The Nine Comments on the RCC Theory

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Abstract

This paper comments on nine aspects of the Region Connection Calculus (the RCC theory) with conclusions as follows: (1) regions in the RCC theory shall not be points; (2) the *identical* relation is a deviation from its classic usage; (3) the ι operator is un-determinate; (4) it is unclear about the notion of *region* in the theory; (5) *atomic* regions have zero thickness, instead of non-zero thickness as is assumed in the theory; (6) there is room for simple distance relations in the RCC theory; (7) there is no room for the *congruence* relation in the theory; (8) properties of the connection relation are not sufficiently characterized; (9) the connection relation formalized in the theory is not primitive for spatial representations in general.

Motivation

The Region Connection Calculus (the RCC theory) has been accepted as one dominant theory in the QSR (Qualitative Spatial Representation) community. However, we found some mysteries in the RCC theory during our research work. In this paper, we list nine mysteries and provide our understanding and comment. The paper is structured as follows: We first very briefly introduce the RCC theory, as we assume that readers are quite familiar with it; then we list nine questions and present our comments and answers, if any.

An outline of the RCC theory

The RCC theory (Randell *et al.* 1992) has two axioms to govern the connection relation '$\mathbf{C}$' between regions: (1) for any region x, x connects with itself; (2) for any regions x and y, if x connects with y, then y also connects with x. The two axioms are formalized as follows.

$$\forall x\ [\mathbf{C}(x,x)]$$
$$\forall xy\ [\mathbf{C}(x,y) \rightarrow \mathbf{C}(y,x)]$$

Several spatial relations are further formalized using the connection relation. For example, that region x disconnects from region y, $\mathbf{DC}(x,y)$, is defined as $\neg\mathbf{C}(x,y)$. That x is a part of y, $\mathbf{P}(x,y)$, is defined as $\forall z[\mathbf{C}(z,x) \rightarrow \mathbf{C}(z,y)]$. That region x is a proper part of region y, $\mathbf{PP}(x,y)$, is defined as $\mathbf{PP}(x,y) \stackrel{\text{def}}{=} \mathbf{P}(x,y) \wedge \neg\mathbf{P}(y,x)$. That region x is identical to region y, $\mathbf{EQ}(x,y)$ (or $x = y$), is defined as $\mathbf{EQ}(x,y) \stackrel{\text{def}}{=} \mathbf{P}(x,y) \wedge \mathbf{P}(y,x)$. That region x overlaps region y, $\mathbf{O}(x,y)$, is defined as $\mathbf{O}(x,y) \stackrel{\text{def}}{=} \exists z[\mathbf{P}(z,x) \wedge \mathbf{P}(z,y)]$. That region x partially overlaps region y, $\mathbf{PO}(x,y)$, is defined as follows.

$$\mathbf{PO}(x,y) \stackrel{\text{def}}{=} \mathbf{O}(x,y) \wedge \neg\mathbf{P}(x,y) \wedge \neg\mathbf{P}(y,x)$$

That region x externally connects with region y, $\mathbf{EC}(x,y)$, is defined as x connects with y and x does not overlap y, i.e.,

$$\mathbf{EC}(x,y) \stackrel{\text{def}}{=} \mathbf{C}(x,y) \wedge \neg\mathbf{O}(x,y)$$

The relation that region x is a tangential proper part of region y, $\mathbf{TPP}(x,y)$, is defined as follows.

$$\mathbf{TPP}(x,y) \stackrel{\text{def}}{=} \mathbf{PP}(x,y) \wedge \exists z[\mathbf{EC}(z,x) \wedge \mathbf{EC}(z,y)]$$

That region x is a non-tangential proper part of region y, $\mathbf{NTPP}(x,y)$, is defined as follows.

$$\mathbf{NTPP}(x,y) \stackrel{\text{def}}{=} \mathbf{PP}(x,y) \wedge \neg\exists z[\mathbf{EC}(z,x) \wedge \mathbf{EC}(z,y)]$$

Can regions be points?

