Abduction and Conversational Implicate 
(Extended Abstract)

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Introduction

In conversation or dialogue, people use abduction to understand reasons behind utterances. Suppose that your colleague says “I will be at my office this weekend”. Then you may surmise that he/she has much work to do. In this dialogue, the utterance is considered an evidence provided by the colleague, then you seek reasons to explain the utterance. You abduce that your colleague would have much work to do if you believe the implication “much_work ⊨ office_weekend”. Given an utterance by a speaker, a hearer could perform two different types of abduction. The first one is to produce a hearer’s belief of a fact that could explain an evidence provided by a speaker. The above example is of this type of abduction. The second one is to produce a hearer’s belief of a speaker’s belief which could explain the speaker’s utterance. The above example, suppose that you believe that the colleague believes “much_work ⊨ office_weekend”. Then you abduce that the colleague believes that he/she has much work to do. In this case, however, you do not necessarily believe yourself that the colleague has much work to do. In this way, a hearer may use abduction not only for generating assumption that accounts for the utterance, but for generating assumption on the belief state of a speaker who makes the utterance.

In conversation or dialogue, the notion of conversational implicature (Grice 1975) is known as pragmatic inference in linguistic phenomena. The principal subject is to investigate the meaning of a sentence more than what is actually said. For instance, if a speaker utters the sentence “I have two children”, it normally implicates “I do not have more than two children”. This is called a scalar implicature (or Q-implicature) which says that a speaker implicates the negation of a semantically stronger proposition than the one asserted (Levinson 1987). Generally, a hearer infers ¬ψ from the utterance ϕ and the implication ψ ⊨ ϕ by Q-implicature. This is in contrast with abduction in dialogue, however. In abduction, a hearer infers ψ from the utterance ϕ and the implication ψ ⊨ ϕ. Thus, two inferences appear to reach opposite conclusions in face of an utterance. In conversational implicature, an alternative implicature, called the I-implicature, implies a semantically stronger sentence than what is actually uttered. The clash between Q- and I-implicatures has been studied in the field of pragmatics that concerns with the meaning of sentences in conversation (Horn 1984; Levinson 1987; Carson 1995). Some researchers have pointed out the utility of abduction in interpreting speech acts (Hobbs et al. 1993; McRoy and Hirst 1993; Jančík 2012). To the best of our knowledge, however, relations or differences between abduction and conversational implicature have never been formally explored in the literature.

In this abstract we argue a formal account of abduction and conversational implicature in human dialogues. We first formulate two different types of abduction in a propositional modal logic. We next formulate conversational implicatures and contrast them with abduction. The results characterize how hearers use abduction or conversational implicatures to figure out what speakers have implicated and show how two commonsense inferences are distinguished.

Abduction in Dialogue

We use a propositional modal logic of knowledge and belief that is standard in the literature. A sentence $B_a \phi$ is read as “an agent $a$ believes a sentence $\phi$”, $K_a \phi$ is read as “$a$ knows $\phi$”, and $C \phi$ is read as “it is common knowledge that $\phi$”.1 It holds that $K_a \phi \supset B_a \phi$, $C \phi \supset \phi \land K_a \phi \land K_b \phi$, etc. The logic has the axioms and inference rules of the system KD45 for $B_a$ and $S5$ for $K_a$. We assume a dialogue between two agents, called a speaker and a hearer. Each agent has a (consistent) set of sentences as background knowledge and believes those sentences. When a speaker utters a (consistent) sentence, a hearer performs abduction to explain reasons behind the utterance.

Definition 1 (objective abduction) Let $a$ be a hearer and $b$ a speaker. When $b$ utters a sentence $\phi (\not= \bot)$, a sentence $\psi$ is inferred by objective abduction (O-abduction) from $\phi$ by $a$ if

$$B_a \phi \land B_a (\psi \supset \phi) \land \neg B_a \neg \psi. \quad (1)$$

In this case, $\psi$ is called an O-explanation of $\phi$. We write $O-abd_a(\phi, \psi)$ if $\psi$ is an O-explanation of $\phi$ by $a$.

