Towards a Preference Formalism for Modular Systems

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Abstract

We propose a framework for combining knowledge bases in modular systems with preferences. In our formalism, each module (knowledge base) may be characterized in a different language. Preferences defined over each module can be specified by different preference formalisms. We define the notion of a preference-based modular system that includes a formalization of meta-preferences. We prove that our formalism is robust in the sense that the operations for combining modules preserve the notion of a preference-based modular system.

Introduction

Combining knowledge bases (KBs) is very important when common sense reasoning is involved. For example, in planning, we may want to combine temporal and spatial reasoning, or reasoning from the point of view of several agents. Here, we focus on search problems, i.e., the problems where some input is given, and we are looking for a solution (e.g. a schedule, a trajectory, a business plan) according to a KB or a combination of KBs. Search problems are formalized as the task of Model Expansion (MX) (Mitchell and Ternovska 2005), which is the task of expanding a given structure (that represents an instance of the problem) with interpretations of new relations and functions (that represent solutions) to satisfy a specification in some logic, e.g. first-order logic, Answer Set Programming, etc. For example, consider the problem of constructing a trajectory of a falling ball. The input structure represents the initial conditions, and it is expanded with interpretation of a function (or a predicate) that represents spatial coordinates of the ball over a time interval, to satisfy an axiomatization of the trajectory.

Modular Systems (MS) (Tasharrofi and Ternovska 2011) is an extension to the MX framework. Each module (and a combination of them) is an MX task. Modules are combined through the operators of composition, union, projection, complementation and feedback. The framework is able to specify multi-component problems where their constituents are characterized in different languages. An algorithm for solving MSs was proposed in (Tasharrofi, Wu, and Ternovska 2011). Connections to Satisfiability Modulo Theory and other systems were discussed in the same paper.

An important aspect of knowledge representation systems is the capability to represent preferences. The literature on preference handling shows a variety of approaches to formalize preferences (Brafman and Domshlak 2009), (Santhanam, Basu, and Honavar 2011), (Brewka, Truszczynski, and Woltran 2010) (Sohrabi, Baier, and McIlraith 2009), (Boutilier et al. 2004), and (Faber, Truszczynski, and Woltran 2013). Several surveys have appeared in recent years categorizing preference formalisms from various perspectives. For example, in (Jorge and McIlraith 2009), a set of formalisms proposed for modelling planning with preferences have been introduced. The authors of (Delgrande et al. 2004) classified preference handling approaches in non-monotonic reasoning. Preferences in database systems have been broadly investigated by different researchers such as (Kiessling 2002) and (Stefanidis, Koutrika, and Pitoura 2011). A well-known preference relation is skyline relation. A skyline query returns a set of all preferred tuples stored in a database table (Borzsony, Kossmann, and Stocker 2001). However, the main principles behind combinations of preference formalisms (possibly written in different languages) have not been fully formalized yet. There have been some attempts to address the problem of composing preferences in modular systems such as (Ross 2007) or (Kiessling 2002). These are not language-independent. The following example clarifies the complexity of formalizing a modular system with preferences.

Example 1 A logistics service provider in a retail company decides how to pack and deliver goods. It takes orders from customers, decides how to pack items in available delivery trucks by solving the multiple-Knapsack problem, and for each truck, solves a TSP problem. The company may have preferences for packing and delivery of products. E.g. if a fragile item is packed in a truck, it may be preferred to exclude heavy items. Among certain routes with equal costs, some of them may be preferred to others. It is possible that preferences in the Knapsack problem are formalized by cnets (Boutilier et al. 2004) and the TSP’s preferences are represented in Answer Set Programming (Brewka, Niemela, and Truszczynski 2003). Formalizing this modular problem with preferences is not easy because the Knapsack and the TSP are axiomatized in different languages and preferences of each module are represented by a different formalism.
Contributions  We propose model-theoretic foundations for combining KBs with preferences in a modular system. On the logic level, each module is represented by a KB in some logic $\mathcal{L}$, and its preferences (and meta-preferences) are represented by a preference formalism. Different logics and preference formalisms can be used for modules in the same system. Semantically, each module is a set of structures, and their preferences are (strict) partial orders on partial structures. Operations for combining modules are generalized to combine preferences of each module. We prove that our formalism is robust in the sense that the operations for combining modules preserve the notion of a preference-based modular system. Our formalism is consistent with (and extends) the model-theoretic semantic of modular systems (Tasharrofi and Ternovska 2011).