In this section, we show that in the RCC theory a region cannot be a point. That two regions overlap ($\mathbf{O}$) is defined in the theory as that there exists a region which is part of both of the two regions, i.e. $\mathbf{O}(x,y) \stackrel{\text{def}}{=} \exists z[\mathbf{P}(z,x) \wedge \mathbf{P}(z,y)]$. That two regions are externally connected ($\mathbf{EC}$) is defined as that they are connected and do not overlap, i.e. $\mathbf{EC}(x,y) \stackrel{\text{def}}{=} \mathbf{C}(x,y) \wedge \neg\mathbf{O}(x,y)$, which means that their closures share a point and there is no region which is shared by both of them. If the theory allows a region to be a point, that *there is no region which is shared by both of them* will further lead to that *there is no points which is shared by both of them*. This is unfortunately equivalent to that the two regions are disconnected, and this makes the definition of $\mathbf{EC}$ self-contradictory. Therefore, a region in the RCC theory cannot be a point. Following the same lines of arguments, we can come to the conclusion that regions in the RCC theory cannot be lines, curves, etc.

What does *identical* mean?

In the RCC theory, two regions x and y being *identical* ($\mathbf{EQ}$ or $=$) is defined as that each is part of the other. This is equivalent to that for any region z, z connects with x, if and only if z connects with y, i.e.,$\forall z[\mathbf{C}(z,x) \leftrightarrow \mathbf{C}(z,y)]$. As $\mathbf{C}(x,y)$ is interpreted in the theory as *closures of x and y share a common point*, two regions being *identical* means their closures are perfectly coincided. That is, if region $x^2+y^2 \leq 4$ and region $x^2+y^2 < 4$ are two regions in the RCC theory, they will be *identical*; if region $x^2+y^2 \leq 4, (x,y) \neq (0,0)$ and region $x^2+y^2 \leq 4$ are two regions in the RCC theory, they will be *identical*. This means that the *identical* relation in the RCC theory is a deviation from its classic usage, which can be stated as follows: *$x = y$ if, and only if, x has every property which y has, and y has every property which x has*, (Tarski 1946, pp. 55). Therefore, in the classic sense, neither region $x^2+y^2 \leq 4$ and region $x^2+y^2 < 4$, nor region $x^2+y^2 \leq 4, (x,y) \neq (0,0)$ and region $x^2+y^2 \leq 4$ are *identical*. They only have the same closure. This questions the unique denotation of the ι operator in the RCC theory.

Which region does the ι operator denote?

In the RCC theory the $\mathtt{sum}(x,y)$[1] function is defined as *the* region w such that for any region z connecting with w, if and only if z connects with x or y.

$$\mathtt{sum}(x,y) \overset{\text{def}}{=} \iota w[\forall z[\mathbf{C}(z,w) \equiv [\mathbf{C}(z,x) \vee \mathbf{C}(z,y)]]]$$

Let x and y be two *non-closed*[2] regions in the RCC theory, then the question is raised: which region can be *the* sum of x and y, the joint region of x and y, the joint region of cl(x)[3] and y, the joint region of x and cl(y), the joint region of cl(x) and cl(y), or others? — there are indeed many regions which are *identical* with the joint region of x and y in the RCC theory. Classically, ιy[*so-and-so*] means '*the* so-and-so' and '*the so-and-so' will be false if there are no so-and-so's or several so-and-so's* (Russell 1956, pp.92). Therefore, the ι operator in the RCC theory is a deviation from the classic usage. Our question is: Can ιy[*so-and-so*] mean that *any* region *y so-and-so* and that all these regions are *identical* in the RCC theory? The advantage of this explanation is that the uniqueness of the ι descriptor is guaranteed inside the RCC theory. Let us examine this explanation by considering one property of the complement function defined in the RCC theory. The $\mathtt{compl}(x)$ function is defined in (Randell *et al.* 1992, pp.168) as *the* region y such that for any region z two conditions hold: (1) z connects with y if and only if z is not non-tangential proper part of x; (2) z overlaps y if and only if z is not part of x.

