

# ABA<sup>+</sup>: Assumption-Based Argumentation with Preferences

Kristijonas Čyras and Francesca Toni  
Imperial College London  
London, UK

## Abstract

We present a novel approach to account for preferences in a well known structured argumentation formalism, Assumption-Based Argumentation (ABA). The new formalism, called ABA<sup>+</sup>, incorporates object-level preferences (over assumptions) directly into the attack relation to reverse attacks. We give several basic desirable properties of ABA<sup>+</sup>.

## Introduction

Argumentation and preferences come a long way, see e.g. (Simari and Loui 1992). While argumentation deals with uncertain and conflicting information, preferences help to discriminate among alternatives. Broadly speaking, argumentation represents information via arguments, and attacks among them reflect conflicts. These may be asymmetric: information  $\beta$  may attack information  $\alpha$ , but  $\alpha$  need not attack  $\beta$ . Preferences can bring a different type of asymmetry by insisting that, say, the alternative  $\alpha$  is preferred over  $\beta$ .

**Example 1.** Zed wants to go out and two of his friends, Alice and Bob, are available. Best, Zed would take them both, but as far as he knows, Bob does not like Alice, although she does not have anything against Bob. If Zed offers to both of them at the same time, Bob may be in the awkward position to refuse Alice’s company. Offering separately, Alice is up for all three going, while Bob insists on cutting Alice out. Zed may opt for the latter option. However, had Zed a preference between the two—say Alice were a better friend of his—then he would go out with her.

A commonality of abstract argumentation (AA) (Dung 1995) and structured argumentation (see (Besnard et al. 2014) for an overview) in approaching the example above is to construct arguments  $A$  for Alice and  $B$  for Bob, and represent the asymmetric conflict by an attack  $B \rightsquigarrow A$ . Without preferences, argumentation semantics (say, stable semantics (Dung 1995)) sanction  $\{B\}$  as a unique acceptable *extension*, i.e. set of arguments. Preferences then play a twofold role (Amgoud and Vesic 2014): modifying the attack relation and selecting the most ‘preferable’ extensions. Intuitively, if  $A$  is preferred over  $B$  (in symbols,  $B < A$ ), then  $\{A\}$  should be chosen as the unique acceptable extension.

Various argumentation formalisms (see e.g. (Amgoud and Cayrol 2002; Bench-Capon 2003; Kaci and van der Torre 2008; Brewka et al. 2013; Besnard et al. 2014)) use preferences on *the argument level* to modify the attack relation: attacks from less preferred arguments are discarded, i.e. if  $B < A$ , then  $B \rightsquigarrow A$  fails. But this may be problematic, for instance, if  $\{A, B\}$  becomes a unique extension, then the intended conflict is lost (see e.g. (Kaci 2011; Amgoud and Vesic 2014) for discussions).

Assumption-Based Argumentation (ABA) (Bondarenko et al. 1997; Toni 2014), a well known structured argumentation formalism, accounts for preferences in one of two ways, as follows. On the one hand, preferences can be implicitly compiled into *the object level*, i.e. by encoding preference information within existing components (assumptions, contraries and rules), as in e.g. (Kowalski and Toni 1996; Fan and Toni 2013; Thang and Luong 2014). Such an approach may, however, produce numerous rules and assumptions from a compact preference relation (Wakaki 2014). On the other hand, in *ABA Equipped with Preferences* (p\_ABA henceforth) (Wakaki 2014), a preference relation is used on *the extension level* to select the most ‘preferable’ extensions. This means that, in Example 1, as  $\{B\}$  is the only extension to begin with, there is no choice to be made and  $\{A\}$  cannot be selected.

We propose a new way to explicitly handle preferences in ABA, advancing an alternative framework for structured argumentation with preferences. Our formalism, called ABA<sup>+</sup>, diverges significantly from other structured argumentation formalisms in two aspects.

First, preference information over defeasible knowledge is used to *reverse attacks* from an attacker that is less preferred than the attackee, instead of discarding them. The intuition is that when  $B$  ‘tries’ to attack  $A$  but fails due to preference  $B < A$ , the conflict between  $B$  and  $A$  is still present, and should be resolved in favour of  $A$  (cf. (Amgoud and Vesic 2014)).

