Revising General Knowledge Bases in Description Logics*

Zhe Wang
Griffith University, Australia
truewz@gmail.com

Kewen Wang
Griffith University, Australia
k.wang@griffith.edu.au

Rodney Topor
Griffith University, Australia
r.topor@griffith.edu.au

Abstract

This paper introduces a new methodology of revising general KBs in DL-Lite. Two specific revision operators are defined, their properties are investigated and algorithms for computing revisions are developed.

Introduction

Recently, there has been significant interest in revision/update of knowledge bases in description logics. In particular, several model-based revision/update approaches to DLs have been proposed (Liu et al. 2006; Giacomo et al. 2007; Qi et al. 2009). However, these revision/update operators for DLs are unable to deal with general KBs. Specifically, approaches in (Liu et al. 2006; Giacomo et al. 2007) can deal with only ABoxe revision/update, while only TBoxes are considered in (Qi et al. 2009).

The key issues in adapting classical model-based approaches to DLs are how to define the distance between models and how to construct the resulting KB (directly or indirectly) from selected models. However, such adaption is difficult for the following reasons. (1) DL models have complex (possibly infinite) structures, which require a complex definition of the distance between two models. (2) Unlike a propositional theory, a DL KB may have infinitely many models, making it impossible to compute the result directly via models. (3) Given a collection $\mathbb{M}$ of models, there may not exist a single KB $\mathcal{K}$ such that $\mathbb{M}$ is exactly the set of models for $\mathcal{K}$. These are also the reasons for the restrictions in previous approaches to DL revision.

In contrast to previous approaches, we focus on DL-Lite, but address the problem of defining and computing revisions for general KBs, consisting of TBoxes *and* ABoxes. We first define *features* for DL-Lite$^{\mathcal{N}}_{bool}$, which precisely capture the most important semantic properties of DL-Lite$^{\mathcal{N}}_{bool}$ KBs, and (unlike models) are always finite. We adapt the techniques of model-based revision in propositional logic to the revision of DL-Lite$^{\mathcal{N}}_{bool}$ KBs, and define two specific revision operators based on two definitions of distance between features. We show that both revision operators possess desirable logical properties, and one of them preserve more knowledge from the original KB and thus yields a better result. A longer version of this paper can be found at `http://hobbit.ict.griffith.edu.au/~kewen/Papers/revision_long.pdf`.

The DL-Lite family(Calvanese et al. 2007; Artale et al. 2007), which forms the basis of OWL 2 QL (one of the three profiles of OWL 2), is a family of lightweight DLs with efficient KB reasoning and query answering algorithms. A *signature* is a finite set $\mathcal{S} = \mathcal{S}_C \cup \mathcal{S}_R \cup \mathcal{S}_I \cup \mathcal{S}_N$ where $\mathcal{S}_C$ is the set of atomic concepts, $\mathcal{S}_R$ is the set of atomic roles, $\mathcal{S}_I$ is the set of individual names and $\mathcal{S}_N$ is the set of natural numbers in $\mathcal{S}$. We assume the number 1 is always in $\mathcal{S}_N$. $\top$ and $\bot$ will not be considered as atomic concepts or atomic roles. Formally, given a signature $\mathcal{S}$, a DL-Lite$^{\mathcal{N}}_{bool}$ language has the following syntax:

$$R \leftarrow P \mid P^-, \qquad B \leftarrow \top \mid \bot \mid A \mid \geqslant n\, R,$$
$$C \leftarrow B \mid \neg C \mid C_1 \sqcap C_2,$$

where $n \in \mathcal{S}_N$, $A \in \mathcal{S}_C$ and $P \in \mathcal{S}_R$. B is called a *basic concept* and C is called a *general concept*. We write $\exists R$ as a shorthand for $\geq 1\ R$.