Preliminaries

A vocabulary is a set of non-logical symbols (predicate and function symbols). A structure is a domain (a set), and interpretation of vocabulary symbols.

Definition 1 (Model Expansion task) Given: a formula $\phi$ in logic $\mathcal{L}$ with a vocabulary $\sigma \cup \varepsilon$, and $\sigma$-structure $A$. Find: structure $B$ that expands $A$ to $\sigma \cup \varepsilon$ and satisfies $\phi$. We call $\sigma$ instance and $\varepsilon$ expansion vocabularies.

A set of model expansion tasks can be integrated in a system and form a new model expansion task that is called a modular system.

Definition 2 A modular system is a set of primitive modules, where each module represents a $\mathcal{M}$X task, combined through a set of operators\footnote{Any logic with model-theoretic semantics can be used, including logic programs.} as follows:

- Projection ($\pi_r(M)$) restricts the vocabulary of $M$ to $\tau$. For modular system $M$ and $\tau \subseteq \mathcal{S}_M \cup \varepsilon_M$, modular system $\pi_r(M)$ is defined such that (a) $\sigma_{\pi_r(M)} = \sigma_M \cap \tau$, (b) $\varepsilon_{\pi_r(M)} = \varepsilon_M \cap \tau$, and (c) $B \in \pi_r(M)$ iff there is a structure $B' \in M$ with $B' \vDash \pi_r(M) = B$.

- Composition ($M_1 \triangleright M_2$) connects outputs of $M_1$ to inputs of $M_2$ such that (a) $\sigma_{M_1 \triangleright M_2} = \sigma_{M_1} \cup (\sigma_{M_2} \setminus \varepsilon_{M_1})$, (b) $\varepsilon_{M_1 \triangleright M_2} = \varepsilon_{M_1} \cup \varepsilon_{M_2}$, and (c) $B \in (M_1 \triangleright M_2)$ iff $B|_{\mathcal{S}_{\sigma_{M_1 \triangleright M_2}}} \in M_1$ and $B|_{\varepsilon_{\sigma_{M_1 \triangleright M_2}}} \in M_2$.

- Union ($M_1 \sqcup M_2$) represents choice. For modular systems $M_1$ and $M_2$ with $\sigma_{M_1} \cap \sigma_{M_2} = \sigma_{M_1} \cap \varepsilon_{M_2} = \varepsilon_{M_1} \cap \sigma_{M_2} = \emptyset$, the expression $M_1 \sqcup M_2$ defines a modular system such that (a) $\sigma_{M_1 \sqcup M_2} = \sigma_{M_1} \cup \sigma_{M_2}$, (b) $\varepsilon_{M_1 \sqcup M_2} = \varepsilon_{M_1} \cup \varepsilon_{M_2}$, and (c) $B \in (M_1 \sqcup M_2)$ iff $B|_{\mathcal{S}_{\sigma_{M_1 \sqcup M_2}}} \in M_1$ or $B|_{\varepsilon_{\sigma_{M_1 \sqcup M_2}}} \in M_2$.

- Feedback($M[R = S]$) which connects output of $M$ to its inputs $R$. For modular system $M$ and $R \in \sigma_M$ and $S \in \varepsilon_M$ being two symbols of similar type (i.e., either both function symbols or both predicate symbols) and of the same arities; expression $M[R = S]$ is a modular system such that (a) $\sigma_{M[R = S]} = \sigma_M \setminus \{R\}$, (b) $\varepsilon_{M[R = S]} = \varepsilon_M \cup \{R\}$, and (c) $B \in M[R = S]$ iff $B \in M$ and $R^B = S^B$.

Partial structures allow interpretation of some vocabulary symbols to be partially specified.