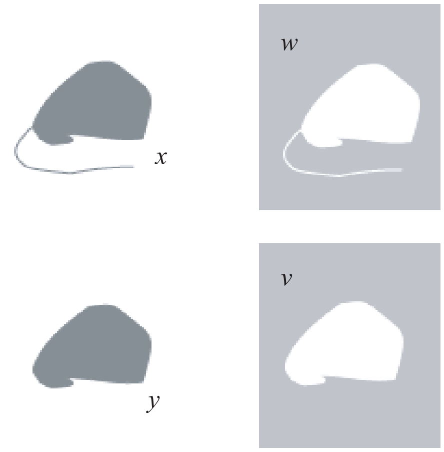

Figure 1: Region x is not *identical* with region y; region w is *identical* with region v; either w or v can be a complement of x or y. Then the complement of the complement of x is not *identical* with x

$$\mathtt{compl}(x) \overset{\text{def}}{=} \iota y[\forall z[[\mathbf{C}(z,y) \leftrightarrow \neg\mathbf{NTPP}(z,x)] \wedge [\mathbf{O}(z,y) \leftrightarrow \neg\mathbf{P}(z,x)]]]$$

It holds that $\forall x[\mathtt{compl}(\mathtt{compl}(x)) = x]$. Let regions x, y, w, and v be regions as illustrated in Figure 1. Region x consists of two parts: an extended part and a curve; region y is the extended part of x; the complement of region x is *identical* with the complement of region y in the RCC theory. And the complement of the complement of region x is region y, not *identical* with region x. The theorem $\forall x[\mathtt{compl}(\mathtt{compl}(x)) = x]$ will not be held. We would conclude that ιy[*so-and-so*] cannot be explained as *the y so-and-so* in the RCC theory, and that if it is explained as *any y so-and-so* with the condition that all these ys are identical inside the RCC theory, there must be some restrictions on the notion of *regions* in the RCC theory, otherwise some theorems in the theory will not be held.

What kinds of regions are *regions*?

The RCC theory stipulates what kinds of regions are *regions* as follows: for every region x there is region y such that y is non-tangential proper part of x, (Randell *et al.* 1992, pp.169).

$$\forall x \exists y[\mathbf{NTPP}(y,x)]$$

From this stipulation we know immediately that neither points nor curves should be called as *regions* in the theory, as we know that they do not have non-tangential part. The question is: Does this stipulation exclude other kinds of regions? Is region x in Figure 1 a *region* in the theory? As region x has a non-tangential proper part, it should be a valid region in the theory. In general, if there is a region r which has non-tangential proper part, then for any region p including points and curves, the sum of r and p has non-tangential proper part. Suppose the RCC theory were a country whose

[1]In (Randell *et al.* 1992), the sum function is written incorrectly as $\mathtt{sum}(x,y) =_{def} \iota y[\forall z[\mathbf{C}(z,y) \leftrightarrow [\mathbf{C}(z,x) \vee \mathbf{C}(z,y)]]]$. The y in $\ldots \iota y[\forall z[\mathbf{C}(z,y)\ldots$ should be replaced with a letter other than x, y, z.

[2]By a *non-closed region*, we mean a region which is not exactly the same as its closure. For example, $x^2+y^2 < 4$ is a *non-closed region*, as it is not exactly the same as its closure $x^2+y^2 \leq 4$.

[3]Following the notion appeared in (Randell *et al.* 1992, pp. 171), the closure of region x is written as 'cl(x)'.

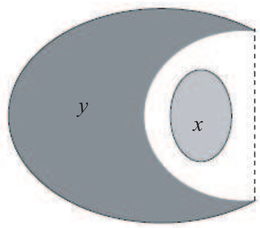

Figure 2: Region x being inside of region y means that x is discrete from y and that x is part of the convex hull of y

citizens are *regions*, the above stipulation serves as a judger who decides whether a person is a citizen of the country or not. This judger would be rather ‘forgiving’ — for any person who is not a citizen of the country, he only needs to find a citizen of the country and say to the judger, “we are together”. Then, the judger will say, “you both as a whole is a citizen of the country”. This may introduce troubles into the RCC theory. For example, suppose region x in Figure 1 is a *region* in the theory, the theorem $\forall x[\texttt{compl}(\texttt{compl}(x)) = x]$ will not be held any more.

