w'\models\psi$. So $M, w\models\phi^{x}\geq^{y}\psi\Leftrightarrow w''\boldsymbol{R}w'\Leftrightarrow M, w\models\phi^{x'}\geq^{y'}\psi$ holds for any $x, y, x', y'\in\{\exists,\forall\}$.
- Axiom 4-7: Suppose that $M, w\models\Diamond\psi\wedge(((\phi^{\forall}\geq^{\exists}\psi)\wedge\gamma)^{\exists}\geq^{\forall}\chi)$. then

  1) $\exists w'\in W$, $M, w'\models\psi$ and $M, w\models((\phi^{\forall}\geq^{\exists}\psi)\wedge\gamma)$, $\forall w'\in W$, $M, w'\not\models\chi$, or

  2) $\exists w'\in W$, $M, w'\models\psi$, $\exists w_1\in W(M, w_1\models((\phi^{\forall}\geq^{\exists}\psi)\wedge\gamma))$ and $\forall w_2\in W(M, w_2\models\gamma)$, $w_1\boldsymbol{R}w_2$.

If 1) holds, then $M, w\models\Diamond\psi$. $M, w\models((\phi^{\forall}\geq^{\exists}\psi)\wedge\gamma)\Rightarrow M, w\models\phi^{\forall}\geq^{\exists}\psi$ and $M, w\models\gamma$. $M, w\models\gamma$ and $\forall w'\in W$, $M, w'\not\models\chi\Rightarrow M, w\models\gamma^{\exists}\geq^{\forall}\chi$. These three conclusions can be combined to derive $M, w\models\Diamond\psi\wedge(\phi^{\forall}\geq^{\exists}\psi)\wedge(\gamma^{\exists}\geq^{\forall}\chi)$.

If 2) holds, then $M, w\models\Diamond\psi$. $M, w_1\models\phi^{\forall}\geq^{\exists}\psi$, $M, w_1\models\gamma$. $M, w\models\Diamond\psi$ and $M, w_1\models\phi^{\forall}\geq^{\exists}\psi\Rightarrow\forall w_0\in W(M, w_0\models\phi)$, $\exists w_2\in W(M, w_2\models\psi)$, $w_0\boldsymbol{R}w_2\Rightarrow M, w\models\phi^{\forall}\geq^{\exists}\psi$. $M, w_1\models\gamma$, $\forall w_2\in W(M, w_2\models\gamma)$, $w_1\boldsymbol{R}w_2\Rightarrow M, w\models\gamma^{\exists}\geq^{\forall}\chi$. These three conclusions can be combined to derive $M, w\models\Diamond\psi\wedge(\phi^{\forall}\geq^{\exists}\psi)\wedge(\gamma^{\exists}\geq^{\forall}\chi)$.

The above proof procedure is reversible, so we can get that $\Diamond\psi\wedge(((\phi^{\forall}\geq^{\exists}\psi)\wedge\gamma)^{\exists}\geq^{\forall}\chi)\leftrightarrow\Diamond\psi\wedge(\phi^{\forall}\geq^{\exists}\psi)\wedge(\gamma^{\exists}\geq^{\forall}\chi)$.

For other interpretations of x, y, x', y', and axiom 5-7, the proof are of the same type.

- Axiom 8: these axioms can be easily derived from the corresponding conclusions in proposition 1.
- Axiom 9: these axioms can be easily derived from the corresponding conclusions in proposition 2.

(2) Completeness. S_{MKPL} is complete iff $\models\phi$ implies $S_{MKPL}\vdash\phi$. Let S be a maximally consistent set containing ϕ. All we need to do is to show that there is a model M, w such that $\forall\psi\in S$: $M, w\models\Diamond\psi$. In S_{MKPL}, all axioms are based on a partial order, we need to construct $M=(W, \boldsymbol{R})$ with $\boldsymbol{R}$ a partial order.

Let $S^{P}=\{\alpha^{x}\geq^{y}\beta|$ both α and β are maximal propositional conjunction, $x, y\in\{\exists,\forall\}\}$, then construct $M=(W, \boldsymbol{R})$ with $W=2^{V}$ and $\boldsymbol{R}=\{(\alpha, \beta)|\alpha^{x}\geq^{y}\beta\in S^{P}\}$. According to axiom 3, $\boldsymbol{R}=\{(\alpha, \beta)|\alpha^{\forall}\geq^{\forall}\beta\in S^{P}\}=\{(\alpha, \beta)|\alpha^{\forall}\geq^{\exists}\beta\in S^{P}\}=\{(\alpha, \beta)|\alpha^{\exists}\geq^{\exists}\beta\in S^{P}\}=\{(\alpha, \beta)|\alpha^{\exists}\geq^{\forall}\beta\in S^{P}\}$. According to axiom 8, 9, $\boldsymbol{R}$ is reflexive and transitive: a partial order.

