Risk Management Systems Must Provide Automatic Decisions According to Crisis Computable Algebras

Olivier Bartheye
CREC St-Cyr
Écoles de Saint-Cyr Coëtquidan
F–56381 Guer Cedex, France
olivier.bartheye@st-cyr.terre-net.defense.gouv.fr

Laurent Chaudron
ONERA
BA 701
F–13661 Salon-de-Provence, France
laurent.chaudron@polytechnique.org

Abstract
The present paper claims that any risk management system (more generally any decision support system) must be designed and modeled in order to provide automatically solutions for crisis issues. That is, one has to understand the abstract decision process in military tactics in order to be able to define why this process necessary converges and what means this kind of property in the frame of automatic plan generation.

Introduction
According to the sociologist Raymond Aron (Aron 1999), our civilization requires a “praxeology”, i.e. a science of action. In the frame of computer-aided risk management, that means that the design phase of any decision support system must provide also action plans thanks to a suitable algebraic representation of actions. In terms of decision support systems, that means that if one wants to classify among “good” and “bad” actions, one ought to use mathematical tools in order to represent the dynamic, i.e. algebras.

Several attempts using algebras as risk assessments in crisis domains exist; in strategic and tactic algebras (Yakovleff 2006), axioms and factors are undoubtedly relevant but they remain qualitative and imprecise.

Since any computer program is to be defined, as far as possible, according to correct algorithms, the paper is devoted to emphasize the influence of human decision processes and their good properties to design automatic decision processes based on formal representations.

Qualification of crisis algebras according to time and transition features
“Crisis” comes from the Greek word “krisis”, which has a common root with the Greek verb krinein (Latin cernere); “krisis” is related to Greek oracles, interpretations of bird flights, dreams, choice of sacrificed victims. “Krisis” denotes the ability to distinguish, to choose, to decide, to separate. It refers to the decisive moment in an uncertain process, one that allows a diagnosis, a prognosis, and, possibly, a solution to the crisis. Thus a state of crisis is inseparable from the experience of time. It is worthwhile to note that, on the contrary, the well-admitted definition or “The” current contemporaneous crisis leads to qualify a crisis domain as an an undecidable framework (Revault D’Allones 2012). At startup, this might be a conceptual confusion: does “The crisis” indicate an exceptional state or a regular or a permanent feature? For any individual, crisis is necessary associated to existence as a living experience; therefore by construction, it implies structurally a transition.

Strategic algebras and tactic algebras
Strategy is an implementation of a political objective (strategy is the manner in which military power should be developed and applied to achieve national objectives or those of a group of nations). Tactic is an implementation of strategy (tactic is the science and art of organizing a military force, and the techniques for using weapons or military units in combination for engaging and defeating an enemy in battle). The doctrine considers the power of current forces and proposes the solution which corresponds to the best use of these forces. If the ratio of forces is unbalanced and if the goal is modest, a single threat can lead to an agreement. Conversely, if means are not sufficient, one looks generally for indirect pressure (diplomatic action, …). Successive situations occur after applications of selected actions; actions are triggered according to valid conditions expressed by situations. Military actions agree with the following properties:

- Sequencing : every system in motion performs sequentially elementary actions in a fixed or a variable order.
- Phasing (planning step) : this step triggers actions with the best timing in order to gain the initiative.

Initiative is the ability to define the action terms all along the battle or the military operation, i.e. the ability to be an initiator when no clear statement holds or when the situation changes. Gaining the initiative means to control the “next” shot. One can taken into account six axioms for the so-called tactic algebra (Yakovleff 2006):

1. relativity: a military chief hasn’t a total freedom in the initiative for a given mission,
2. interdependency: initiative is shared with the enemy,
3. subjectivity: initiative depends on the information each opponent has,

4. unicity: each echelon has a unique commander able to gain initiative

5. double transitivity: all along the echelon chain, initiative is transferable downwards and usable upwards.

6. mortality: unused initiative is given to the opponent.

