

Qualitative Spatial Representation Based on Connection Pattern and Convexity*

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Abstract

We present an extended PLCA that represents the shape of an object qualitatively. PLCA, a framework for qualitative reasoning, is based on the simple components: *points(P)*, *lines(L)*, *circuits(C)* and *areas(A)*, and the entire figure is represented as a combination of these components. The entire space is considered to be partitioned into disjoint regions, and the connection patterns of regions can be distinguished. We extend PLCA to represent the qualitative shape of a region with a hierarchy of convex hulls. We formalize our approach, present an algorithm to generate the symbolic expression for a given figure, and discuss the properties that should be satisfied by this expression. Our goal is to represent not only the shapes of the outer circuits of single regions, but also those of the boundaries between regions.

1. Introduction

Qualitative Spatial Reasoning(QSR) treats images or figures qualitatively, by representing only the information required for a user's purpose, such as mereological relationships, relative positional relationships, and relative sizes between regions (Cohn and Hazarika 2001; Stock 1997). In QSR systems, figures are represented not numerically but symbolically to reduce the amount of data and computation.

RCC (Randell and Cui 1992) is a logical theory that considers a space to be a set of regions, in which the entire figure is represented by a set of binary relations between regions. The 9-intersection is another method which uses a matrix to describe the relationships between objects (Egenhofer 1991; Egenhofer and Franzosa 1991; 1995). PLCA is a framework for qualitative spatial reasoning that utilizes simple components: *points(P)*, *lines(L)*, *circuits(C)* and *areas(A)*, and describes connection patterns between regions (Takahashi and Sumitomo 2005). In PLCA, pairs of areas, circuits and lines never cross. Intuitively, the entire space is partitioned into disjoint regions. As shown in Figure 1, PLCA can explicitly represent the fact that the two objects touch at two points, while other QSR methods are limited to representing the property that the two objects have the common part.

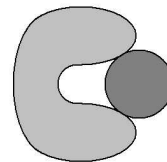


Figure 1: Objects touched by two points

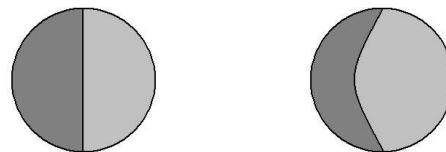


Figure 2: The same figures

In PLCA, only the connection patterns between regions are represented, and shape information is ignored. The two figures in Figure 2, for example, are regarded as the same one, since both show two objects that are connected with a line.

Shape representation, however, is necessary in many fields: for example, in recognizing maps or geological changes, in designing and building objects, and in using Geographic Information Systems (GIS).

Qualitative shape representation has been studied in several QSR frameworks. However, most of these have focused on representing the shape of a single object and they cannot handle the shapes of multiple objects connected with each other (Figure 3). Consider, for example, the shapes of the borders of countries. Most borders of European countries are curved, while those of African countries are straight. To represent borders, it is necessary to express not only the shape of each single border, but also the pattern of connections between borders. In this paper, we treat both of these problems with a single solution.

We extend PLCA to describe the convex hull of each area, a new approach designated PLCA+. The convexity of each object is represented by its convex hull, and we extract any

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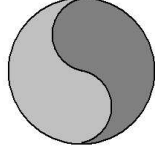


Figure 3: Figures including multiple objects

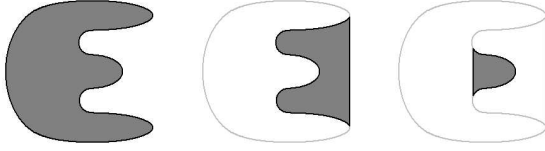


Figure 4: Hierarchical representation of convexity

concavity as the difference between the area and its convex hull. We construct the convex hulls of the extracted parts recursively. Using this approach, it is possible to express the detailed shapes of regions (Figure 4). Our goal is to represent not only the shape of the boundaries of a single object, but also the detailed connection between objects and the convexity of areas.

We present an algorithm for generating a PLCA+ expression from a given figure in a two-dimensional plane, and discuss the properties the expression satisfies. We also discuss the expressive power of PLCA+.