Second, unlike *Preference-based Argumentation Frameworks* (PAFs) (Amgoud and Vesic 2014) that use attack reversal with preferences given over arguments, ABA<sup>+</sup> uses attack reversal with preferences given over assumptions (on the object level), that form the support of arguments. This is, moreover, done by integrating preferences directly into the attack relation, without lifting preferences from the ob-

ject level to either the argument or the extension levels, thus dispensing with preference aggregation mechanisms (cf. (Brewka, Truszczynski, and Woltran 2010; Modgil and Prakken 2013; Amgoud and Vesic 2014; Wakaki 2014)).

In this short exposition, after background on ABA, we will provide the essentials of  $ABA^+$  and indicate some of its basic properties: conflict preservation; being a conservative extension of ABA; and satisfaction of rationality postulates (Caminada and Amgoud 2007).

## Background

Background on ABA is based on (Toni 2014).

An *ABA framework* is a tuple  $(\mathcal{L}, \mathcal{R}, \mathcal{A}, \neg)$ , where:

- $(\mathcal{L}, \mathcal{R})$  is a deductive system with a language  $\mathcal{L}$  and a set  $\mathcal{R}$  of rules of the form  $\varphi_0 \leftarrow \varphi_1, \dots, \varphi_m$  with  $m \geq 0$  and  $\varphi_i \in \mathcal{L}$  for  $i \in \{0, \dots, m\}$ ;  $\varphi_0$  is referred to as the *head* of the rule, and  $\varphi_1, \dots, \varphi_m$  is referred to as the *body* of the rule; if  $m = 0$ , then the rule  $\varphi_0 \leftarrow \varphi_1, \dots, \varphi_m$  is written as  $\varphi_0 \leftarrow \top$  and is said to have an empty body;
- $\mathcal{A} \subseteq \mathcal{L}$  is a non-empty set, whose elements are referred to as *assumptions*;
- $\neg : \mathcal{A} \rightarrow \mathcal{L}$  is a total map: for  $\alpha \in \mathcal{A}$ , the  $\mathcal{L}$ -formula  $\bar{\alpha}$  is referred to as the *contrary* of  $\alpha$ .

We focus on *flat* ABA frameworks, where no assumption is the head of any rule from  $\mathcal{R}$ . Flat ABA frameworks capture, as instances, widely used paradigms of KR, such as Logic Programming and Default Logic (see e.g. (Bondarenko et al. 1997)).

A *deduction* for  $\varphi \in \mathcal{L}$  supported by  $S \subseteq \mathcal{L}$  and  $R \subseteq \mathcal{R}$ , denoted by  $S \vdash^R \varphi$ , is a finite tree with the root labelled by  $\varphi$ , leaves labelled by  $\top$  or elements from  $S$ , the children of non-leaf nodes  $\psi$  labelled by the elements of the body of some rule from  $\mathcal{R}$  with head  $\psi$ , and  $R$  being the set of all such rules. For  $E \subseteq \mathcal{L}$ , the *conclusions*  $Cn(E)$  is the set of elements with deductions supported by  $S \subseteq E$  (and  $R \subseteq \mathcal{R}$ ), i.e.  $Cn(E) = \{\varphi \in \mathcal{L} : \exists S \vdash^R \varphi, S \subseteq E, R \subseteq \mathcal{R}\}$ .

Assumption-level attacks in ABA are defined thus. A set  $A \subseteq \mathcal{A}$  *attacks* a set  $B \subseteq \mathcal{A}$ , denoted  $A \rightsquigarrow B$ , if there is a deduction  $A' \vdash^R \bar{\beta}$ , for some  $\beta \in B$ , supported by some  $A' \subseteq A$  and  $R \subseteq \mathcal{R}$ . For an *extension*  $E \subseteq \mathcal{A}$ , we say that:  $E$  is *conflict-free* if  $E \not\rightsquigarrow E$ ;  $E$  *defends*  $\alpha \in \mathcal{A}$  if for all  $B \rightsquigarrow \{\alpha\}$  it holds that  $E \rightsquigarrow B$ ;  $E$  is *admissible* if  $E$  is conflict-free and defends all  $\alpha \in E$ .

The most standard ABA semantics are defined as follows. A conflict-free extension  $E \subseteq \mathcal{A}$  is: *stable*, if  $E \rightsquigarrow \{\beta\}$  for every  $\{\beta\} \subseteq \mathcal{A} \setminus E$ ; *complete* if  $E$  is admissible and contains every assumption it defends; *preferred* if  $E$  is  $\subseteq$ -maximally admissible; *grounded* if  $E$  is  $\subseteq$ -minimally complete.