Similar problem also happens to the so-called *convex-hull* of region: the *convex-hull* of region x, written as $\texttt{conv}(x)$, is characterized by a group of axioms, (Randell *et al.* 1992, pp. 169), as follows.

$$
\begin{array}{ll}
(1) & \forall x[\mathbf{P}(x, \texttt{conv}(x))] \\
(2) & \forall x[\mathbf{P}(\texttt{conv}(\texttt{conv}(x)), \texttt{conv}(x))] \\
(3) & \forall x \forall y \forall z[[\mathbf{P}(x, \texttt{conv}(y)) \wedge \mathbf{P}(y, \texttt{conv}(z))] \\
 & \qquad \rightarrow \mathbf{P}(x, \texttt{conv}(z))] \\
(4) & \forall x \forall y[[\mathbf{P}(x, \texttt{conv}(y)) \wedge \mathbf{P}(y, \texttt{conv}(x))] \rightarrow \mathbf{O}(x, y)] \\
(5) & \forall x \forall y[[\mathbf{DR}(x, \texttt{conv}(y)) \wedge \mathbf{DR}(y, \texttt{conv}(x))] \\
 & \qquad \rightarrow \mathbf{DR}(\texttt{conv}(x), \texttt{conv}(y))]
\end{array}
$$

The above five axioms are also very ‘forgiving’ in stipulation, as for any region x, let $\texttt{conv}(x)$ be x, the five axioms are held as follows.

$$
\begin{array}{ll}
(1)' & \forall x[\mathbf{P}(x, x)] \\
(2)' & \forall x[\mathbf{P}(x, x)] \\
(3)' & \forall x \forall y \forall z[[\mathbf{P}(x, y) \wedge \mathbf{P}(y, z)] \rightarrow \mathbf{P}(x, z)] \\
(4)' & \forall x \forall y[[\mathbf{P}(x, y) \wedge \mathbf{P}(y, x)] \rightarrow \mathbf{O}(x, y)] \\
(5)' & \forall x \forall y[[\mathbf{DR}(x, y) \wedge \mathbf{DR}(y, x)] \rightarrow \mathbf{DR}(x, y)]
\end{array}
$$

This may introduce troubles into the RCC theory, when we consider the *inside* relation: region x being *inside* of region y is defined as that x is discrete from y and x is part of the *convex full* of y, (Randell *et al.* 1992, p. 169), as illustrated in Figure 2.

$$\texttt{INSIDE}(x, y) \stackrel{\text{def}}{=} \mathbf{DR}(x, y) \wedge \mathbf{P}(x, \texttt{conv}(y))$$

When $\texttt{conv}(x)$ is taken as x, the definition of $\texttt{INSIDE}(x, y)$ will turn out to be $\mathbf{DR}(x, y) \wedge \mathbf{P}(x, y)$, which would be interpreted as *that x is inside of y is defined as x is discrete from y and x is part of y*. Region x might be called a *ghost region* of y, as it can be both inside of and discrete from y.

Does an *atomic region* have *non-zero thickness*?

(Randell *et al.* 1992, p. 173) discussed the notion of *atomic* regions or *atoms*. *Atomic* regions are assumed as being ‘very small’ and with *non-zero thickness*. That region x is an *atomic* region is defined as follows.

$$\mathsf{ATOM}(x) \stackrel{\text{def}}{=} \forall y[\mathbf{P}(y, x) \rightarrow y = x]$$

We proof a theorem as follows.