This paper is organized as follows. In section 2, we present the formal definition of PLCA+, an extended PLCA expression, and the conditions that are to be satisfied. In section 3, we describe an algorithm for generating the symbolic representation of a given figure in a two-dimensional plane, and show the properties that are satisfied. In section 4, we compare our approach with the other works, and discuss the ability of PLCA+. Finally, in section 5, we show the conclusion.

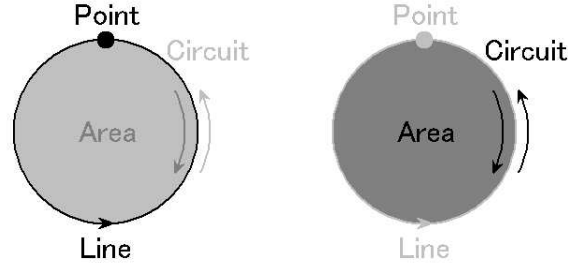
2. Definition of Extended PLCA

Definition of Classes

PLCA has four basic components: *points*(P), *lines*(L), *circuits*(C) and *areas*(A) (Figure 5). We add a new component *subPLCA* to represent the convex shape of an area.

Point is defined as a primitive class.

Line is defined as a class that satisfies the following condition: for an arbitrary instance l of *Line*, $l.ps$ is a pair $[p_1, p_2]$ where $p_1, p_2 \in Point$. A line has an inherent orientation. When $l.ps = [p_1, p_2]$, l^+ and l^- mean $[p_1, p_2]$ and $[p_2, p_1]$, respectively. l^* denotes either l^+ or l^- , and l_{re}^* denotes the line with direction inverse of that of l^* . Intuitively, a line is the edge connecting two (not always different) points. No two lines are allowed to cross. Note that multiple distinct lines may be defined by the same pair of points. In Fig. 6(a), the arrows denote the orientations of the lines. All of the



(a) Point and Line

(b) Circuit and Area

Figure 5: PLCA components

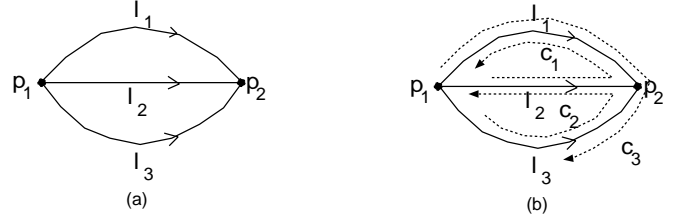


Figure 6: Multiple lines with the same definition and the associated circuits

lines $l_1.ps$, $l_2.ps$ and $l_3.ps$ are defined to be $[p_1, p_2]$, but they are distinguished by the circuits to which they belong.

In this paper, we assume that each line in the figure is curved, although PLCA permits straight lines.

Circuit is defined as a class that satisfies the following condition: for an arbitrary instance c of *Circuit*, $c.ls$ is a sequence $[l_1^*, \dots, l_n^*]$ where $l_1, \dots, l_n \in Line$ ($n \geq 1$), $l_i.ps = [p_i, p_{i+1}]$ ($1 \leq i \leq n$) and $p_{n+1} = p_1$. $[l_1^*, \dots, l_n^*]$ and $[l_j^*, \dots, l_n^*, l_1^*, \dots, l_{j-1}^*]$ denote the same circuit for any j ($1 \leq j \leq n$). In Fig. 6(b), we have three circuits: $c_1.ls = \{l_1^-, l_2^+\}$, $c_2.ls = \{l_2^-, l_3^+\}$, $c_3.ls = \{l_3^-, l_1^+\}$.

For $c_1, c_2 \in Circuit$, we introduce two new predicates lc and pc to denote that two circuits share line(s) and point(s), respectively. $lc(c_1, c_2)$ is *true* iff there exists $l \in Line$ such that $(l^+ \in c_1.ls) \wedge (l^- \in c_2.ls)$. $pc(c_1, c_2)$ is *true* iff there exists $p \in Point$ such that $(p \in l_1.ps) \wedge (p \in l_2.ps) \wedge (l_1^* \in c_1.ls) \wedge (l_2^* \in c_2.ls)$. A circuit is the boundary between an area and its adjacent areas viewed from the side of that area.