**Example 2.** Recall Example 1. Zed's knowledge can be represented in ABA as follows. Let  $\mathcal{L} = \{\alpha, \beta, \bar{\alpha}, \bar{\beta}\}$ ,  $\mathcal{R} = \{\bar{\alpha} \leftarrow \beta\}$  and  $\mathcal{A} = \{\alpha, \beta\}$ , where  $\alpha$  and  $\beta$  stand for (choosing) Alice and Bob, respectively, and  $\bar{\alpha} \leftarrow \beta$  indicates that Bob does not like Alice. In  $(\mathcal{L}, \mathcal{R}, \mathcal{A}, \neg)$ ,  $\{\beta\}$  attacks both  $\{\alpha\}$  and  $\{\alpha, \beta\}$ , while  $\{\alpha, \beta\}$  attacks itself and  $\{\alpha\}$ .  $(\mathcal{L}, \mathcal{R}, \mathcal{A}, \neg)$  can be graphically represented via its *assumption framework*, pictured below (in illustrations of assumption frameworks, nodes hold sets of assumptions and directed edges indicate attacks):



This  $(\mathcal{L}, \mathcal{R}, \mathcal{A}, \neg)$  has a unique complete extension  $\{\beta\}$ , which is also grounded, preferred and stable, and has conclusions  $Cn(\{\beta\}) = \{\bar{\alpha}, \beta\}$ .

## $ABA^+$

We extend standard ABA frameworks  $(\mathcal{L}, \mathcal{R}, \mathcal{A}, \neg)$  with a preference ordering  $\leq$  on the set  $\mathcal{A}$  of assumptions to obtain  *$ABA^+$  frameworks*  $(\mathcal{L}, \mathcal{R}, \mathcal{A}, \neg, \leq)$ , as follows.

**Definition 3.** An  **$ABA^+$  framework** is any  $(\mathcal{L}, \mathcal{R}, \mathcal{A}, \neg, \leq)$ , where  $(\mathcal{L}, \mathcal{R}, \mathcal{A}, \neg)$  is an ABA framework and  $\leq$  is a pre-order (i.e. reflexive and transitive binary relation) on  $\mathcal{A}$ .

From now on, unless stated differently, we consider a fixed, but otherwise arbitrary  $ABA^+$  framework  $(\mathcal{L}, \mathcal{R}, \mathcal{A}, \neg, \leq)$ , and implicitly assume  $(\mathcal{L}, \mathcal{R}, \mathcal{A}, \neg)$  to be its underlying ABA framework. The strict counterpart  $<$  of  $\leq$  is defined as  $\alpha < \beta$  iff  $\alpha \leq \beta$  and  $\beta \not\leq \alpha$ , for any  $\alpha$  and  $\beta$ .

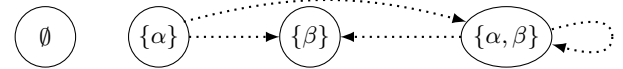
We next define the attack relation in  $ABA^+$ . The idea is that preferences reverse attacks such that the attacker contains an assumption less preferred than the one attacked.

**Definition 4.** A set  $A \subseteq \mathcal{A}$  of assumptions  **$<$ -attacks** a set  $B \subseteq \mathcal{A}$  of assumptions, written as  $A \rightsquigarrow_{<} B$ , just in case:

- either there is a deduction  $A' \vdash^R \bar{\beta}$ , for some  $\beta \in B$ , supported by  $A' \subseteq A$ , such that  $\forall \alpha' \in A' \alpha' < \beta$ ;
- or there is a deduction  $B' \vdash^R \bar{\alpha}$ , for some  $\alpha \in A$ , supported by  $B' \subseteq B$ , such that  $\exists \beta' \in B'$  with  $\beta' < \alpha$ .

The first type of attack is called *normal*, and the second one *reverse*. The following example illustrates.

