Proceedings of the Twelfth International Conference on the Principles of Knowledge Representation and Reasoning (KR 2010)

A Layered Graph Representation for Complex Regions * †

Sanjiang Li[1,2]
[1]Centre for Quantum Computation and Intelligent Systems,
Faculty of Engineering and Information Technology,
University of Technology, Sydney, Broadway NSW 2007, Australia
sanjiang.li@uts.edu.au
http://sites.google.com/site/lisanjiang/
[2]Key Laboratory of Symbolic Computation and Knowledge Engineering of Ministry of Education,
Jilin University, Changchun, 130012, China

Abstract

This paper proposes a layered graph model for representing the internal structure of complex plane regions, where each node represents the closure of a connected component of the interior or exterior of a complex region. The model provides a complete representation in the sense that the (global) nine-intersections between the interiors, the boundaries, and the exteriors of two complex regions can be determined by the (local) RCC8 relations between associated simple regions.

Keywords: qualitative spatial reasoning; topological relation; internal structure; layered graph; hole

I. Introduction

Among the many aspects of space, topology is perhaps the most important one. A major part of Qualitative Spatial Reasoning (QSR) research focuses on topological relation models and topological properties. The Region Connection Calculus (RCC) (Randell, Cui, and Cohn 1992) is the most well-known logic-based approach to topological information, which supports definitions of many topological relations, including the well-known RCC8 relations. Topological properties, such as a region '*has a hole*' or '*has up to k-components*' are also expressible in the RCC and similar logical-based formalisms (Cohn and Hazarika 2001).

This paper will not propose new relation model. Instead, we focus on the internal topological structure of complex regions and propose a graph model for representing complex regions. The new model distinguishes, for example, between each pair of regions in Fig.s 1 and 2, which are considered to be same in (Schneider and Behr 2006) and (Worboys and Bofakos 1993), respectively.

The rest of this paper proceeds as follows. Section II recalls some basic notions, and Section III proposes the graph model. A complete characterization of the (*global*) 9IM relations between two complex regions *locally* is given in Section IV, and the generalization technique is introduced in Section V. The last section concludes the paper.

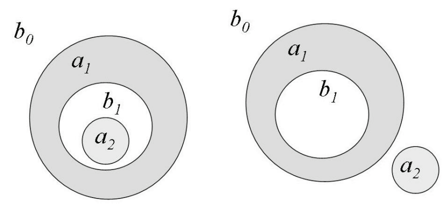
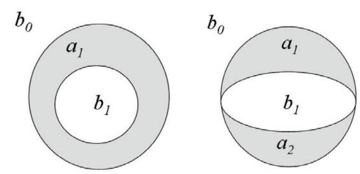


Figure 1: Two complex regions



(a) (b)

Figure 2: Two complex regions

II. Backgrounds

In this section, we introduce the basic notions and preliminary results needed in this paper.

For a set A of a topological space X, the *interior, closure, boundary* of A are denoted by A°, $\overline{A}$, and ∂A, respectively. A closed set A is *regular* if $\overline{A^\circ} = A$. We say a subset A of X is *disconnected* if there exist two disjoint open sets U, V such that $U \cap A$ and $V \cap A$ are nonempty and $A \subseteq U \cup V$. We say A is a *connected set* if it is not disconnected. A connected set A is called a *connected component* of an open set U if A is a maximal connected subset of U.

As usual, we define a *plane region* as a regular closed set in the real plane (with the usual topology). For a bounded plane region A, we call the closure of a connected component of A° (the interior of A) a *positive* component of A, and call the closure of a connected component of A^e (the exterior of A) a *negative* component of A. Since A is bounded, A^e has a unique unbounded connected component. We call its

*This work was partially supported by an Australian Research Council (ARC) Future Fellowship (FT0990811), the National Natural Science Foundation of China (60673105), the National Basic Research Program of China (2007CB311003), and an open project program of the MOE Key Laboratory of Symbolic Computation and Knowledge Engineering (93K-17-2009-K03).

†The complete version is available via http://arxiv.org/abs/0909.0109

closure *the* unbounded component of A and denote the closure as b_0. We call the closure of each bounded connected component of A^e a *hole* component of A, or simply a *hole* of A. As a consequence, each component of A is a plane region, *i.e.* a regular closed set.

Clearly, each bounded region has a unique unbounded component and at least one positive component. For practical applications, it is reasonable to require a complex region to have only finitely many (positive or hole) components.

In this paper, we plan to establish a valid representation scheme for plane regions. A primary requirement for such a representation is that *each region is finitely representable*. We hope each object can be reconstructed by applying finite basic operations (*e.g.* union, intersection, and difference) upon a finite set of atomic regions.