Area is defined as a class that satisfies the following condition: for an arbitrary instance a of *Area*, $a.cs$ is a set $\{c_1, \dots, c_n\}$ where $c_1, \dots, c_n \in Circuit$ ($n \geq 1$), and $\forall c_i, c_j \in a.cs; (i \neq j) \rightarrow (\neg pc(c_i, c_j) \wedge \neg lc(c_i, c_j))$. Intuitively, an area is a connected region which consists of exactly one piece. No two areas are allowed to cross. The final condition means that any pair of circuits that belong to the same area cannot share a point or a line. For areas a_1 and a_2 , if there exist circuits c_1 and c_2 such that $c_1 \in a_1.cs$ and $c_2 \in a_2.cs$, respectively, and $lc(c_1, c_2)$ holds, then a_1 and a_2 are said to be *line-connected*.

We assume that there exists a circuit along the outermost extremity of the figure called *om* (outermost). This implies

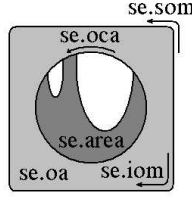


Figure 7: Important components of convex hull

that the target figure is finite and bounded, and the space can be partitioned into a number of areas that do not overlap with each other.

SubPLCA is defined as a class that satisfies several conditions, for an arbitrary instance *se* of *SubPLCA*, these are as follows:

definition 1 (*SubPLCA*)

$$\begin{aligned}
 se.ps &= \{p_0, p_1, \dots, p_{n-1}\} \\
 &\text{where } p_0, p_1, \dots, p_{n-1} \in \text{Point} \\
 se.ls &= \{l_0, l_1, \dots, l_{n-1}\} \\
 &\text{where } l_0, l_1, \dots, l_{n-1} \in \text{Line} \\
 se.cs &= \{c_0, c_1, \dots, c_{n-1}\} \\
 &\text{where } c_0, c_1, \dots, c_{n-1} \in \text{Circuit} \\
 se.as &= \{a_0, a_1, \dots, a_{n-1}\} \\
 &\text{where } a_0, a_1, \dots, a_{n-1} \in \text{Area} \\
 se.area &= a \quad \text{where } a \in se.as \\
 se.som &= c \quad \text{where } c \in se.cs
 \end{aligned}$$

We call this expression *SubPLCA* of Area *a*.

Intuitively, *SubPLCA* *se* represents a restricted frame in which the extracted area *se.area* from the source figure is pasted. There exists a circuit along the outermost extremity of the frame called *som*(suboutermost).

We also define three components of *se*. *se.iom* is the inner circuit of the frame, *se.oca* is the outer circuit of the convex hull of the extracted area, and *se.oa*, which is called the *background area*, is the area between the convex hull of the extracted area and the outer boundary of the frame. These components are illustrated for a particular figure in Figure 7, and their formal definitions are given below.

definition 2 (*the inner circuit of the frame*) *se.iom* is a Circuit *c* that satisfies:

$$\begin{aligned}
 &\forall l^* \in se.som.ls (l_{re}^* \in c.ls) \\
 \wedge &\forall l^* \in c.ls (l_{re}^* \in se.som.ls) \\
 \wedge &c \in se.cs
 \end{aligned}$$

It means that for each line l^* that belongs to the suboutermost circuit, the line oriented opposite to l^* belongs to *se.iom*, and vice versa.

definition 3 (*the outer circuit of the convex hull of the extracted area*) *se.oca* is a Circuit *c* that satisfies:

$$c \in se.oa.cs \wedge c \neq se.iom$$

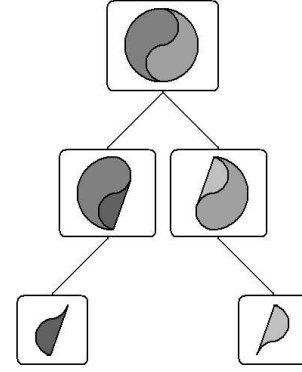


Figure 8: A tree structure of $e^+.ses$

definition 4 (*the background area*) *se.oa* is an Area *a* that satisfies:

$$\begin{aligned}
 &se.iom \in a.cs \\
 \wedge &|a.cs| = 2 \\
 \wedge &a \in se.as \\
 \wedge &a \neq se.area
 \end{aligned}$$

It means that the only circuits belonging to the background area are the inner circuit of the frame and the outer circuit of the convex hull of the extracted area.