**Example 5.** Recall  $(\mathcal{L}, \mathcal{R}, \mathcal{A}, \neg)$  from Example 2. Suppose there is a total preference order  $<$  on  $\mathcal{A}$  given by  $\beta < \alpha$ , representing that Zed prefers Alice over Bob. In the  $ABA^+$  framework  $(\mathcal{L}, \mathcal{R}, \mathcal{A}, \neg, \leq)$ ,  $\{\beta\}$  'tries' to attack  $\{\alpha\}$ , but is prevented by the preference  $\beta < \alpha$ . Instead,  $\{\alpha\}$   $<$ -attacks  $\{\beta\}$ , and likewise  $\{\alpha, \beta\}$ , via reverse attack, and the latter  $<$ -attacks both itself and  $\{\beta\}$  via reverse attack.  $(\mathcal{L}, \mathcal{R}, \mathcal{A}, \neg, \leq)$  can be represented graphically via its assumption framework as follows (where dotted arrows indicate reverse attacks):



In contrast with the ABA framework, where  $\{\beta\}$  is unattacked and generates an attack on  $\alpha$ , in the  $ABA^+$  framework,  $\{\alpha\}$  is  $<$ -unattacked and  $<$ -attacks all sets of assumptions that contain  $\beta$ . This concurs with the intended meaning of the preference  $\beta < \alpha$ , that the conflict should be resolved in favour of  $\alpha$ .

Normal attacks follow the standard notion of attack in ABA, preventing the attack to succeed when the attacker uses assumptions less preferred than the one attacked. Reverse attacks, meanwhile, manifest the conflict between two sets of assumptions by favouring the one which is contradicted (i.e. contains an assumption whose contrary is deduced using assumptions from the other set) over the one which contradicts using less preferred assumptions.

We next define the notions of conflict-freeness and defence w.r.t.  $\rightsquigarrow_{<}$ , and then introduce  $\text{ABA}^+$  semantics.

**Definition 6.** For  $E \subseteq \mathcal{A}$  we say that:  $E$  is **<-conflict-free** if  $E \not\rightsquigarrow_{<} E$ ;  $E$  **<-defends**  $\alpha \in \mathcal{A}$  if for all  $B \rightsquigarrow_{<} \{\alpha\}$  it holds that  $E \rightsquigarrow_{<} B$ ;  $E$  is **<-admissible** if  $E$  is <-conflict-free and <-defends every  $\alpha \in E$ .

$\text{ABA}^+$  semantics can be defined by replacing, in the standard ABA semantics definition, the notions of attack and defence with those of <-attack and <-defence.

**Definition 7.** A <-conflict-free extension  $E \subseteq \mathcal{A}$  is:

- **<-stable** if  $E \rightsquigarrow_{<} \{\alpha\}$  for every  $\{\alpha\} \subseteq \mathcal{A} \setminus E$ ;
- **<-complete** if  $E$  is <-admissible and contains every assumption it <-defends;
- **<-preferred** if  $E$  is  $\subseteq$ -maximally <-admissible;
- **<-grounded** if  $E$  is  $\subseteq$ -minimally <-complete.

From now on, we assume  $\sigma \in \{\text{grounded, stable, preferred, complete}\}$ , and likewise <- $\sigma$ , to denote a fixed semantics.

In Example 5,  $\{\alpha\}$  is a unique <-complete, <-preferred, <-stable and <-grounded extension.

### Basic Properties of $\text{ABA}^+$

The following property of ‘monotonicity of  $\rightsquigarrow_{<}$ ’ follows immediately from Definition 4.

**Lemma 1.** Let  $A' \subseteq A \subseteq \mathcal{A}$  and  $B' \subseteq B \subseteq \mathcal{A}$  be given. If  $A' \rightsquigarrow_{<} B'$ , then  $A \rightsquigarrow_{<} B$ .

We next consider how  $\text{ABA}^+$  and ABA attacks relate.

**Lemma 2.** For any  $A, B \subseteq \mathcal{A}$ :

- if  $A \rightsquigarrow B$ , then either  $A \rightsquigarrow_{<} B$  or  $B \rightsquigarrow_{<} A$ ;
- if  $A \rightsquigarrow_{<} B$ , then either  $A \rightsquigarrow B$  or  $B \rightsquigarrow A$ .

*Proof.* Let  $A, B \subseteq \mathcal{A}$  be arbitrary. Suppose first  $A \rightsquigarrow B$ . Then  $\exists A' \vdash^R \bar{\beta}$ ,  $\beta \in B$ ,  $A' \subseteq A$ , and either (i)  $\forall \alpha' \in A'$  we have  $\alpha' \not\prec \beta$ , or (ii)  $\exists \alpha' \in A'$  with  $\alpha' \prec \beta$ . In case (i),  $A' \rightsquigarrow_{<} B$ , and hence  $A \rightsquigarrow_{<} B$ , by Lemma 1. In case (ii),  $\{\beta\} \rightsquigarrow_{<} A'$ , and hence  $B \rightsquigarrow_{<} A$ , by Lemma 1 as well.