A *simple region* is a region which is homeomorphic to a closed disk. For our purpose, it is reasonable to choose simple regions as atomic regions. A *simple region with holes* (Egenhofer, Clementini, and Di Felice 1994) (*srh* for short) is a bounded connected region with several holes. The union of a srh and all its holes is called the *generalized region*. We assume, as usual, the holes and the generalized region of a srh are all simple regions.

A complex region is defined as follows.

Definition 1 (complex region). *A complex region A is a bounded regular closed subset of the real plane that has a finite set of positive components and a finite set of holes, where each positive component of A and each hole of A is a simple region with holes.*

For a bounded component c, suppose $a_1, a_2, \cdots, a_k$ are its holes. Let $a_0 = c \cup \bigcup_{i=1}^{k} a_i$ be the *generalized region* of c. For clarity, we often write c as $(a_0; a_1, \cdots, a_k)$ and write $\widehat{c}$ for the generalized region a_0 of c.

III. A Layered Graph Representation

We say two components are *strongly connected* or *linked* if the intersection of their closures contains a simple curve. Each component is linked with at least one other component. This is because its boundary is contained in the union of the boundaries of all other components.

For a complex region A, we introduce a level function for the components of A. For each component c of A, we define $\mathsf{lev}(c)$, the level of component c, inductively as follows:

- The level of b_0, the unbounded component of A, is 0;
- For an undefined c^*, if there exists a previously defined node c which is linked to c^*, then define $\mathsf{lev}(c^*) = \mathsf{lev}(c) + 1$.

If c_1, c_2 are two linked components, then $\mathsf{lev}(c_1) - \mathsf{lev}(c_2) = \pm 1$. For the complex region in Figure 2(b), we have $\mathsf{lev}(b_0) = 0$, $\mathsf{lev}(a_1) = \mathsf{lev}(a_2) = 1$, and $\mathsf{lev}(b_1) = 2$.

Definition 2 (link graph). *The link graph $\mathcal{L}_A$ of a complex region A is the directed graph $(N(A), E(A))$ defined as:*

- *$N(A)$ is the set of all components (positive, hole, or unbounded) of A;*
- *For $c_1, c_2 \in N(A)$, $(c_1, c_2) \in E(A)$ if they are linked and $\mathsf{lev}(c_2) = \mathsf{lev}(c_1) + 1$.*

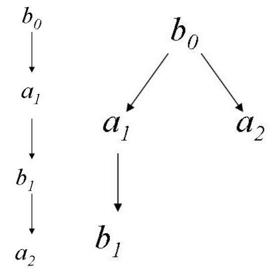


Figure 3: The link graphs of regions in Fig. 1

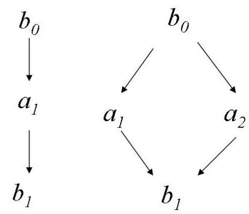


Figure 4: The link graphs of regions in Fig. 2

In this way, each complex region is represented by a layered graph. Fig.s 3 and 4 show the link graphs for the regions in Fig.s 1 and 2, respectively. Regions in these two figures have different link graphs. This suggests that link graph model is more expressive than the models of (Schneider and Behr 2006) and (Worboys and Bofakos 1993).

A complex region is called a *composite region* if its positive components are all simple regions and it has no hole components (Clementini, Di Felice, and Califano 1995).

Proposition 1. *Let b_0 be the unbounded component of a complex region A. Then $-b_0$, the closure of the complement of b_0 is a composite region.*

Recall that the generalized region of a srh is the union of all its bounded components. For a complex region A, we also call the union of all its bounded components the *generalized region* of A, written $\widehat{A}$. By Prop. 1, we know $\widehat{A} = -b_0$ is a composite region.

We now define the atom set of a complex region.

Definition 3 (atom set of complex region). *A simple region a is called an atom of a complex region A if one of the following conditions is satisfied:*

- *a is the generalized region or a hole of some bounded component c of A;*
- *a is a positive component of the composite region $-b_0$, where b_0 is the unbounded component of A.*

We write $\mathsf{ATOM}(A)$ for the atom set of A.

IV. Local Characterization of 9IM Relations

The 9IM relation (Egenhofer and Franzosa 1991) between two complex regions A and A' is defined as

$$M(A, A') = \begin{pmatrix} A^\circ \cap A'^\circ & A^\circ \cap \partial A' & A^\circ \cap A'^e \\ \partial A \cap A'^\circ & \partial A \cap \partial A' & \partial A \cap A'^e \\ A^e \cap A'^\circ & A^e \cap \partial A' & A^e \cap A'^e \end{pmatrix}.$$