Tree Structure of SubPLCAs If the source figure contains *n* areas, then *n* SubPLCAs *se*'s are defined independently. Moreover, if a concave part of the source figure has concave sub-parts, we use hierarchical representation to capture its shape. Each SubPLCA is a PLCA+ expression that includes SubPLCA recursively. As a result, for a PLCA+ expression e^+ , $e^+.ses$ has a tree structure (Figure 8). For a SubPLCA *se*, if $se.area = a \wedge se.as = \{a, a_1, a_2, \dots, a_n\}$, Area *a* is said to be a *parent Area* of Area a_1, a_2, \dots, a_n , and a_1, a_2, \dots, a_n are said to be *child Areas* of Area *a*.

PLCA+ Expression

PLCA+ expression is defined as a class that satisfies the following condition: for an arbitrary instance e^+ of PLCA+ expression, $e^+.ps$, $e^+.ls$, $e^+.cs$, $e^+.as$ and $e^+.ses$ are sets of Points, Lines, Circuits, Areas and subPLCA expressions, respectively, and $e^+.om \in e^+.cs$ is the outermost circuit of the whole figure.

definition 5 (*element*) (i) Let *p*, *l*, *c* and *a* be Point, Line, Circuit and Area, respectively. If $p \in l.ps$, then *p* is said to be an *element of l*. If $l \in c.ls$, then *l* is said to be an *element of c*. If $c \in a.cs$, then *c* is said to be an *element of a*. (ii) Let o_1, o_2 and o_3 be each a Point, Line, Circuit or Area. If o_1 is an element of o_2 and o_2 is an element of o_3 , then o_1 is an element of o_3 .

Consistency of PLCA+

definition 6 (*consistency*) A PLCA+ expression e^+ is said to be *consistent* iff the following constraints are satisfied.

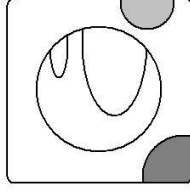


Figure 9: A figure corresponding to a SubPLCA that does not satisfy the uniqueness of $se.iom$

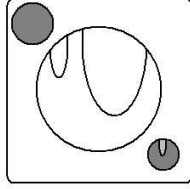


Figure 10: A figure corresponding to a SubPLCA that does not satisfy the uniqueness of $se.oe$

Constraint on Point-Line Each Point belongs to some line. Each Point in $l.ps$ should belong to $e^+.ps$ where l belongs to $e^+.ls$. For each SubPLCA, the same constraints hold.

Constraint on Line-Circuit Each Line belongs to exactly two distinct Circuits. Each Line in $c.ls$ should belong to $e^+.ls$ where c belongs to $e^+.cs$. For each SubPLCA, the same constraints hold.

Constraint on Circuit-Area Each *Circuit* besides *outermost* and *suboutermost* belongs to exactly one Area. Each Circuit in $a.cs$ should belong to $e^+.cs$ where a belongs to $e^+.as$. For each SubPLCA, the same constraints hold.

Due to these three constraints, neither isolated lines nor points are allowed.

Constraint on SubPLCA There exist a unique $se.area$, $se.som$, $se.iom$, $se.oa$ and $se.oe$ for each subPLCA. Moreover, the extracted area $se.area$ and the background area $se.oe$ should be line-connected.

The uniqueness of $se.iom$ eliminates the case shown in Figure 9. The uniqueness of $se.oe$ eliminates the case shown in Figure 10. The line-connectedness of $se.area$ and $se.oe$ eliminates the case shown in Figure 11.

Planarity of PLCA+

We investigated the planarity condition for PLCA expressions (Takahashi and Sumitomo 2008). For a PLCA+ expression, since e^+ and each SubPLCA se are also regarded as PLCA expressions, they satisfy the planarity condition:

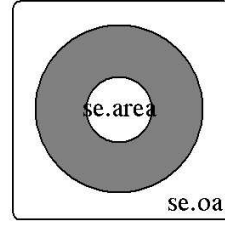


Figure 11: A figure corresponding to a SubPLCA that does not satisfy the line-connectedness between $se.area$ and $se.oe$

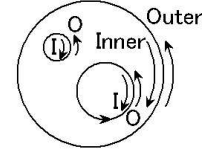


Figure 12: Inner Circuit and Outer Circuit

$$|e^+.ps| - |e^+.ls| - |e^+.cs| + 2|e^+.as| = 0$$

$$\forall se \in e^+.ses \left(\begin{aligned} &|se.ps| - |se.ls| - |se.cs| + 2|se.as| = 0 \end{aligned} \right)$$

We require additional planarity constraints between Area and SubPLCA, and between SubPLCAs.