Theorem 1 *Given an* atomic *region x, it holds that for any regions, z and w, if z connects with x, w connects with x, then z connects with w.*

$$\forall z, w[\mathbf{C}(z, x) \rightarrow [\mathbf{C}(w, x) \rightarrow \mathbf{C}(z, w)]]$$

Proof 1

$$
\begin{array}{ll}
 & \mathsf{ATOM}(x) \\
(1) & \forall y[\mathbf{P}(y, x) \rightarrow y = x] \\
 & (1) \\
(2) & \forall y[\forall z[\mathbf{C}(z, y) \rightarrow \mathbf{C}(z, x)] \\
 & \qquad \rightarrow \forall w[\mathbf{C}(w, y) \leftrightarrow \mathbf{C}(w, x)]] \\
 & (2), \mathit{Let\ y\ be\ z} \\
(3) & [\forall z[\mathbf{C}(z, z) \rightarrow \mathbf{C}(z, x)] \\
 & \qquad \rightarrow \forall w[\mathbf{C}(w, z) \leftrightarrow \mathbf{C}(w, x)]] \\
 & (3), \forall z[\mathbf{C}(z, z)] \\
(4) & \forall z[\mathbf{C}(z, x) \rightarrow \forall w[\mathbf{C}(w, z) \leftrightarrow \mathbf{C}(w, x)]] \\
 & (4) \\
(5) & \forall z[\mathbf{C}(z, x) \rightarrow \forall w[\mathbf{C}(w, x) \rightarrow \mathbf{C}(w, z)]]
\end{array}
$$

This theorem shows that for any regions w and z, if they connect with an *atomic* region, they will connect with each other. Formula (5) $\forall z[\mathbf{C}(z, x) \rightarrow \forall w[\mathbf{C}(w, x) \rightarrow \mathbf{C}(w, z)]]$ is equivalent with $\forall z[\mathbf{C}(z, x) \rightarrow \mathbf{P}(x, z)]$, which means that for any region z, if an *atomic* region connects with z, then it is part of z. All these imply that an *atomic* region has *zero thickness*, instead of *non-zero thickness*. Including regions with *zero thickness* into the theory would cause inconsistency. And including *atomic* regions into the theory indeed introduced inconsistency, as mentioned in (Randell *et al.* 1992, pp. 173-174). What we would mention here is that in some work the notion of *point* is characterized just the same as the notion of the *atomic* region. For example in (Smith 1996, pp. 289), a point x, written as $\mathbf{Pt}(x)$ is defined as $\forall y(y\mathbf{P}x \rightarrow y = x)$, which means for any y, if y is part of x, then y is identical to x. From this point, we will be easier to understand why introducing *atomic* regions into the theory would cause inconsistency — *atomic* regions are in fact *points*, and *regions* in the theory cannot be *points*.

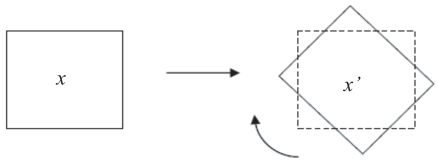

Figure 3: Region x and region x' are congruent, as x can be perfectly placed on x' by moving and rotation

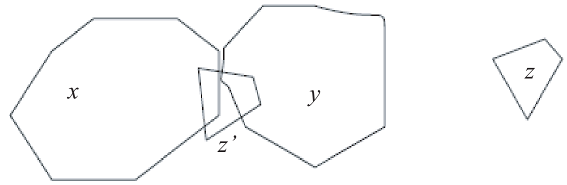

Figure 4: Region x connects region y, for any region z, there is region z' which is congruent with z and which connects with x and y

Is there room for *distance* in the RCC theory?

In this section, we discuss whether there is room for distance relations in the RCC theory. The answer seems positive, as this is at least evidenced in (Cohn and Bennett 1997, 7): *An alternative interpretation of* $\mathbf{C}$ *might be given informally by saying the distance between the two regions is zero*. That is, the RCC theory can also be viewed as a calculus whose primitive relation is *the distance between two regions is zero*. Further, let us think about the distance relation as follows: 'distance-less-than-or-equal-to-one-foot(x, y)', read as "region x is less than or equal to one foot away from region y". This distance relation is reflexive and symmetrical: x is less than or equal to one foot away from itself (reflexive); if x is less than or equal to one foot away from y, then y is less than or equal to one foot away from x (symmetrical). This is also a valid interpretation for the $\mathbf{C}$ relation. Generally, let p be a non-negative real number, 'distance-less-than-or-equal-to-p-meter(x,y)' is a reflexive, and symmetrical relation and therefore can be an interpretation of $\mathbf{C}$. When p is zero, the theory is a *connection* calculus; when p is greater than zero, the theory would be a simple *distance* calculus. *The distance between* x *and* y *being more than* p *meter* can be represented by $\neg\mathbf{C}(x, y)$. We conclude that although the RCC theory is named as a calculus with the connection relation, it can also be used as a calculus for simple distance relations.