A Decision Procedure

In this section, we propose a decision procedure for SAT of *MKPL*, that is, a procedure to decide whether there is a preference model for a given formula ϕ of *MKPL*.

Definition 5 (Preference Formula). A *MKPL* formula ϕ is a **preference formula** iff it has the form $\alpha^{x}\geq^{y}\beta$. $\alpha^{x}\geq^{y}\beta$ is a **basic preference formula** iff both α and β are propositional formulas. $\alpha^{x}\geq^{y}\beta$ is a **normal preference formula** iff both α and β are maximal propositional conjunction.

Lemma 2. Conjunction expansion principles.

- (1) $\Diamond(\phi\wedge\gamma)\wedge\Diamond(\phi\wedge\neg\gamma)\wedge(\phi^{\forall}\geq^{\forall}\psi)\rightarrow((\phi\wedge\gamma)^{\forall}\geq^{\forall}\psi)\wedge((\phi\wedge\neg\gamma)^{\forall}\geq^{\forall}\psi)$
  - (2) $\phi^{\forall}\geq^{\forall}\psi\rightarrow(\phi^{\forall}\geq^{\forall}(\psi\wedge\gamma))\wedge(\phi^{\forall}\geq^{\forall}(\psi\wedge\neg\gamma))$
  - (3) $\phi^{\exists}\geq^{\forall}\psi\rightarrow((\phi\wedge\gamma)^{\exists}\geq^{\forall}\psi)\vee((\phi\wedge\neg\gamma)^{\exists}\geq^{\forall}\psi)$
  - (4) $\phi^{\exists}\geq^{\forall}\psi\rightarrow(\phi^{\exists}\geq^{\forall}(\psi\wedge\gamma))\wedge(\phi^{\exists}\geq^{\forall}(\psi\wedge\neg\gamma))$
  - (5) $\Diamond(\phi\wedge\gamma)\wedge\Diamond(\phi\wedge\neg\gamma)\wedge(\phi^{\forall}\geq^{\exists}\psi)\rightarrow((\phi\wedge\gamma)^{\forall}\geq^{\exists}\psi)\wedge((\phi\wedge\neg\gamma)^{\forall}\geq^{\exists}\psi)$
  - (6) $\phi^{\forall}\geq^{\exists}\psi\rightarrow(\phi^{\forall}\geq^{\exists}(\psi\wedge\gamma))\vee(\phi^{\forall}\geq^{\exists}(\psi\wedge\neg\gamma))$
  - (7) $\phi^{\exists}\geq^{\exists}\psi\rightarrow((\phi\wedge\gamma)^{\exists}\geq^{\exists}\psi)\vee((\phi\wedge\neg\gamma)^{\exists}\geq^{\exists}\psi)$
  - (8) $\phi^{\exists}\geq^{\exists}\psi\rightarrow(\phi^{\exists}\geq^{\exists}(\psi\wedge\gamma))\vee(\phi^{\exists}\geq^{\exists}(\psi\wedge\neg\gamma))$

Proof.(1) can be derived from axiom 2 and ∀∀**R1**, (2) can be derived from axiom 2 and ∀∀**R4**, (3) can be derived from axiom 2 and ∃∀**R1**, (4) can be derived from axiom 2 and ∃∀**R4**, (5) can be derived from axiom 2 and ∀∃**R2**, (6) can be derived from axiom 2 and ∀∃**R4**, (7) can be derived from axiom 2 and ∃∃**R2**, (8) can be derived from axiom 2 and ∃∃**R4**.

Lemma 3. Disjunction expansion principles.