Inner and Outer Before presenting the additional planarity conditions, we introduce concepts of Inner and Outer.

First, we define the predicates ioc (is Outer Circuit) and iic (is Inner Circuit) for a Circuit c as follows.

definition 7 (*Inner/Outer Circuit*)

$$ioc(c) = \begin{cases} true & \begin{aligned} &c = e^+.om \\ &\exists se \in e^+.ses (c = se.som) \\ &\forall l^* \in c.ls (l_{re}^* \in c'.ls \wedge \neg ioc(c')) \\ &\forall a \in \{a | c \in a.cs\} \left(\begin{aligned} &\exists c' \in a.cs \setminus \{c\} (\neg ioc(c')) \end{aligned} \right) \end{aligned} \\ false & otherwise \end{cases}$$

$$iic(c) = \begin{cases} true & \neg ioc(c) \\ false & otherwise \end{cases}$$

Intuitively, an Outer Circuit is a Circuit that describes the outer boundary of an Area, while an Inner Circuit describes the inner boundary (Figure 12). For a consistent PLCA expression, it is decidable whether a Circuit is an Inner Circuit or an Outer Circuit (Takahashi and Sumitomo 2008).

Next, we define the predicate io (Inner Object) for components o_1 and o_2 as follows.

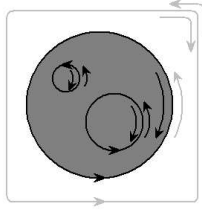


Figure 13: Inner Objects in an Area

definition 8 (*Inner Object*)

$$io(o_1, o_2) = \begin{cases} o_1 \in As \wedge o_2 \in o_1.cs \\ o_1 \in Cs \wedge o_2 \in o_1.ls \\ o_1 \in Ls \wedge o_2 \in o_1.ps \\ iic(o_1) \wedge o_1 \in o_2.cs \\ ioc(o_1) \wedge l^* \in o_1.ls \\ \quad \wedge \exists c \in o_2.cs (l_{re}^* \in c.ls) \\ io(o_1, o_3) \wedge io(o_3, o_2) \end{cases}$$

Let o_1 and o_2 each be a Point, Line, Circuit or Area. Intuitively, $io(o_1, o_2)$ is *true* if o_2 is contained within o_1 , and *false*, otherwise (Figure 13).

Constraint on Area-SubPLCA For each Area a other than the background area in an se , there exists an Area in some se' the parent Area of which is a . Background areas have no children. These constraints are formalized as follows.

Let $As_{se.oa} = \bigcup_{se \in e^+.ses} se.oa$.

$$\forall a \in As \setminus As_{se.oa} \left(|\{se | se.area = a\}| = 1 \right)$$

$$\forall a \in As_{se.oa} \left(\begin{array}{l} |\{se | a \in se.as\}| = 1 \\ \wedge a \notin \bigcup_{se \in e^+.ses} se.area \\ \wedge a \notin e^+.as \end{array} \right)$$

An Area a and all of its Inner Objects are included in the interior of the background area of the SubPLCA of a . This constraint is formalized as follows.

$$\begin{aligned} se.area &= a \\ se.ps &\supseteq \left\{ p \mid \begin{array}{l} io(a, p) \wedge p \in Ps \\ \vee \exists l \in \{l \mid l^* \in se.som.ls\} (p \in l.ps) \end{array} \right\} \\ se.ls &\supseteq \left\{ l \mid io(a, l) \wedge l \in Ls \vee l^* \in se.som.ls \right\} \\ se.cs &\supseteq \left\{ c \mid \begin{array}{l} io(a, c) \wedge c \in Cs \\ \vee c = se.som \\ \vee c = se.iom \\ \vee c = se.oa \end{array} \right\} \\ se.as &\supseteq \left\{ a' \mid \begin{array}{l} io(a, a') \wedge a' \in As \\ \vee c = se.oa \\ \vee c = se.area \end{array} \right\} \end{aligned}$$

Constraint on SubPLCA-SubPLCA Consider the SubPLCAs of areas that are line-connected. The line shared by these areas should not connect to the background areas of the SubPLCAs (Figure 14). This reflects the fact that when