Given a set $\mathbb{M}$ of interpretations and a signature $\mathcal{S}$, in most cases there does not exist a KB $\mathcal{K}$ over $\mathcal{S}$ such that the models of $\mathcal{K}$ is exactly $\mathbb{M}$. To tackle this inexpressibility problem, a notion of best approximation is introduced in (Giacomo et al. 2007). A KB $\mathcal{K}$ is said to be a *maximal approximation* of $\mathbb{M}$ over $\mathcal{S}$ if (1) $\mathsf{Sig}(\mathcal{K}) \subseteq \mathcal{S}$, (2) $\mathbb{M} \subseteq \mathsf{Mod}(\mathcal{K})$, and (3) there exists no KB $\mathcal{K}'$ over $\mathcal{S}$ such that $\mathbb{M} \subseteq \mathsf{Mod}(\mathcal{K}') \subset \mathsf{Mod}(\mathcal{K})$. It is shown in (Giacomo et al. 2007) that maximal approximation may not exist for some DLs. However, we can show that maximal approximations always exist in DL-Lite$^{\mathcal{N}}_{bool}$. When it exists, maximal approximation of $\mathbb{M}$ is unique up to KB equivalence.

A *disjunctive knowledge base* (DKB) (Meyer et al. 2005) is a set $\mathbb{K}$ of KBs, defined in such a way that $\mathsf{Mod}(\mathbb{K}) = \bigcup_{\mathcal{K}\in\mathbb{K}} \mathsf{Mod}(\mathcal{K})$.

Features in DL-Lite$^{\mathcal{N}}_{bool}$

In this section, we introduce the concept of features in DL-Lite$^{\mathcal{N}}_{bool}$, which provides an alternative semantic characterization for DL-Lite$^{\mathcal{N}}_{bool}$. An advantage of semantic features

*This work was partially supported by the Australia Research Council (ARC) Discovery Project 0666107.

over models is that the number of all features for a DL-Lite$_{bool}^{\mathcal{N}}$ knowledge base is finite and each feature is finite. These finiteness properties make it possible to recast key approaches to revision for classical propositional logic into DL-Lite$_{bool}^{\mathcal{N}}$.

Features for DL-Lite$_{bool}^{\mathcal{N}}$ are based on the notion of *types* defined in (Kontchakov et al. 2008). Types are enough to capture the semantics of TBoxes, but insufficient for ABoxes. We need to extend the notion of types and thus define *Herbrand sets* for ABoxes.

Definition 1 *A* $\mathcal{S}$-Herbrand set *(or* Herbrand set *when $\mathcal{S}$ is clear from the context) $\mathcal{H}$ is a finite set of assertions of the form $B(a)$ or $P(a,b)$, where $a,b \in \mathcal{S}_I$, $P \in \mathcal{S}_R$ and B is a basic concept over $\mathcal{S}$, satisfying the following conditions*

1. *For each $a \in \mathcal{S}_I$, $\top(a) \in \mathcal{H}$, $\bot(a) \notin \mathcal{H}$, and $\geqslant n\ R(a) \in \mathcal{H}$ implies $\geqslant m\ R(a) \in \mathcal{H}$ for $m, n \in \mathcal{S}_N$ with $m < n$.*
2. *For each $P \in \mathcal{S}_R$, $P(a, b_i) \in \mathcal{H}$ $(i = 1, \ldots, n)$ implies $\geqslant m\ P(a) \in \mathcal{H}$ for any $m \in \mathcal{S}_N$ such that $m \leq n$.*
3. *For each $P \in \mathcal{S}_R$, $P(b_i, a) \in \mathcal{H}$ $(i = 1, \ldots, n)$ implies $\geqslant m\ P^-(a) \in \mathcal{H}$ for any $m \in \mathcal{S}_N$ such that $m \leq n$.*