Can the *congruence* relation between regions be delineated in the RCC theory?

The *congruence* relation (the *geometric equivalence*) is one of the main topics in geometry, (Flegg 1974). Two objects being *congruent* means that one can be perfectly placed on the other, as illustrated in Figure 3. In the RCC theory the *congruence* relation between regions should be interpreted as the closure of one region can be perfectly placed on the closure of the other. Our question is: can the *congruence* relation between regions be represented in the RCC theory? This is equivalent to ask: is there such a formula $\mathbf{G}(x, y)$ that $\mathbf{G}(x, y)$ holds if and only if regions x and y are congruent, where $\mathbf{G}(x, y)$ is constructed by the relation $\mathbf{C}$ and notions in the first order logic. If $\mathbf{G}(x, y)$ is such a formula, then $\neg\mathbf{G}(x, y)$ is such that when y is congruent with x, it does not hold, and that when y is not congruent with x, it holds. This is also a valid formula to determine whether two regions are congruent, but in a non-conventional way – just like, in some countries people give positive attitude by *shaking* the head, and give negative attitude by *nodding* the head. Therefore, the property of $\mathbf{G}(x, y)$ can be stated as follows: either $\mathbf{G}(x, y)$ holds, if and only if x and y are congruent, or $\mathbf{G}(x, y)$ holds, if and only if x and y are not congruent.

We are not so optimistic about the existence of such a formula, for the reason that the RCC theory talks about 'locations' of regions in terms of the connection relation, while 'congruence' is about the shape and the size of regions which is independent on 'locations'. Two regions is (or is not) congruent, while they can be located anywhere, e.g. they can be connected, externally connected, partially overlapped, or disconnected, etc. This hypothesize can be proved by structural induction through the construction of formula. For the simplest case, we first consider $\mathbf{C}(z_1, z_2)$, $\mathbf{C}(x, z)$, $\mathbf{C}(z, y)$, and $\mathbf{C}(x, y)$. Their values are *independent* from whether x and y are congruent. Suppose $\mathbf{D}(x, y)$, $\mathbf{F}(x, y)$ are any two formula whose values are *independent* from whether x and y are congruent. We need to show that values of $\neg\mathbf{D}(x, y)$, $\mathbf{D}(x, y) \vee \mathbf{F}(x, y)$, $\mathbf{D}(x, y) \wedge \mathbf{F}(x, y)$, $\mathbf{D}(x, y) \rightarrow \mathbf{F}(x, y)$, $\forall z\mathbf{D}(x, y)$, and $\exists z\mathbf{D}(x, y)$ are all *independent* from whether x and y are congruent. Let v_1 and v_2 be the values of $\mathbf{D}(x, y)$ and $\mathbf{F}(x, y)$, respectively. v_1 and v_2 can be t (true) or f (false). Then, values of $\neg\mathbf{D}(x, y)$ $\mathbf{D}(x, y) \vee \mathbf{F}(x, y)$, $\mathbf{D}(x, y) \wedge \mathbf{F}(x, y)$, $\mathbf{D}(x, y) \rightarrow \mathbf{F}(x, y)$ are all determined by v_1 and v_2. $\forall z\mathbf{D}(x, y)$ is equivalent to $\mathbf{D}_{z_1}(x, y) \wedge \mathbf{D}_{z_2}(x, y) \wedge \ldots \wedge \mathbf{D}_{z_n}(x, y)$, where z_i $(1 < i < n)$ is in the domain of z; $\exists z\mathbf{D}(x, y)$ is equivalent to $\mathbf{D}_{z_1}(x, y) \vee \mathbf{D}_{z_2}(x, y) \vee \ldots \vee \mathbf{D}_{z_n}(x, y)$, where z_i $(1 < i < n)$ is in the domain of z, therefore their values are not related with whether x and y are congruent or not.