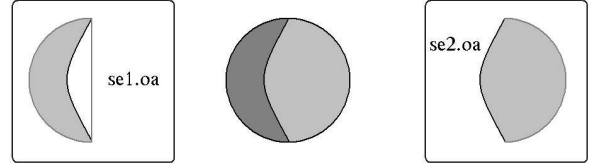


Figure 14: Convexity of Line and constraints on SubPLCAs

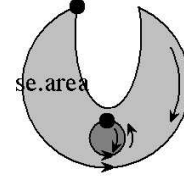


Figure 15: Inside of Area a

areas are line-connected, one is convex and the other is concave.

$$\begin{aligned} \forall l^* \in se_1.oa.ls (l_{re}^* \notin se_2.oa.ls) \\ (se_1, se_2 \in e^+.ses \wedge se_1 \neq se_2) \end{aligned}$$

3. Generation of PLCA+

Constructing PLCA+ from PLCA and Figure

For a given figure F in a two-dimensional plane, we have already described the construction of a PLCA expression e for F (Takahashi and Sumitomo 2008). Here, we describe the generation of a PLCA+ expression e^+ from F and e . Note that A and A' denote the parts in the figure F corresponding to the expression a and the convex hull of A . In this algorithm, for each area in F , we build a surrounding frame for that area's SubPLCA, construct expressions corresponding to the inside of A and to the background in the frame, and combine these expressions. If the extracted area has a concave part, this process is repeated recursively.

Here, we present the outline of the algorithm.

Initially, $e^+.ps, e^+.ls, e^+.cs, e^+.as$ are set to be $\{\}$.

function : $generate(F, e)$

(a) Set $e^+.ps = e^+.ps \cup e.ps$, $e^+.ls = e^+.ls \cup e.ls$, $e^+.cs = e^+.cps \cup e.cs$ and $e^+.as = e^+.as \cup e.as$.

(b) Set $e^+.ses = \{\}$ and $Areas = e^+.as$.

(c) Repeat (d) until $Areas = \{\}$.

(d) Select an arbitrary Area a from $Areas$, and proceed the followings.

(d1) **The inside of the Area** Each element of a is added to $se.ps$, $se.ls$, $se.cs$ and $se.as$, depending on its class. In addition, set $se.area = a$ (Figure 15).

(d2) **The outside of the Area** Make a SubPLCA expression se which consists of the only one area a' containing one Point, one Line, and two Circuits $se.som$ and $se.iom$ (Figure 16).

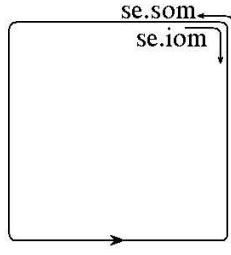


Figure 16: The background in the frame

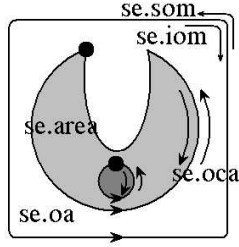


Figure 17: Combining

(d3) Combining expressions Make a new Circuit expression corresponding to the circuit that encircles the outer part of Area a , and add it to both $se.cs$ and $se.oa.cs$ (Figure 17).

(d4) Generating expression for concavity If the convex hull A' is not filled by the area A in F , obtain the concave part of A by comparing A and A' in F . To do this, we use the Line-division and Area-generation operators (Sumitomo and Takahashi 2007) (Figure 18). Otherwise, do nothing.

(d5) Updating Areas Add se to $e^+.ses$, and add all Areas in $se.areas$ other than $se.area$ and $se.oa$ to $Areas$.

Judgment of Line Convexity

For a given PLCA+ expression, assume that a and a' are the expressions corresponding to the Area A and its convex hull A' in the figure. If A and A' are matched, that is, A' is fully occupied by A , then A is said to be *convex*. Otherwise, it is said to be *concave*.