Definition 2 (Features) *Given a signature $\mathcal{S}$, an* $\mathcal{S}$-feature *(or simply* feature *when $\mathcal{S}$ is clear) is defined as a pair $\mathcal{F} = \langle \Xi, \mathcal{H} \rangle$, where Ξ is a non-empty set of $\mathcal{S}$-types and $\mathcal{H}$ a $\mathcal{S}$-Herbrand set, satisfying the following conditions:*

1. *$\exists P \in \bigcup \Xi$ iff $\exists P^- \in \bigcup \Xi$, for each $P \in \mathcal{S}_R$.*
2. *$\tau \in \Xi$, for each $a \in \mathcal{S}_I$ and τ the type of a in $\mathcal{H}$.*

The satisfaction relation of an inclusion or assertion *w.r.t.* a feature can be defined in an intuitive way.

A feature $\mathcal{F}$ is a *model feature* of KB $\mathcal{K}$ if $\mathcal{F}$ satisfies every inclusion and assertion in $\mathcal{K}$. We use $\mathsf{MF}(\mathcal{K})$ to denote the set of all model features of $\mathcal{K}$.

From the definition of features, as we only consider finite signatures, a model feature is always finite in structure, and the number of model features of a KB is also finite.

Given an inclusion or assertion α, define $\mathcal{K} \models_f \alpha$ if all features in $\mathsf{MF}(\mathcal{K})$ satisfy α. Given two KBs $\mathcal{K}_1, \mathcal{K}_2$ and $\mathcal{S} = \mathsf{Sig}(\mathcal{K}_1 \cup \mathcal{K}_2)$, define $\mathcal{K}_1 \models_f \mathcal{K}_2$ if $\mathsf{MF}(\mathcal{K}_1) \subseteq \mathsf{MF}(\mathcal{K}_2)$, and $\mathcal{K}_1 \equiv_f \mathcal{K}_2$ if $\mathsf{MF}(\mathcal{K}_1) = \mathsf{MF}(\mathcal{K}_2)$.

The model features do capture the semantic properties of DL-Lite KBs.

Proposition 1 *Let $\mathcal{K}$ be a DL-Lite$_{bool}^{\mathcal{N}}$ KB and $\mathcal{S} = \mathsf{Sig}(\mathcal{K})$. Then*

- *$\mathcal{K}$ is consistent iff $\mathcal{K}$ has a model feature.*
- *$\mathcal{K} \models (C_1 \sqsubseteq C_2)$ iff $\mathcal{K} \models_f (C_1 \sqsubseteq C_2)$ for any $C_1 \sqsubseteq C_2$ over $\mathcal{S}$.*
- *$\mathcal{K} \models \alpha$ iff $\mathcal{K} \models_f \alpha$ for any assertion α over $\mathcal{S}$.*
- *$\mathcal{K} \equiv \mathcal{K}'$ iff $\mathcal{K} \equiv_f \mathcal{K}'$ where $\mathcal{K}'$ is another KB.*

A KB $\mathcal{K}$ is a *maximal approximation* of a set $\mathbb{F}$ of $\mathcal{S}$-features iff (1) $\mathsf{Sig}(\mathcal{K}) \subseteq \mathcal{S}$, (2) $\mathbb{F} \subseteq \mathsf{MF}(\mathcal{K})$, and (3) there exists no KB $\mathcal{K}'$ over $\mathcal{S}$ such that $\mathbb{F} \subseteq \mathsf{MF}(\mathcal{K}') \subset \mathsf{MF}(\mathcal{K})$.

Feature Distance and Revision

In this section, we define a distance between features, in the spirit of Hamming distance for propositional models. The distance is defined as the set of concept and role names interpreted differently in the two features.

Given a set Σ of concept and role names and $\mathcal{S}$-types τ_1, τ_2, denote $\tau_1 \sim_\Sigma \tau_2$ if for all basic concepts B over $\mathcal{S} - \Sigma$, $B \in \tau_1$ iff $B \in \tau_2$.