Are properties of the connection relation sufficiently characterized by the axioms?

The RCC theory uses the connection relation $\mathbf{C}$ as the primitive relation. Two axioms characterize properties of the connection relation as *reflexive* and *symmetrical*: $\forall x[\mathbf{C}(x, x)]$ and $\forall xy[\mathbf{C}(x, y) \rightarrow \mathbf{C}(y, x)]$. We claim that the two properties do not sufficiently characterize properties of the connection relation. If it were, the two axioms should be able to deduce any property of the connection relation. Let us think about one property as follows: if two regions are connected, then for any region p, we can find a place for it, so that p can connect with the two regions. This property even holds, when p is a point — if p is located at any of the shared part of the closures of the two connected regions, p will connect with both of them. This property can also be stated as follows: for any regions x, y, and z, if x connects with y, then there is region z' such that z' is *congruent* with

z, and that z' connects with x and y, as illustrated in Figure 4. As the *congruence* relation can not be delineated in the RCC theory, this property of the connection relation cannot be represented either. So, we conclude that the RCC theory does not delineate all primitive properties of the connection relation. However, this will not be a problem, if those delineated properties of the connection relation can construct all spatial properties that the theory wishes to represent.

Is C *primitive* for spatial relations?

The relation **C** is primitive in the RCC theory, as all the spatial relations appear in the theory are defined through it. However, the fact that the axioms in the RCC theory can neither characterize nor deduce one property of the connection relation leads to the question: are there spatial relations which can be interpreted through the connection relation but cannot be represented in the RCC theory? If yes, we will have to conclude that **C** is not *primitive* for spatial relations, even the connection relation is. In the above section, we have shown that the congruence relation cannot be developed through **C**. In the following part, we will show that the **C** relation in the RCC theory cannot describe more than two distance relations in one interpretation. By doing so we argue that the **C** relation in the RCC theory is not a *primitive* relation for spatial relations, in the sense that it cannot develop sufficient distance relations.

The definition of distance in general has been discussed in mathematical philosophy and general topology, (Russell 1903), (Kelley 1955). However, such mathematical or philosophical perspectives to view the notion of distance relation do not meet the motivation for grounding the RCC theory which took *the naive point of view*: *From the standpoint of our naive understanding of the world, this topological structure is arguably too rich for our purposes, . . . Given the choice between two possible theories used for formal representing space, the ease by which a person can understand and use the theory must be taken into account. The basic part of the original theory . . . required the user to be familiar with general topology . . . . We thought this restriction could be eased . . .* (Randell *et al.* 1992, pp. 166). We therefore will take a naive point of view to analyze one understanding of distance relations, and show why such understanding is beyond the descriptive power of the **C** relation. Ancient Egyptians used the notion of *Royal Egyptian Cubit* to describe distances. The length of a *Cubit* is defined as the length of the forearm from the bent elbow to the tip of the extended middle finger plus the width of the palm of the hand of the Pharaoh or King ruling at that time. The distance between two objects being no more than one *Cubit* can be understood as there is a cubit which can connect with the two objects. If **C** is interpreted as one kind of distance relation, e.g. $\mathbf{C}(x, y)$ is interpreted as the distance between x and y is no more than one *Cubit*, there will be two basic distance relations distinguished: the distance between two regions is less than or equal to one *Cubit*, **C**, and the distance between two regions is more than one *Cubit*, $\neg\mathbf{C}$. The question is raised: Can distance relations, such as less than two/ three/four *Cubits* be described in the RCC theory? To answer this question, we shall first make clear the meaning of *the distance being less than two Cubits*. If *the distance between x and y being less than or equal to one Cubit* means that there is a region which is congruent with *Cubit* such that this region connects with x and y, *the distance between x and y being less than or equal to two Cubits* shall mean that there are two regions which are *congruent* with *Cubit* and such that when they connect with one another, they shall connect with x and y. However, as we have shown that there is no room for the *congruence* relation in the RCC theory, there is also no room for relations which are based on the *congruence* relation, such as distance relations.

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