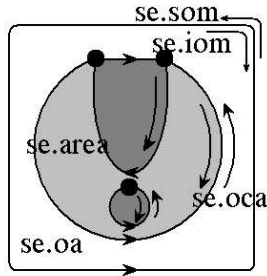


Figure 18: Generation of concavity

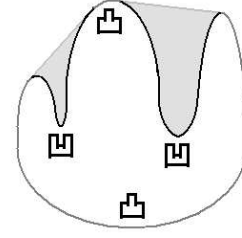


Figure 19: Convexity of Line

The Line expression corresponding to the overlap between A and A' when they are superimposed is said to be *convex*, and that not corresponding to the overlap is said to be *concave* (Figure 19). Note that the convexity of a Line is defined with respect to the inside of A and it is inverted with respect to the outside of A . For a directed Line l^* , which is an element of an Area a , $convex(l^*)$ denotes that Line l is convex with respect to an Area a ; and $concave(l^*)$ denotes that Line l is concave with respect to an Area a .

Each line in the figure is curved. Therefore, the Line in $se.oa$ is concave, since $se.oa$ corresponds to the outer circuit of the convex hull. Hence, the following properties hold.

$$convex(l^*) \leftrightarrow concave(l_{re}^*)$$

$$\forall se \in e^+.ses \left(\forall l^* \in se.oa.ls (concave(l^*)) \right)$$

An Algorithm for Judging the Convexity of Line We give an algorithm for determining the convexity of a Line which is not included in the outermost or sub-outermosts.

function : $getConvexity(l^*)$

Consider SubPLCA se such that $l^* \in se.ls$ holds.

(a) If $l^* \in se.oa.ls$, then $concave(l^*)$.

(b) If $l^* \notin se.oa.ls$, consider $getConvexity(l_{re}^*)$.

(b1) If $convex(l_{re}^*)$ is obtained as the result of $getConvexity(l_{re}^*)$, then $concave(l^*)$.

(b2) If $concave(l_{re}^*)$ is obtained as the result of $getConvexity(l_{re}^*)$, then $convex(l^*)$.

The convexity of each Line can be determined by this algorithm (see Appendix).

4. Discussion

There have been several previous studies of qualitative shape representations.

Some authors have taken a logic-based approach in which the relationships between the regions are represented using predicates. Gotts provided a qualitative representation of the shape of a region in the RCC framework (Gotts 1994). He used a predicate that signifies a connected relation and showed that various types of qualitative shape reference can be represented using Clark's C operator in first-order logic. Cohn proposed a symbolic representation for the shapes of figures (Cohn 1995), and extended RCC to represent differences between the shapes of regions in first-order logic. In

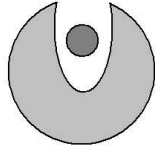


Figure 20: Geometric inside

Cohn’s framework, concavity of a region is defined by the difference between the original shape of the region and its convex hull. He represented subtle qualitative shape differences using the relative positional relationships of the concave parts of a region, and used a hierarchical representation of regions to capture complicated shapes. Although these studies demonstrated the expressive power of RCC or the C operator, new predicates and axioms must be defined every time a new distinction is introduced, and there have been no discussions of well-definedness. Pratt investigated shape representation in an algebraic manner (Pratt 1999). In PLCA+, we also utilize convex hulls and a hierarchical representation of complicated shapes. However, our approach to representation is object-oriented, rather than relying on predicates. Moreover, the above-mentioned frameworks address only the shape of a single object, and not relationships between multiple objects. For example, the connection between two objects with concave parts shown in Figure 3 cannot be represented in these other methods, while it can be represented in PLCA+.

Another approach to qualitative shape representation focuses on the shapes of the lines between regions. In (Museros and Escrig 2004), each line is divided into several segments, and properties such as the qualitative shape, angle and size of each segment are represented. This method, requires a large amount of information even for a single segment. In (Nedas and Egenhofer 2004), each line is also divided into segments, and the relationships between these segments are represented. In (Schlieder 1996), the shape of the line is represented by a positional ordering of points on the line. In these methods, it is sometimes difficult to choose the locations of the representational points, with potential for a large amount of redundancy. Moreover, it is impossible to represent the locations of the objects shown in Figure 20 called the “geometric inside” using only this line information. Therefore, we focused on convex hulls in PLCA+. Although PLCA+ does not currently represent a case of “geometric inside,” it is possible with a little extension.

5. Conclusion

We have proposed a qualitative spatial representation called PLCA+, which extends PLCA to handle qualitative shape representation using convex hulls. We formally defined PLCA+ and presented an algorithm to generate the PLCA+ expression for a given figure.