Let $\mathcal{F}_1 = \langle \Xi_1, \mathcal{H}_1 \rangle$ and $\mathcal{F}_2 = \langle \Xi_2, \mathcal{H}_2 \rangle$ be two $\mathcal{S}$-features, and $\Sigma \subseteq \mathcal{S}_C \cup \mathcal{S}_R$. Define $\mathcal{F}_1 \leftrightarrow_\Sigma \mathcal{F}_2$ if the following conditions are satisfied:

1. For each $\tau_1 \in \Xi_1$, there exists $\tau_2 \in \Xi_2$ s.t. $\tau_1 \sim_\Sigma \tau_2$; conversely, for each $\tau_2 \in \Xi_2$, there exists $\tau_1 \in \Xi_1$ s.t. $\tau_1 \sim_\Sigma \tau_2$.
2. For each $a \in \mathcal{S}_I$, $\tau_1 \sim_\Sigma \tau_2$, where τ_i $(i = 1, 2)$ is the type of a in $\mathcal{H}_i$; and $P(a, b) \in \mathcal{H}_1$ iff $P(a, b) \in \mathcal{H}_2$ for each $P \in \mathcal{S}_R - \Sigma$ and $a, b \in \mathcal{S}_I$.

Intuitively, the minimal sets Σ such that $\mathcal{F}_1 \leftrightarrow_\Sigma \mathcal{F}_2$ are the sets of concept and role names on whose interpretations $\mathcal{F}_1, \mathcal{F}_2$ disagree.

Given two KBs $\mathcal{K}_1, \mathcal{K}_2$ and $\mathcal{S} = \mathsf{Sig}(\mathcal{K}_1 \cup \mathcal{K}_2)$, define the distance between $\mathcal{K}_1$ and $\mathcal{K}_2$ as the set of all minimal distances between model features of $\mathcal{K}_1$ and $\mathcal{K}_2$:

$d_f(\mathcal{K}_1, \mathcal{K}_2) = \min_\subseteq(\{\ \Sigma \subseteq \mathcal{S}_C \cup \mathcal{S}_R \mid \exists \mathcal{F}_1 \in \mathsf{MF}(\mathcal{K}_1), \exists \mathcal{F}_2 \in \mathsf{MF}(\mathcal{K}_2)$ *s.t.* $\mathcal{F}_1 \leftrightarrow_\Sigma \mathcal{F}_2\ \})$.

To define a revision operator in analogy to classical model-based revision, we need to specify the subset of $\mathsf{MF}(\mathcal{K}')$ that is *closest* to $\mathsf{MF}(\mathcal{K})$ (*w.r.t.* feature distance).

Definition 3 (S-Revision) *Let $\mathcal{K}, \mathcal{K}'$ be two DL-Lite$_{bool}^{\mathcal{N}}$ KBs and $\mathcal{S} = \mathsf{Sig}(\mathcal{K} \cup \mathcal{K}')$. Define the* s-revision *of $\mathcal{K}$ by $\mathcal{K}'$, denoted $\mathcal{K} \circ_s \mathcal{K}'$, such that $\mathsf{MF}(\mathcal{K} \circ_s \mathcal{K}') = \mathsf{MF}(\mathcal{K}')$ if $\mathsf{MF}(\mathcal{K}) = \emptyset$, and otherwise,*

$$\mathsf{MF}(\mathcal{K} \circ_s \mathcal{K}') = \{\ \mathcal{F}' \in \mathsf{MF}(\mathcal{K}') \mid \exists \mathcal{F} \in \mathsf{MF}(\mathcal{K}) \text{ s.t. } \mathcal{F} \leftrightarrow_\Sigma \mathcal{F}' \text{ and } \Sigma \in d_f(\mathcal{K}_1, \mathcal{K}_2)\ \}.$$