Taking into account the behavior of the enemy leads to player/opponent algebraic strategies with polarities “+/−” (Winskel 2012). Games and strategies are represented inside a pure algebraic framework by event structures with polarity, where events carry a polarity “+/−” respected by maps. Simple parallel composition of games $G/H$ is defined by juxtaposition and for any game $G$, the dual of a game $G^+$ reversing the roles of Player and Opponent is defined (an event structure $G^+$ with polarity is a copy of the event structure $G$ with a reversal of polarities, i.e. $g ∈ G^+$ is complement of $g ∈ G$, and vice versa). The arrow $G → H$ is an “asymmetric intuitionist entailment” as $G^+ || H$; the intuitive idea is that the player can represent the arrow from the knowledge he has about an opponent play $g ∈ G^+$ and he can define his play as $h ∈ H$ but not the other way round. That is, a strategy $σ : G → H$ is a strategy in $G^+ || H$; strategies compose with identities $A → A$. A pre-strategy is a total map of event structures with polarity $σ : S → G^+ || H$ and must agree with supplementary properties: receptivity (completeness), a strategy should be receptive to all possible moves of opponent and innocence (correctness), a strategy only adjoints immediate causal dependencies “−−” → “++”. A strategy is a receptive, innocent pre-strategy. The winning condition $W ⊆ C^∞(A)$ is a strategy $σ : S → A$ in $(A,W)$; $σ$ is winning for Player if and only if any maximal play against a counter-strategy results in a win for Player if and only if $σx ∈ W$, for all $+-$ maximal configurations $x ∈ C^∞(S)$.

**Automatic plan generation and crisis issues**

If one wants to provide automatic generation of actions plans, one is faced with two well-known issues already encountered in Artificial Intelligence since fifty years:

1. Consistency issue: for a given unit $u$, one has to separate “good” $u$-actions plans $T_u$ and “bad” $u$-actions plans $⊥_u$ with $T_u ∩ ⊥_u = ∅$.

2. Sequencing issue: the set of actions $Σ$ has a monoid structure with infinitely many neutral elements (identity sequences): $∀ξ, η ∈ Σ^2, ψ = ξ ∘ η ∧ ψ ∈ Σ$.

For sake of completeness and correctness, the topology of deductive systems is Hausdorff, i.e. discrete and separable. Take an action system $A$, in which selected instances of generic action patterns can be ordered along an affine $ℤ$-time line; if a deductive system holds in order to compute consistent action sequences, infinite action loops are perfectly valid. The lack of reaction provided by the affine $ℤ$-time line prevents to solve the crisis issue. In effect, infinite sequences although semantically valid must be considered as “bad” $u$-actions plans $⊥_u$.

Therefore the crucial issue is the following “Why does a military chief converge in his decision (even a wrong one or a null one) whereas an automatic planning process usually diverges”? The answer in expressed in terms of mathematical representation of reaction (i.e. of automorphisms groups inside the tactic algebra). A convergent decision process must handle compact information in order to solve the crisis issue. This process would converge if it would based on linear operators, i.e., tactical operators could be defined as a linear action on a vector $ξ$, an element of a $k$-vector space $V$ where $k$ is a field. Unfortunately, plan execution is conditional; i.e. inconsistency may be everywhere. A possible solution to converge is to accept definitely inconsistency in order to perform a transition; i.e. to forbid totally any kind of operator sequence in order to ensure innocence.

**Conclusion**

It turns out that in the frame of military actions, the knowledge required in order to provide an outcome is difficult to exploit by an automatic decision procedure. According to tactic axioms, an opponent $o$, depending on the side, is defined as the pair $(ξ,≺)_+$ or $(ξ,≺)_−$ and is a subtle balance between an equalizer “$≡_c$” of opponents capacitive forces (information stored on a compact support : intelligence services, network, structures) and an order relation “≺” based on the gain of the initiative and which corresponds to pure action, temporal dependency and uncertainty. “$≡$” requires a commutative valuation function with compact values, e.g. the norm of an operator $A^*: A → k$ in order to quantify the capacity of forces. Amalgamated forces of that kind are reactive and “defensive only”. They are opposite to active forces of positive polarity “≺” which are consistent without a compact support (i.e. without feedback) and are nevertheless crucial in order to perform a transition. Assume that any transition “≺” is performed according to simple non-commutative representation of information (i.e. without any intermediate state in between since there is no feedback); one obtains a convergent algorithm: “$≺_c^+$” and “$≺_c^−$” fight against, i.e. generate entropy in order to defeat and transform structural amalgamated forces “$≡_c^+$” and “$≡_c^−$” and to compute a central unit support $∅$ federating “$≺_c^+$ and “$≺_c^−$”.

**References**