(1) means that a hearer $a$ believes the utterance $\phi$, and $a$ believes the implication $\psi \supset \phi$ and disbelieves $\neg \psi$ in his/her

1For the precise definition of $C \phi$, see (Fagin et al. 1995).
background knowledge. In this case, a hearer $a$ infers $\psi$ as an explanation for the utterance $\varphi$. It is called “objective” abduction because abduction is performed based on the objective fact of an utterance. Note that every explanation is consistent ($\psi \not\equiv \bot$). If $\psi \equiv \bot$, then $\neg B_a \neg \psi$ in (1) is false. There exist multiple explanations $\psi$ for an utterance $\varphi$ in general, and a hearer selects best explanations to be believed. We do not address the issue further in this paper.

**Definition 2 (subjective abduction)** Let $a$ be a hearer and $b$ a speaker. When $b$ utters a sentence $\varphi (\not\equiv \bot)$, a sentence $B_b \psi$ is inferred by *subjective abduction (S-abduction)* from $\varphi$ by $a$ if

$$B_a B_b \varphi \land B_a B_b (\psi \supset \varphi) \land \neg B_a \neg B_b \psi. \quad (2)$$

In this case, $B_b \psi$ is called an *S-explanation* of $\varphi$. We write $S-abd_{ab}(\varphi, \psi)$ if $B_b \psi$ is an S-explanation of $\varphi$ by $a$.

(2) means that a hearer $a$ believes that a speaker $b$ believes his/her utterance $\varphi$, and $a$ believes that $b$ believes the implication $\psi \supset \varphi$, and $a$ disbelieves that $b$ disbelieves $\psi$. In this case, a hearer $a$ infers $B_b \psi$ as an explanation for the utterance $\varphi$. It is called “subjective” abduction because abduction is performed based on the hearer’s subjective view on the speaker’s belief state.

In objective abduction, a hearer may believe an O-explanation $\psi$ which accounts for an utterance $\varphi$ by a speaker. In subjective abduction, on the other hand, a hearer may believe $B_b \psi$ but does not necessarily believe $\psi$ by himself/herself. A connection between O-abduction and S-abduction is as follows.

**Proposition 1** $S-abd_{ab}(\varphi, \psi)$ implies $O-abd_{ab}(B_b \varphi, B_b \psi)$.

The differences between $S-abd_{ab}(\varphi, \psi)$ and $O-abd_{ab}(B_b \varphi, B_b \psi)$ are twofold. First, a speaker $b$ utters a sentence $\varphi$ in $S-abd_{ab}(\varphi, \psi)$, while $b$ utters his/her belief $B_b \varphi$ in $O-abd_{ab}(B_b \varphi, B_b \psi)$. Second, a hearer $a$ believes that a speaker $b$ believes the implication $\psi \supset \varphi$ in $S-abd_{ab}(\varphi, \psi)$, while $a$ believes a weaker implication $B_b \psi \supset B_b \varphi$ (if a speaker $b$ believes $\psi$ then he believes $\varphi$) in $O-abd_{ab}(B_b \varphi, B_b \psi)$. One could also use O-abduction to abduce the belief state of a hearer by $O-abd_{ab}(\varphi, B_b \psi)$ if $B_a \varphi \land B_a (B_b \psi \supset \varphi) \land \neg B_a \neg B_b \psi$. In this case, a speaker utters a sentence $\varphi$ and a hearer abduces the belief state of the speaker which explains the utterance. All $S-abd_{ab}(\varphi, \psi)$, $O-abd_{ab}(B_b \varphi, B_b \psi)$ and $O-abd_{ab}(\varphi, B_b \psi)$ abduce the belief state of a speaker in different ways.

Suppose that a hearer $a$ infers an O-explanation $\psi$ or an S-explanation $B_b \psi$ in face of a speaker $b$’s utterance $\varphi$. If it is in fact $\psi$ or $B_b \psi$, the hearer successfully understands a reason behind the utterance $\varphi$. On the other hand, if it is in fact $\neg \psi$ or $\neg B_b \psi$, the hearer misunderstands a reason behind the utterance. In the introductory example, a colleague says “I will be at my office this weekend” and you conjecture “he/she has much work to do” by O-abduction. If you say “You seem to have much work to do” in response to the colleague, and he/she says “Not at all, I just come to my office to surf the net”, then you realize your incorrect abduction. As such, abduction is nonmonotonic in the sense that an explanation might be withdrawn if it turns out incorrect.