Definition 3 $B$ is a $\tau_p$-partial structure over vocabulary $\tau$ if: (1) $\tau_p \subseteq \tau$, (2) $B$ gives a total interpretation to symbols in $\tau \setminus \tau_p$, and (3) for each $n$-ary symbol $R$ in $\tau_p$, $B$ interprets $R$ using two sets $R^+$ and $R^-$ such that $R^+ \cap R^- = \emptyset$, and $R^+ \cup R^- \neq [\mathsf{dom}(B)]^n$.

Definition 4 For two partial structures $B$ and $B'$ over the same vocabulary and domain, we say that $B$ extends $B'$ if all undefined symbols in $B$ are also undefined in $B'$.

$\mathcal{P}$-$\mathcal{M}$S: Preference-based Modular Systems

In this section we introduce Preference-based Modular Systems ($\mathcal{P}$-$\mathcal{M}$S). In the model-theoretic semantics, a module can be viewed as a set of structures. To have a formalism compatible with these semantics, our goal is to define preference statements based on the concept of structures. However, using structures to model preferences is not always practical. Formally speaking, some interpreted symbols may be preferred to others and there is not enough information to decide about the rest. Unlike structures, partial structures interpret a subset of vocabulary symbols while interpretation of other symbols is unknown. The idea of partial structures originated from the notion of three-valued logic that a truth value of a statement can be true, false, or unknown (Kleene 1952). In our proposed formalism, a preference statement can be represented by a partial order over a set of partial structures when certain conditions hold. First, we explain the meaning of strict partial order.

Definition 5 A strict partial order $O$ over a set $S$ is a pair $O := (S, \prec)$ such that $\prec$ is a binary relation over elements of $S$ with the following properties: anti-reflexivity, asymmetry and transitivity.

Now, we define one preference statement for a single model expansion specified by a primitive module.

Definition 6 Let $M$ be a primitive module and $\mathcal{S}_{\mathcal{M}}(M) = \tau$. A preference $P = (O, \Gamma)$ in $M$ is a pair where $O = (S, \prec)$ is a strict partial order, $\Gamma$ is the set of all partial structures in $M$ as well, $\Gamma = \{C_1, C_2, ..., C_m\}$ is a set of $\tau_p_i$-partial structures, $1 \leq i \leq m$, in $M$ that $\tau_{pi} \subseteq \tau$.

In practical domains, preferences are usually expressed by conditional statements. In the above definition, we utilize a set of partial structures $\Gamma$ to express the the hypothesis of a preference statement and $O$ represents the conclusion. Once the preference has been defined, a preferred structure is introduced as follows:

Definition 7 Let $M$ be a primitive module, and $B, B'$ be two structures in $M$. Given preference $P = (O, \Gamma)$ in $M$, let $\Delta$ be a set of all structures in $M$ that extend at least one member of $\Gamma$. We say structure $B$ is preferred to $B'$ with respect to $P$ (denoted by $B \succ_p B'$) if

- $B, B' \in \Delta$ and
- There are partial structures $B_i$ and $B_j$ over $\mathcal{S}_{\mathcal{M}}(M)$ that can be extended to structures in $M$ such that $B_i \succ_j B_j$, and $B$ is an extension of $B_i$, where $B' \equiv B_j$.
This definition states that when a part of $B$ is preferred to $B'$, if a condition specified by $\Gamma$ is satisfied by both structures, we can conclude that $B$ is preferred to $B'$. It makes no difference how the rest of the vocabulary is interpreted because it is irrelevant to $\mathcal{P}$.

**Meta-Preferences** In practice, each module may have more than one preference statements. Some of them may be preferred to others. The question then arises how a preferred structure is defined in this case. The notion of meta-preference addresses this question.