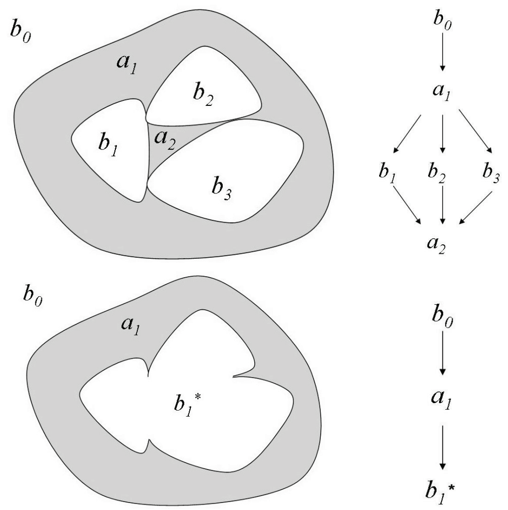


Figure 5: A complex region (upper) and its generalization (lower), where $b_1^* = a_2 \cup b_1 \cup b_2 \cup b_3$.

It has been proved that there are all together 34 different 9IM relations between (bounded) complex regions (Schneider and Behr 2006; Li 2006).

In this section, we characterize the 9IM relations in terms of the components of the two complex regions.

Theorem 1 (sufficiency). *The 9IM relation between two complex regions A, A' can be uniquely determined by the RCC8 relations between the atoms of A and A'.*

Are these atoms also *necessary*? That is, suppose we delete an atom o from the atom set $\mathsf{ATOM}(A)$. Do there exist two complex regions $B \neq C$, such that $M(A,B) \neq M(A,C)$ but $M(\rho,B) = M(\rho,C)$ for all $\rho \in \mathsf{ATOM}(A)$ such that $\rho \neq o$? The answer is NO!

Theorem 2 (necessary). *Each atom of a complex region A is necessary in locally determining the 9IM relation of A and some other region.*

A similar result has been obtained for simple regions with holes in (McKenney, Praing, and Schneider 2008).

V. Generalization by Dropping

Map generalization is a very important technique used in cartography and GISs. We now propose a new method to generalize a complex region into simpler ones.

For a simple region with holes $A = (a_0; a_1, \cdots, a_k)$, we obtain its generalized region $\widehat{A}$ by dropping all its holes. Similarly, for a complex region, we obtain its generalized region $\widehat{A}$ by merging all its bounded components. We could also obtain less complicated regions by dropping *some* components (together with their holes) from a complex region. Take Figure 5 as example, merging a_2 into its parents b_1, b_2, b_3 will obtain a simpler complex region.

Note that after each step of dropping, we obtain a complex region that has fewer components. Step by step, we will obtain a region that cannot be generalized. This is exactly $\widehat{A}$, the *generalized region* of A.

VI. Conclusion

In this paper we have established a qualitative model for representing the internal structure of complex plane regions. For the first time, we introduced the notions of holes, generalized regions, and atoms for general complex regions. We have proved that the generalized region of a complex region is a composite region, which equals to the closure of the set complement of the unbounded component.

We have also proved that the atoms of complex regions are necessary and sufficient for determining the nine-intersection relation between complex regions. We believe this provides a partial justification for the rationality for applying the 9IM to complex regions. It also suggests that the 9IM relation between complex regions can be implemented through the implementation of the RCC8 (or the 9IM) relation between simple regions.

References

Clementini, E.; Di Felice, P.; and Califano, G. 1995. Composite regions in topological queries. *Information Systems* 20(7):579–594.

Cohn, A., and Hazarika, S. 2001. Qualitative spatial representation and reasoning: An overview. *Fundamenta Informaticae* 46(1-2):1–29.

Egenhofer, M., and Franzosa, R. 1991. Point-set topological spatial relations. *International Journal of Geographical Information Systems* 5(1):161–174.

Egenhofer, M.; Clementini, E.; and Di Felice, P. 1994. Topological relations between regions with holes. *International Journal of Geographical Information Systems* 8(2):129–142.

Li, S. 2006. A complete classification of topological relations using the 9-intersection method. *International Journal of Geographical Information Science* 20(6):589–610.

McKenney, M.; Praing, R.; and Schneider, M. 2008. Deriving topological relationships between simple regions with holes. In *Proceedings of the 13th International Symposium on Spatial Data Handling (SDH-08)*, 1863–2246.

Randell, D.; Cui, Z.; and Cohn, A. 1992. A spatial logic based on regions and connection. In *Proceedings of the Third International Conference on Principles of Knowledge Representation and Reasoning (KR-92)*, 165–176.

Schneider, M., and Behr, T. 2006. Topological relationships between complex spatial objects. *ACM Transactions on Database Systems* 31(1):39–81.

Worboys, M., and Bofakos, P. 1993. A canonical model for a class of areal spatial objects. In *Proceedings of the Third International Symposium on Advances in Spatial Databases (SSD'93)*, 36–52. London, UK: Springer-Verlag.