**Conversational Implicature**

*Conversational implicature* (Grice 1975) is a pragmatic inference to an implicit meaning of a sentence that is not actually uttered by a speaker. In his *maxims of conversation*, Grice introduces two maxims of quantity:

1. Make your contribution as informative as is required (for the current purposes of the exchange).
2. Do not make your contribution more informative than is required.

Based on these two maxims, two principles are introduced from the speaker’s viewpoint (Horn 1984; Levinson 1987):

**Q-principle**: Say as much as you can.

**I-principle**: Say no more than you must.

These two principles correspond to the next implicatures from the hearer’s viewpoint.

**Q-implicature**: Imply the negation of a semantically stronger sentence than what is actually uttered.

**I-implicature**: Imply a semantically stronger (or more specific) sentence than what is actually uttered.

These two implicatures apparently conflict because “the Q-implicatures induce the *negation* of the very sort of stronger interpretation that the I-implicatures actually appear to be promoting” (Levinson 1987). Various attempts to resolve the clash are proposed in pragmatics (Horn 1984; Levinson 1987; Carson 1995). In what follows, we formulate these two implicatures in our logic.

**Definition 3 (Q-implicature)** Let $a$ be a hearer and $b$ a speaker. When $b$ utters a sentence $\varphi (\not\equiv \bot)$, a sentence $B_b \neg \psi$ is inferred by *Q-implicature* from $\varphi$ by $a$ if

$$B_a B_b \varphi \land C (\psi \supset \varphi) \land \neg B_a B_b \psi. \quad (3)$$

We write $Q-imp_{ab}(\varphi, \psi)$ if $B_b \neg \psi$ is inferred by Q-implicature from $\varphi$ by $a$.

(3) is explained as follows. First, a hearer $a$ believes that a speaker $b$ believes his/her utterance $\varphi$. Otherwise, it would be meaningless to infer the implicit meaning behind the utterance. Second, $\psi \supset \varphi$ is not just a private belief of the hearer, but a common knowledge that is shared by the speaker and the hearer. Third, the hearer disbelieves that the speaker believes a sentence $\psi$ which is stronger than $\varphi$. In this case, the hearer infers that the speaker believes $\neg \psi$. There exist multiple sentences $\psi$ satisfying (3) in general, and a hearer selects an appropriate one from them in the context. The I-implicature is defined in a similar way.

**Definition 4 (I-implicature)** Let $a$ be a hearer and $b$ a speaker. When $b$ utters a sentence $\varphi (\not\equiv \bot)$, a sentence $B_b \psi$ is inferred by *I-implicature* from $\varphi$ by $a$ if

$$B_a B_b \varphi \land C (\psi \supset \varphi) \land \neg B_a \neg B_b \psi. \quad (4)$$

We write $I-imp_{ab}(\varphi, \psi)$ if $B_b \psi$ is inferred by I-implicature from $\varphi$ by $a$.

In both (3) and (4), conversational implicature is based on common knowledge, that is, both a speaker and a hearer
know the truth of the implication $\psi \supset \varphi$, and each one also knows that the other party knows the truth of the sentence. The reason of using common knowledge here is explained as follows. In Q-implicature (resp. I-implicature), a hearer believes that a speaker implies $\neg \psi$ (resp. $\psi$) by an utterance $\varphi$. In this case, the hearer knows the implication $\psi \supset \varphi$ and, at the same time, the hearer knows that the speaker knows the same implication. If the hearer does not know whether or not the speaker knows the implication, then the hearer cannot infer the intended meaning of the speaker’s utterance. Conversely, a speaker implies $\neg \psi$ (resp. $\psi$) by an utterance $\varphi$ in terms of his/her knowledge of $\psi \supset \varphi$. If the speaker does not know whether or not the hearer knows the implication, then the speaker cannot expect the hearer’s reasoning by Q-implicature (resp. I-implicature). So if the speaker utters $\varphi$, he/she knows that the hearer knows the implication $\psi \supset \varphi$.