Suppose now  $A \rightsquigarrow_{<} B$ . Then (i) either  $\exists A' \vdash^R \bar{\beta}$ ,  $\beta \in B$ ,  $A' \subseteq A$  and  $\forall \alpha' \in A'$  we have  $\alpha' \not\prec \beta$ , or (ii)  $\exists B' \vdash^R \bar{\alpha}$ ,  $\alpha \in A$ ,  $B' \subseteq B$  and  $\exists \beta' \in B'$  with  $\beta' \prec \alpha$ . In the first case,  $A' \rightsquigarrow \{\beta\}$ , and so  $A \rightsquigarrow B$ , whereas in the second case,  $B' \rightsquigarrow \{\alpha\}$ , so that  $B \rightsquigarrow A$ .  $\square$

**Conflict Preservation.** One of the most prominent criticisms of various formalisms of argumentation with preferences (see e.g. (Kaci 2011; Amgoud and Vesic 2014)), is that preferences may disable some attacks, resulting in so-called *defeat* relations, which may fail to capture the original conflicts. As argued by e.g. (Kaci 2011), it is highly undesirable for extensions to be ‘defeat-free’ but not conflict-free with respect to the original attack relation.  $\text{ABA}^+$  avoids this shortcoming by preserving conflicts (due to Lemma 2):

**Proposition 3.**  $E \subseteq \mathcal{A}$  is <-conflict-free in  $(\mathcal{L}, \mathcal{R}, \mathcal{A}, \neg, \leq)$  iff  $E$  is conflict-free in  $(\mathcal{L}, \mathcal{R}, \mathcal{A}, \neg)$ .

Example 5 is a simple illustration of the need to preserve conflicts in argumentation: reversing the attack  $\{\beta\} \rightsquigarrow \{\alpha\}$  into  $\{\alpha\} \rightsquigarrow_{<} \{\beta\}$  preserves the conflict between  $\alpha$  and  $\beta$ , whereas simply discarding the attack would yield  $\{\alpha, \beta\}$  as a unique unintended  $\sigma$  extension (for any  $\sigma$ ).

**$\text{ABA}^+$  as a Conservative Extension of ABA.** Note that any ABA framework can be viewed as an  $\text{ABA}^+$  framework with an empty strict preference ordering. Trivially, if there are no preferences, then there are no reverse attacks, whence  $\text{ABA}^+$   $\emptyset$ -attack relation coincides with ABA attack relation:

**Lemma 4.** For any  $A, B \subseteq \mathcal{A}$ :  $A \rightsquigarrow B$  iff  $A \rightsquigarrow_{\emptyset} B$ .

So  $(\mathcal{L}, \mathcal{R}, \mathcal{A}, \neg, \emptyset)$  behaves exactly like  $(\mathcal{L}, \mathcal{R}, \mathcal{A}, \neg)$ :

**Theorem 5.**  $E \subseteq \mathcal{A}$  is a  $\sigma$ -extension of  $(\mathcal{L}, \mathcal{R}, \mathcal{A}, \neg)$  iff  $E$  is an  $\emptyset$ - $\sigma$  extension of  $(\mathcal{L}, \mathcal{R}, \mathcal{A}, \neg, \emptyset)$ .

**Rationality Postulates.** (Caminada and Amgoud 2007) formulated desirable properties, called *rationality postulates*, of argumentation systems. Following generalization in (Modgil and Prakken 2013), we reformulate the postulates for  $\text{ABA}^+$ . First, an auxiliary definition is required.

**Definition 8.**  $S \subseteq \mathcal{L}$  is: **directly consistent** if there are no  $\varphi, \psi \in S$  with  $\varphi = \bar{\psi}$ ; **indirectly consistent** if  $\text{Cn}(S)$  is directly consistent.

We next formulate the rationality postulates for  $\text{ABA}^+$ .

**Definition 9.** Let  $E_1, \dots, E_n$  be all the <- $\sigma$  extensions of  $(\mathcal{L}, \mathcal{R}, \mathcal{A}, \neg, \leq)$  and  $I = \{1, \dots, n\}$ . Then  $(\mathcal{L}, \mathcal{R}, \mathcal{A}, \neg, \leq)$  fulfils (for <- $\sigma$  semantics) **the Postulate of**

- **Closure** if  $\text{Cn}(E_i) = \text{Cn}(\text{Cn}(E_i)) \forall i \in I$ ;
- **Consistency** if  $E_i$  is directly consistent  $\forall i \in I$ ;
- **Indirect Consistency** if  $E_i$  is indirectly consistent  $\forall i \in I$ .