**Definition 8** Given a module $M$ and a set of preferences $\Pi = \{\pi_1, \pi_2, ..., \pi_n\}$, assume $\mathcal{O}_{MP} = (\Pi, \prec)$ is a strict partial order over elements of $\Pi$. A Meta-preference $MP$ is characterized as $MP := \mathcal{O}_{MP}$. Let $\Omega_i := \{\pi_j \in \Pi | (\pi_j \succ_{\Pi} \pi_i) \lor (\pi_j \succ_{\Pi} \pi_i)\}$ be a subset of $\Pi$ that its elements have order relation with $\pi_i$. We say structure $B$ is preferred to $B'$ with respect to $MP$ (notation $B \succ_{MP} B'$), if there is a preference $\pi_i \in \Pi$ such that $B \succ_{\pi_i} B'$ and

- (a) There does not exist $\pi_j \in \Omega_i$, that $\pi_j \succ_{\pi_i} \pi_i$ with respect to $OM_{MP}$ and $B \succ_{\pi_i} B'$.
- (b) There is no a preference $\pi_k \in \Pi \setminus \Omega_i$, such that $B \succ_{\pi_k} B'$.
- (c) For a finite number $n$, if $B \succ_{MP} B', B \succ_{MP} B^2, ... \succ_{MP} B^n$, where $B^n_1 \leq i \leq n$, is a structure in $M$, there is no a structure $B'$ such that $B' \succ_{MP} B$.

This definition states that structure $B$ is preferred to $B'$ with respect to $MP$ if we can find a preference such as $\pi_i$, that $B \succ_{\pi_i} B'$ and there is no preference that makes $B'$ preferred to $B$. If there is a preference $\pi_j$ such that $B \succ_{\pi_j} B$ then $B$ is not preferred to $B'$ with respect to the meta-preference unless $\pi_j$ is preferred to $\pi_i$. Also, condition (c) guarantees transitivity of meta-preferences. If $B$ is preferred to $B'$ with respect to $MP$, and if $B'$ is preferred to $B''$, then $B'$ cannot be preferred to $B$ with respect to $MP$.

**Definition 9** Let $\Pi = \{\pi_1, ..., \pi_n\}$ be a set of preferences in module $M$ and $MP$ be a meta-preference over elements of $\Pi$. Two structures $B, B' \in M$ are called equally preferred with respect to $MP$, represented by $B \equiv_{MP} B'$, if none of them is preferred to the other one with respect to $MP$. The symbol $\equiv_{MP}$ is equivalent to $\succ_{MP}$ or $\approx_{MP}$.

**Proposition 1** Given a module $M$, that is a set of structures, and a set of preferences $\Pi = \{\pi_1, \pi_2, ..., \pi_n\}$ and a meta-preference $MP := \mathcal{O}_{MP}$, the pair $(M, \equiv_{MP})$ is a non-strict partial order.

**Preference-based Modular Systems** Up to now, we defined a preference $P$ in a single primitive module. In what follows, we study how a preference in a modular system is modelled when preferences of its components are given.

**Definition 10** Let $M = M_1 \triangleright M_2$ and let’s assume that $\text{vocab}(M_1) = \tau_1$ and $\text{vocab}(M_2) = \tau_2$. Assume that $\Pi_1$ is a set of preferences in $M_1$ and $MP_1$ is meta-preference over $\Pi_1$. Similarly, $\Pi_2$ and $MP_2$ are a set of preferences and meta-preference respectively in $M_2$. For $B, B' \in M$, $B$ is preferred to $B'$ with respect to $MP_1$ and $MP_2$, and is represented by $B \succ_{MP_1, MP_2} B'$ when $B|\tau_1 \succ_{MP_1} B'|\tau_1$ and $B|\tau_2 \succ_{MP_2} B'|\tau_2$.

Informally, $B$ is preferred to $B'$ with respect to $MP = MP_1 \triangleright MP_2$, if $B$ is preferred to $B'$ with respect to $MP_1$, when they are restricted to the vocabulary of $M_1$, and with respect to $MP_2$ when they are restricted to the vocabulary of $M_2$. We proceed to the union operator.