Example 1 *Let $\mathcal{K} = \langle \{ PhD \sqsubseteq Student \sqcap Postgrad, Student \sqsubseteq \neg \exists teaches, \exists teaches^- \sqsubseteq Course, Student \sqcap Course \sqsubseteq \bot \}, \{ PhD(Tom) \} \rangle$. Suppose PhD students are actually allowed to teach, and we want to revise $\mathcal{K}$ with $\mathcal{K}' = \langle \{ PhD \sqsubseteq \exists teaches \}, \emptyset \rangle$. Then $\mathcal{K} \circ_s \mathcal{K}'$ is a DKB $\{\mathcal{K}_1, \mathcal{K}_2\}$ where*

$$\begin{aligned}\mathcal{K}_1 = \langle\ &\{ PhD \sqsubseteq \exists teaches,\ Student \sqsubseteq \neg\exists teaches,\\ &\quad \exists teaches^- \sqsubseteq Course,\ Student \sqcap Course \sqsubseteq \bot \},\\ &\{ Student(Tom),\ Postgrad(Tom) \} \rangle, \text{ and}\\ \mathcal{K}_2 = \langle\ &\{ PhD \sqsubseteq \exists teaches,\ PhD \sqsubseteq Student \sqcap Postgrad,\\ &\quad Student \sqcap Course \sqsubseteq \bot \},\ \{ PhD(Tom) \} \rangle.\end{aligned}$$

An interesting observation is that $\mathcal{K} \circ_s \mathcal{K}'$ can be computed by query-based forgetting. In particular, let $\mathsf{forget}(\mathcal{K}, \Sigma)$ denote a result of $\mathcal{Q}_{\mathcal{L}}^u$-forgetting about Σ in $\mathcal{K}$ (Wang et al. 2010), we have the following connection between revision and forgetting.

Proposition 2 *Let $\mathcal{K}, \mathcal{K}'$ be two consistent DL-Lite$_{bool}^{\mathcal{N}}$ KBs and $\mathcal{S} = \mathsf{Sig}(\mathcal{K} \cup \mathcal{K}')$. Then*

$$\mathcal{K} \circ_s \mathcal{K}' = \{\ \mathsf{forget}(\mathcal{K}, \Sigma) \cup \mathcal{K}' \mid \Sigma \in d_f(\mathcal{K}, \mathcal{K}')\ \},$$

where $\mathsf{forget}(\mathcal{K}, \Sigma)$ is a result of $\mathcal{Q}_{\mathcal{L}}^u$-forgetting about Σ in $\mathcal{K}$.

Revision under Approximation

For many applications, it is desirable to have the revision as a single DL-Lite$^{\mathcal{N}}_{bool}$ KB rather than a DKB, *i.e.*, the maximal approximation of the revision is desired. However, in most cases, $\mathcal{K} \circ_s \mathcal{K}'$ is too weak in preserving knowledge of the original KB, as shown in the following example.

Example 2 *In Example 1, $\mathcal{K} \circ_s \mathcal{K}'$ is a DKB, whose maximal approximation is the following KB,* $\langle \{ \mathit{PhD} \sqsubseteq \exists \mathit{teaches}, \mathit{Student} \sqcap \mathit{Course} \sqsubseteq \bot \}, \{ (\mathit{Student}(\mathit{Tom}), \mathit{Postgrad}(\mathit{Tom}), \big(\mathit{PhD} \sqcup (\neg\exists \mathit{teaches} \sqcap \neg\exists \mathit{teaches}^-)\big)(\mathit{Tom}) \} \rangle$.

Note that in the above example, knowledge in $\mathcal{K}$ about concept PhD and about role $\mathit{teaches}$ are totally lost after revision and approximation. In particular, $\mathit{PhD} \sqsubseteq \mathit{Postgrad}$ and $\exists \mathit{teaches}^- \sqsubseteq \mathit{Course}$ are eliminated, though they have nothing to do with the inconsistency.

We argue that the reason the revision operator $\circ_s$ performs poorly under approximation is that the distance defined on concept and role names is too simple to reflect differences between model features. To obtain a better definition of KB revision, we need to introduce a more complex notion of feature distance, which extends the definition of symmetric difference $\triangle$.