We note that (Caminada and Amgoud 2007) originally intended the postulates to account for classical negation in logic-based argumentation, which is absent in (Modgil and Prakken 2013)’s and our formulations. The original intention can be accounted for by, for instance, incorporating negation into the contrary mapping and imposing certain rational conditions on a framework (see e.g. (Modgil and Prakken 2013)). Alternatively, it can be accounted for by guaranteeing that conflict-freeness amounts to (classical) consistency, e.g. by appropriately modifying the original ABA framework as in (Toni 2007). To prove that  $\text{ABA}^+$  satisfies (our formulations of) the rationality postulates, we first show that <-conflict-free sets are (in)directly consistent.

**Lemma 6.** Let  $E \subseteq \mathcal{A}$  be a <-conflict-free extension. Then  $E$  is both directly and indirectly consistent.

*Proof.* If  $E$  is <-conflict-free, then it is conflict-free, by Proposition 3. If  $E$  were not directly consistent, there would be  $\alpha, \beta \in E$  such that  $\alpha = \bar{\beta}$ . But since  $\{\alpha\} \vdash^{\emptyset} \alpha$  is a deduction supported by  $\{\alpha\} \subseteq E$  and  $\emptyset \subseteq \mathcal{R}$ , we would get  $E \rightsquigarrow E$ , contradicting conflict-freeness of  $E$ .

Likewise, if  $E$  were not indirectly consistent, there would be  $\varphi, \beta \in \text{Cn}(E)$  such that  $\varphi = \bar{\beta}$ . But then there would be a deduction  $\Phi \vdash^R \varphi$  supported by some  $\Phi \subseteq E$  and  $R \subseteq \mathcal{R}$ , so that  $E \rightsquigarrow E$ , which is a contradiction.  $\square$

$\text{ABA}^+$ ’s ability to preserve conflicts sanctions satisfaction of the rationality postulates, as our next result indicates.

**Theorem 7.**  $(\mathcal{L}, \mathcal{R}, \mathcal{A}, \neg, \leq)$  fulfils the postulates of Closure, Consistency and Indirect Consistency, for any <- $\sigma$ .

*Proof.* Satisfaction of the Postulate of Closure is immediate, and fulfilment of the postulates of Direct and Indirect Consistency follows from Lemma 6.  $\square$

## Related and Future Work, Conclusions

We presented a new formalism,  $ABA^+$ , conservatively extending ABA with a novel technique for dealing with explicit preferences over assumptions.  $ABA^+$  is the first structured argumentation formalism to effectively reverse, rather than discard, attacks due to preference information.  $ABA^+$  incorporates preferences directly into its attack relation, dispensing with preference aggregation mechanisms on the argument level and/or the extension level.

$ABA^+$  differs from other formalisms of argumentation with preferences in various ways. For instance, p\_ABA employs preferences on the extension level to discriminate among extensions; but then, in Example 5, the unique extension  $\{\beta\}$  is ‘preferable’, and in case of non-existence of extensions (e.g. due to odd cycles), no ‘preferable’ extensions exist. On the other hand, ASPIC<sup>+</sup> (Modgil and Prakken 2013), as well as many other approaches, e.g. (Amgoud and Cayrol 2002; Bench-Capon 2003; Kaci and van der Torre 2008; Besnard et al. 2014), uses preferences to render some attacks obsolete, thus losing conflicts (cf. Proposition 3). PAFs (Amgoud and Vesic 2014) instead utilize attack reversal in the AA setting, but with a preference relation over arguments taken for granted. By contrast,  $ABA^+$  takes care of preferences on the object level (i.e. assumptions), by incorporating them into the attack relation.

Future research directions include: formally comparing  $ABA^+$  to various formalisms of argumentation with preferences (e.g. (Modgil 2009; Baroni et al. 2011; Dunne et al. 2011) in addition to the ones mentioned in the paper); examining other semantics (e.g. *ideal* (Dung, Mancarella, and Toni 2007)) and relationships among them; generalizing to non-flat frameworks (Bondarenko et al. 1997); accommodating *dynamic preferences* (as in e.g. (Prakken and Sartor 1996)); studying properties of preference handling (as given in e.g. (Brewka and Eiter 1999)).

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