- (1) $((\phi\vee\gamma)^{\forall}\geq^{\forall}\psi)\wedge\Diamond\phi\wedge\Diamond\gamma\rightarrow(\phi^{\forall}\geq^{\forall}\psi)\wedge(\gamma^{\forall}\geq^{\forall}\psi)$
- (2) $\phi^{\forall}\geq^{\forall}(\psi\vee\gamma)\rightarrow(\phi^{\forall}\geq^{\forall}\psi)\wedge(\phi^{\forall}\geq^{\forall}\gamma)$
- (3) $(\phi\vee\gamma)^{\exists}\geq^{\forall}\psi\rightarrow(\phi^{\exists}\geq^{\forall}\psi)\vee(\gamma^{\exists}\geq^{\forall}\psi)$
- (4) $\phi^{\exists}\geq^{\forall}(\psi\vee\gamma)\rightarrow(\phi^{\exists}\geq^{\forall}\psi)\wedge(\phi^{\exists}\geq^{\forall}\gamma)$
- (5) $(\phi\vee\gamma)^{\forall}\geq^{\exists}\psi\wedge\Diamond\phi\wedge\Diamond\gamma\rightarrow(\phi^{\forall}\geq^{\exists}\psi)\wedge(\gamma^{\forall}\geq^{\exists}\psi)$
- (6) $\phi^{\forall}\geq^{\exists}(\psi\vee\gamma)\rightarrow(\phi^{\forall}\geq^{\exists}\psi)\vee(\phi^{\forall}\geq^{\exists}\gamma)$
- (7) $(\phi\vee\gamma)^{\exists}\geq^{\exists}\psi\rightarrow(\phi^{\exists}\geq^{\exists}\psi)\vee(\gamma^{\exists}\geq^{\exists}\psi)$
- (8) $\phi^{\exists}\geq^{\exists}(\psi\vee\gamma)\rightarrow(\phi^{\exists}\geq^{\exists}\psi)\vee(\phi^{\exists}\geq^{\exists}\gamma)$

Proof. (1) can be derived from axiom 2 and ∀∀**R1**, (2) can be derived from axiom 2 and ∀∀**R4**, (3) can be derived from axiom 2 and ∃∀**R1**, (4) can be derived from axiom 2 and ∃∀**R4**, (5) can be derived from axiom 2 and ∀∃**R2**, (6) can be derived from axiom 2 and ∀∃**R4**, (7) can be derived from axiom 2 and ∃∃**R2**, (8) can be derived from axiom 2 and ∃∃**R4**.

Our decision procedure is inspired by von Wright's decision procedure in his preference logic, and similarly includes two steps: normalization and consistency checking. The normalization step to transform a well-formed formula into its normal form is based on axioms 4-7, conjunction expansion principles, and disjunction expansion principles. In the normal form, all preference formulas are normal preference formulas. The consistency checking step is based on axiom 8, axiom 9, and truth tables. Following the normalization step, one assigns truth-values to these normal preference formulas and computes the resulting truth-tables. We describe the procedure using an example as follows.

Example 2. Let $V=\{p, q\}$, $\phi\equiv p\wedge q\wedge(((p^{\forall}\geq^{\forall}\neg p)\wedge q)^{\exists}\geq^{\forall}(\neg q\vee p))$. Check the satisfiability of ϕ as follows:

1. Normalization

(1)Decompose preference formulas of ϕ into basic preference formulas by axiom 4. Transform ϕ into ϕ_1.

$$\phi_1\equiv p\wedge q\wedge(p^{\forall}\geq^{\forall}\neg p)\wedge(q^{\exists}\geq^{\forall}(\neg q\vee p))$$

(2)Remove disjunction in preference formulas based on disjunction expansion principles (4), and transform ϕ_1 into ϕ_2.

$$\phi_2\equiv p\wedge q\wedge(p^{\forall}\geq^{\forall}\neg p)\wedge(q^{\exists}\geq^{\forall}\neg q)\wedge(q^{\exists}\geq^{\forall}p)$$

(3) Suppose $p\in V$, but does not occur in β or α of a basic preference formula $\alpha^{x}\geq^{y}\beta$. According to conjunction expansion principles, transform basic preference formulas into normal preference formulas. For ϕ_2:

- Use conjunction expansion principles (1) and (2) on $p^{\forall}\geq^{\forall}\neg p$, we get:

$$((p\wedge q)^{\forall}\geq^{\forall}(\neg p\wedge q))\wedge((p\wedge q)^{\forall}\geq^{\forall}(\neg p\wedge\neg q))\wedge((p\wedge\neg q)^{\forall}\geq^{\forall}(\neg p\wedge q))\wedge((p\wedge\neg q)^{\forall}\geq^{\forall}(\neg p\wedge\neg q))$$