Recall that $S_1 \triangle S_2 = (S_1 - S_2) \cup (S_2 - S_1)$ for any two sets S_1 and S_2. Given two $\mathcal{S}$-features $\mathcal{F}_1 = \langle \Xi_1, \mathcal{H}_1 \rangle$ and $\mathcal{F}_2 = \langle \Xi_2, \mathcal{H}_2 \rangle$, we define the *distance* between $\mathcal{F}_1$ and $\mathcal{F}_2$, denoted $\mathcal{F}_1 \triangle \mathcal{F}_2$, as a pair $\langle \Xi_1 \triangle \Xi_2, \mathcal{H}_1 \triangle \mathcal{H}_2 \rangle$. Note that we do not require $\mathcal{H}_1 \triangle \mathcal{H}_2$ to be a Herbrand set.

To compare two distances, given $\mathcal{F}_i = \langle \Xi_i, \mathcal{H}_i \rangle$ for $i = 1, 2, 3, 4$, we could define $\mathcal{F}_1 \triangle \mathcal{F}_2 \subseteq_f \mathcal{F}_3 \triangle \mathcal{F}_4$ if $\Xi_1 \triangle \Xi_2 \subseteq \Xi_3 \triangle \Xi_4$ and $\mathcal{H}_1 \triangle \mathcal{H}_2 \subseteq \mathcal{H}_3 \triangle \mathcal{H}_4$; and $\mathcal{F}_1 \triangle \mathcal{F}_2 \subset_f \mathcal{F}_3 \triangle \mathcal{F}_4$ if $\mathcal{F}_1 \triangle \mathcal{F}_2 \subseteq \mathcal{F}_3 \triangle \mathcal{F}_4$ and $\mathcal{F}_3 \triangle \mathcal{F}_4 \not\subseteq \mathcal{F}_1 \triangle \mathcal{F}_2$. However, our effort shows that such a measure is too weak to preserve enough knowledge of the original KB, as many features are still incomparable under such measure. Instead, we set a preference on Herbrand sets over type sets: $\mathcal{F}_1 \triangle \mathcal{F}_2 \subset_f \mathcal{F}_3 \triangle \mathcal{F}_4$ iff

- $\mathcal{H}_1 \triangle \mathcal{H}_2 \subset \mathcal{H}_3 \triangle \mathcal{H}_4$, or
- $\mathcal{H}_1 \triangle \mathcal{H}_2 = \mathcal{H}_3 \triangle \mathcal{H}_4$ and $\Xi_1 \triangle \Xi_2 \subset \Xi_3 \triangle \Xi_4$.

Definition 4 (F-Revision) *Let $\mathcal{K}, \mathcal{K}'$ be two DL-Lite$^{\mathcal{N}}_{bool}$ KBs and $\mathcal{S} = \mathsf{Sig}(\mathcal{K} \cup \mathcal{K}')$. Define the* f-revision *of $\mathcal{K}$ by $\mathcal{K}'$, denoted $\mathcal{K} \circ_f \mathcal{K}'$, such that $\mathsf{MF}(\mathcal{K} \circ_f \mathcal{K}') = \mathsf{MF}(\mathcal{K}')$ if $\mathsf{MF}(\mathcal{K}) = \emptyset$, and otherwise*

$$\begin{aligned} \mathsf{MF}(\mathcal{K} \circ_f \mathcal{K}') = \{ \mathcal{F}' \in \mathsf{MF}(\mathcal{K}') \mid \exists \mathcal{F} \in \mathsf{MF}(\mathcal{K}) \textit{ s.t.} \\ \forall \mathcal{F}_i \in \mathsf{MF}(\mathcal{K}), \forall \mathcal{F}'_j \in \mathsf{MF}(\mathcal{K}'), (\mathcal{F}_i \triangle \mathcal{F}'_j) \not\subset_f (\mathcal{F} \triangle \mathcal{F}') \}. \end{aligned}$$

The next example shows that $\circ_f$ performs better under maximal approximation.