Thus conversational implicature is in effect if and only if a speaker and a hearer share the same knowledge and each one knows that the other party also shares the same knowledge. The difference between two implicatures comes from a speaker’s belief of whether a speaker believes a stronger sentence or not. Given an utterance $\varphi$, if a hearer disbelieves that a speaker believes a stronger sentence $\psi$, then the hearer interprets that the utterance Q-implicates $\neg \psi$. By contrast, if a hearer disbelieves that a speaker disbelieves a stronger sentence $\psi$, then the hearer interprets that the utterance I-implicates $\psi$. Since $B_a \psi \land B_b \neg \psi \supset \bot$, the conclusions derived by two implicatures contradict with each other. On the other hand, it may happen that $\neg B_a \neg B_b \psi \land \neg B_a \neg B_b \psi$, so it is a hearer’s option to decide which implicature is to be applied in the context where $B_a B_b \psi \land C (\psi \supset \varphi)$ holds.

Abduction vs. Conversational Implicature

Both abduction and conversational implicature infer information behind an utterance. According to our formulation, an essential difference between the two lies in the use of implication $\psi \supset \varphi$. In abduction, it is a hearer’s private belief: a hearer believes $\psi \supset \varphi$ in O-abduction while a hearer believes that a speaker believes $\psi \supset \varphi$ in S-abduction. This is because abduction is a process of private reasoning, and one can reason abductively without knowing the belief state of the other party. By contrast, in conversational implicature the implication is common knowledge: a speaker and a hearer share the same implication. This is because a speaker may use common knowledge for the purpose of abduction when one observes wet grass. On the other hand, it is inappropriate to use one’s private belief to infer conversational implicature. A hearer’s reasoning based on his/her private belief does not always reflect a speaker’s intention. Formally, the following relation holds between S-abduction and I-implication.

**Proposition 2** $I$-implication implies S-abduction. For instance, suppose a dialogue in which a speaker ($a$) utters $\neg a \psi$ but a hearer ($b$) believes $\neg a \psi$. Then mother would infer that her son believes $\psi$ by S-abduction if she disbelieves that her son disbelieves $\psi$ (i.e., $\neg B_a \neg B_b \psi$). Else if she disbelieves that her son believes $\psi$ (i.e., $\neg B_a B_b \psi$), mother would infer that her son believes $\psi \supset \neg a \psi$ (he does not solve at most one question) by Q-implicature.

A hearer believes a speaker’s utterance ($B_a \psi$) in O-abduction, and believes that a speaker believes his/her utterance ($B_a B_b \psi$) in S-abduction. What happens if a hearer believes the falsity of the utterance by a speaker? Suppose a dialogue in which a speaker ($b$) utters $\varphi$ but a hearer ($a$) believes $\neg \varphi$. In this case, it holds that $B_a \neg B_b \psi \land B_a \psi$, and the hearer believes $\neg \psi$ if he/she believes $\psi \supset \varphi$. Likewise, it holds that $B_a \neg B_b \psi \land B_a B_b \psi$, so if a hearer $a$ believes that a speaker $b$ believes the implication $\psi \supset \varphi$, and the hearer also believes that the speaker is lying, i.e., the hearer believes that the speaker believes the falsity of his/her utterance $\varphi$, then a believes that $b$ believes $\neg \psi$. In this way, when a hearer believes the falsity of an utterance $\varphi$, the hearer would believe the negation of a sentence that explains $\varphi$. Moreover, if a hearer believes that a speaker is lying, then the hearer could infer reasons behind the act (a speaker lies $\varphi$ to make a hearer believe $\psi$ by abduction, but in fact the speaker believes $\neg \psi$). In conversational implicature, if a hearer $a$ believes that a speaker $b$ is lying, it holds that $B_a \neg B_b \psi \land C (\psi \supset \varphi) \supset B_a B_b \psi$. So a hearer does not believe the conclusion $B_a \psi$ of I-implicature while still believes the conclusion $B_a \psi$ of Q-implicature. For instance, if a speaker says “I have one million dollars” and a hearer does not believe it, the hearer does not infer that the speaker has at least one million dollars by I-implicature but still infers that the speaker does not have more than one million dollars by Q-implicature. This is an interesting contrast between two implicatures. From a speaker’s viewpoint, a speaker will decide what to say by considering the effect of his/her utterance on the hearer’s side. A speaker may use this to mislead a hearer to reach a wrong assumption. Further discussion on this topic will be done in the full version of this paper.

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1One may consider a weaker definition of implicature using “common belief” instead of “common knowledge”, but we do not address such alternative definitions here.
References