PLCA provides a symbolic expression to describe figures

in a two-dimensional plane by representing the connection patterns of regions using the simple components of Point, Line, Circuit and Area. PLCA+ can additionally represent the convex shapes of regions. We can reason about the convexity of a single region, and the connectivity of multiple regions with the convexity information.

We have given the properties to be satisfied by a PLCA+ expression generated from a figure, which are considered to be necessary and sufficient conditions for the planarity of a PLCA+ expression. In future, we will attempt to prove this property.

Extensions of PLCA+ are also under consideration. We would like to treat figures that use straight lines and also to represent other relationships between regions, including the “geometric inside.”

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Appendix. Decidability of Line Convexity

For a consistent planar PLCA+ expression e^+ , let L_{se} be $L_s \setminus \bigcup_{se \in e^+, ses} \{l | l^* \in se.som.ls\}$, which is equivalent to $\bigcup_{se \in e^+, ses} se.ls \setminus \{l | l^* \in se.som.ls\}$.

We show that the convexity of each Line in each SubPLCA is decidable. We prove this by induction on the tree structure of SubPLCA given in Section 2.

lemma 1 *The convexity of each Line in SubPLCA se at the leaf node is decidable.*

$$\forall l^* \in L_{se, leafnode} (convex(l^*) \wedge concave(l_{re}^*)).$$

Proof)

In this case,

$$se.as = \{se.oa, se.area\}$$

holds, since the SubPLCA of $se.area$ is equivalent to se itself and there is no child node.

The number of other components of se is fully determined, since it is consistent and planar. Therefore, the PLCA+ expression for se is as follows (Figure 21):

$$\begin{array}{ll} se.ps = \{p_1, p_2\} & l_1.ps = [p_1, p_1] \\ se.ls = \{l_1, l_2\} & l_2.ps = [p_2, p_2] \\ se.cs = \{c_1, c_2, c_3, c_4\} & c_1.ls = [l_1^+] \\ se.as = \{a_1, a_2\} & c_2.ls = [l_1^-] \\ se.area = a_2 & c_3.ls = [l_2^+] \\ se.som = c_1 & c_4.ls = [l_2^-] \\ se.iom = c_2 & a_1.cs = \{c_2, c_3\} \\ se.oa = a_1 & a_2.cs = \{c_4\} \\ se.oa = c_3 & \end{array}$$

In this case, it is sufficient to determine the convexity of the directed Lines of l_2^+ and l_2^- .

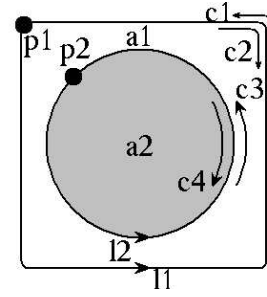


Figure 21: The SubPLCA for a leaf node

$concave(l_2^+)$ holds since $l_2^+ \in se.oa.ls$. Therefore, $convex(l_2^-)$ holds. Thus, the lemma holds. Q.E.D.

lemma 2 *Assume that the convexity of each line in all of the SubPLCAs se_i ($1 \leq i \leq n$), which are the SubPLCAs of Areas in $se.as$ is decidable. Then, the convexity of each line in SubPLCA se is decidable.*

Proof)

Let $L_{se, internal} = se.ls \setminus \{l | l^* \in se.som.ls\}$. It is sufficient to prove that

$$\forall l^* \in L_{se, internal} (convex(l^*) \wedge concave(l_{re}^*))$$

$L_{se, internal}$ can be divided into three subsets: the directed Lines in $se.oa$, the directed Lines belonging to the Circuit in $se.area.cs$, and the directed Lines belonging to the Circuit in $se_i.cs$.

$\forall l^* \in se.oa.ls (convex(l^*))$ holds and the convexity of each Line in se_i is decidable from the induction hypothesis. The shape of the Line belonging to the circuit in $se.area.cs$ is determined by the definition of line convexity. That is, the convexity of every Line in $L_{se, internal}$ can be obtained. Moreover, the convexity of each Line is decidable, since e^+ is a consistent planar expression. Therefore, the lemma holds.

Q.E.D.

theorem 1 *For a consistent planar PLCA+ expression, the convexity of each Line in the expression is decidable.*

Proof) The theorem holds from the above two lemmas.

Q.E.D.