**Definition 11** Let $M = M_1 \cup M_2$ be a modular system. Suppose $\text{vocab}(M_1) = \tau_1$ and $\text{vocab}(M_2) = \tau_2$. Let $\Pi_1$ be a set of preferences in $M_1$ and $MP_1$ is a meta-preference over $\Pi_1$, and let $\Pi_2$ be a set of preferences in $M_2$ and $MP_2$ a meta-preference over $\Pi_2$. For $B, B' \in M$, $B$ is preferred to $B'$ with respect to $MP_1$ and $MP_2$ (notation $B \succ_{MP_1, MP_2} B'$) when $B|\tau_1 \succ_{MP_1} B'|\tau_1$ or $B|\tau_2 \succ_{MP_2} B'|\tau_2$.

Let us comment briefly on the feedback operator. Let $M$ be a $\sigma \cup e$ modular system, $R \in \sigma$, and $S \in e$. If $R$ and $S$ are two vocabulary symbols of the same type and arity, then $M[R = S]$ is a $(\sigma \setminus \{R\}) \cup (e \cup \{R\})$ modular system. The feedback operator does not change the vocabulary of a module. Hence, definition of a preference remains unchanged.

When $B \succ_{MP} B'$ holds in $M$, if $B$ and $B'$ are also structures of $M'$, we conclude that $B$ is preferred to $B'$ in $M'$.

**Definition 12** Assume $M' = M[R = S]$. $\Pi$ is a set of preferences in $M$, and $MP$ is a meta-preference over $\Pi$. Assume that $B$, $B' \in M$, and $B, B' \in M'$. If $B \succ_{MP} B'$ in $M$, then $B \succ_{MP} B'$ in $M'$.

This definition says that if two structures $B$ and $B'$ are in $M$, and $B$ is preferred to $B'$ with respect to $MP$ then $B$ remains preferred to $B'$ in module $M'$ that is module $M$ with feedback operator.

In the model expansion task, in a general sense, there are vocabulary symbols (notation $\varepsilon_{ik}$) that are hidden from outer observers while they are interpreted by the structures of the module. By considering the fact that projection operator hides some visible vocabulary symbols of the module, we present the following definition.

**Definition 13** Let $M' = \pi_\tau(M)$; $\text{vocab}(M') = \tau$, and $\text{vocab}(M') = \tau'$ ($\tau$ and $\tau'$ are visible vocabularies). Assume $B_{\pi}, B'_{\pi} \in M'$, if there are structures $B$ and $B'$ in $M$ such that $B|\pi = B_{\pi}$ and $B'|\pi = B'_{\pi}$ and $B \succ_{MP} B'$, then we say $B \succ_{MP} B'_{\pi}$ on the condition that for all vocabulary symbols $\varepsilon \in (\tau, \tau') \subseteq \varepsilon_{ik}$ the following holds: $R_{\varepsilon} = R'_{\varepsilon}$ and $R_{\varepsilon_{ik}} = R'_{\varepsilon_{ik}}$.

Intuitively, given two structures $B_{\pi}$ and $B'_{\pi}$ in $M'$, if we can find two structures $B$ and $B'$ in $M$ such that they expand $B_{\pi}$ and $B'_{\pi}$, if $B$ is preferred to $B'$ with respect to $MP$, we can conclude that $B_{\pi}$ is also preferred to $B'_{\pi}$.

**Definition 14** A modular system $MS$ is a preferred modular system and called $PM-MS$ if it is specified by a pair $(\equiv_{MP}, MS)$ where $MP$ is a meta-preference in $MS$.

The following statement shows the robustness of our notions and is proven by structural induction.

**Theorem 1** Assume for some $n$, a modular system $MS$ is obtained from $M_1, M_2, ..., M_n$, where $M_i, 1 \leq i \leq n$ are modular systems, by using operations in modular systems including composition, union, feedback, and projection. For all $1 \leq i \leq n$, if $M_i$ is $PM-MS$ then $M$ is also $PM-MS$. 

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Conclusion and Future Work

We proposed an abstract framework for unifying preference languages in modular systems. We introduced the notion of preference-based modular systems ($\mathcal{P}$-MS). We demonstrated that a system constructed by combination of some $\mathcal{P}$-MS is also a $\mathcal{P}$-MS. Our future work will address expressivity and computational issues of the framework. We will continue our study on practical aspects of our framework in AI applications, in particular, Business Processes that have complex modular structures and different users may communicate with different formal languages.

References


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