- Use conjunction expansion principles (3) and (4) on $q^{\exists}\geq^{\forall}\neg q$, we get: $(((p\wedge q)^{\exists}\geq^{\forall}(p\wedge\neg q))\wedge((p\wedge q)^{\exists}\geq^{\forall}(\neg p\wedge\neg q)))\vee(((\neg p\wedge q)^{\exists}\geq^{\forall}(p\wedge\neg q))\wedge((\neg p\wedge q)^{\exists}\geq^{\forall}(\neg p\wedge\neg q)))$
- Use conjunction expansion principles (3) and (4) on $q^{\exists}\geq^{\forall}p$, we get: $(((p\wedge q)^{\exists}\geq^{\forall}(p\wedge\neg q))\wedge((p\wedge q)^{\exists}\geq^{\forall}(p\wedge q)))\vee(((\neg p\wedge q)^{\exists}\geq^{\forall}(p\wedge\neg q))\wedge((\neg p\wedge q)^{\exists}\geq^{\forall}(p\wedge q)))$

Combine these results, transform ϕ_2 into ϕ_3

$$\phi_3\equiv p\wedge q\wedge((p\wedge q)^{\forall}\geq^{\forall}(\neg p\wedge q))\wedge((p\wedge q)^{\forall}\geq^{\forall}(\neg p\wedge\neg q))\wedge((p\wedge\neg q)^{\forall}\geq^{\forall}(\neg p\wedge q))\wedge((p\wedge\neg q)^{\forall}\geq^{\forall}(\neg p\wedge\neg q))\wedge((((p\wedge q)^{\exists}\geq^{\forall}(p\wedge\neg q))\wedge((p\wedge q)^{\exists}\geq^{\forall}(\neg p\wedge\neg q)))\vee(((\neg p\wedge q)^{\exists}\geq^{\forall}(p\wedge\neg q))\wedge((\neg p\wedge q)^{\exists}\geq^{\forall}(\neg p\wedge\neg q))))\wedge((((p\wedge q)^{\exists}\geq^{\forall}(p\wedge\neg q))\wedge((p\wedge q)^{\exists}\geq^{\forall}(p\wedge q)))\vee(((\neg p\wedge q)^{\exists}\geq^{\forall}(p\wedge\neg q))\wedge((\neg p\wedge q)^{\exists}\geq^{\forall}(p\wedge q))))$$

2. Consistency checking

Assign truth-values to the normal preference formulas and computes the resulting truth-tables in the following way:

- The constituent of the form $\alpha^{x}\geq^{y}\beta$ is assigned *true*.
- If both $\alpha^{x}\geq^{y}\beta$ and $\beta^{x'}\geq^{y'}\gamma$ are assigned *true*, the constituent $\alpha^{x''}\geq^{y''}\gamma$ is also assigned *true*.
- Finally, ones compute the truth-table in a standard way.

Check ϕ_3, we can get $M=(\{p\wedge q, p\wedge\neg q, \neg p\wedge q, \neg p\wedge\neg q\}, \boldsymbol{R})$, $w= p\wedge q$ is a model of ϕ_3, where $\boldsymbol{R}$ is as showed in Fig.2.

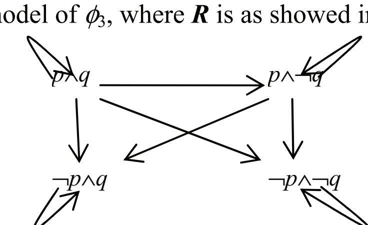

Fig.2. ***R***

Conclusion

In this paper, we introduce *MKPL*: a many-kinds-preferences logic with finite set of propositional variables, to formalize four kinds of preference. We hope to give the user(s) the possibility to represent and reason her(their) varying kinds of preferences simultaneously in a more natural, concise and flexible manner by four approaches: (1)Defining four more general kinds of preference based on partial order on W than those based on total pre-order on *W* in [S. Kaci and L. van der Torre 2005]. (2) Ignoring von Wright's expansion principle in the new preference types. (3)Regarding preference connectives as general propositional connectives, like $\wedge,\vee,\neg$ etc. (4) Studying a whole proof system. Moreover, we propose a decision procedure with *MKPL*.

We are currently extending the work in this paper in the following directions.

- We are investigating expressive power of *MKPL* for preference representation by comparing with other preference definitions.
- We plan to add ceteris paribus constraint in *MKPL*, and plan to investigate computational issue related the approach. In particular, it would be interesting to see the application of varying kinds of preferences in logic programming.
- We are trying to contain more combination strategies in *MKPL*, and to find general principles of reasoning with varying kinds of preferences simultaneously.

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