Example 3 *Consider the KBs $\mathcal{K}, \mathcal{K}'$ in Example 1. We can show that the maximal approximation of $\mathcal{K} \circ_f \mathcal{K}'$ is*

$$\begin{aligned} \langle \, \{ & \mathit{PhD} \sqsubseteq \mathit{Student} \sqcap \mathit{Postgrad}, \mathit{PhD} \sqsubseteq \exists \mathit{teaches}, \\ & \mathit{Student} \sqcap \exists \mathit{teaches} \sqsubseteq \mathit{PhD}, \exists \mathit{teaches}^- \sqsubseteq \mathit{Course}, \\ & \mathit{Student} \sqcap \mathit{Course} \sqsubseteq \bot \}, \\ \{ & \mathit{Student}(\mathit{Tom}), \mathit{Postgrad}(\mathit{Tom}) \} \rangle. \end{aligned}$$

Note that $\mathit{Student} \sqsubseteq \neg\exists \mathit{teaches}$ is revised (and weakened) to $\mathit{Student} \sqcap \exists \mathit{teaches} \sqsubseteq \mathit{PhD}$. In this way, consistency is restored, as well as coherence. Also, the knowledge in $\mathcal{K}$ is well preserved.

Theorem 1 *Both revision operators $\circ_s$ and $\circ_f$ satisfy the following AGM postulates:*

(R1) $\mathcal{K} \circ \mathcal{K}' \models_f \mathcal{K}'$;

(R2) *if* $\mathcal{K} \cup \mathcal{K}'$ *is consistent, then* $\mathcal{K} \circ \mathcal{K}' = \mathcal{K} \cup \mathcal{K}'$;

(R3) *if* $\mathcal{K}'$ *is consistent, then* $\mathsf{MF}(\mathcal{K} \circ \mathcal{K}') \neq \emptyset$;

(R4) *if* $\mathcal{K}_1 \equiv \mathcal{K}_2$ *and* $\mathcal{K}'_1 \equiv \mathcal{K}'_2$, *then* $\mathcal{K}_1 \circ \mathcal{K}'_1 \equiv_f \mathcal{K}_2 \circ \mathcal{K}'_2$;

(R5) $(\mathcal{K} \circ \mathcal{K}') \cup \mathcal{K}'' \models_f \mathcal{K} \circ (\mathcal{K}' \cup \mathcal{K}'')$;

(R6) *if* $(\mathcal{K} \circ \mathcal{K}') \cup \mathcal{K}''$ *is consistent, then* $\mathcal{K} \circ (\mathcal{K}' \cup \mathcal{K}'') \models_f (\mathcal{K} \circ \mathcal{K}') \cup \mathcal{K}''$.

We have developed an algorithm for computing the maximal approximation of revision syntactically but have to omit it here due to space constraint.

Conclusion

We have developed a formal framework for revising general KBs (with no specific restriction) in DL-Lite$^{\mathcal{N}}_{bool}$, based on the notion of features. Two specific revision operators are defined, their properties are investigated and algorithms are developed. We note that other propositional revision operators, e. g., Dalal's revision, belief contraction and update can also be easily defined in our framework. It would be interesting to extend the approach proposed in this paper to KB revisions in more expressive DLs. Another problem is to look at applications of our revision operator in nonmonotonic reasoning problems in DLs.

References

Artale, A.; Calvanese, D.; Kontchakov, R.; and Zakharyaschev, M. 2007. DL-Lite in the light of first-order logic. In *Proc. AAAI*, 361–366.

Calvanese, D.; Giacomo, G. D.; Lembo, D.; Lenzerini, M.; and Rosati, R. 2007. Tractable reasoning and efficient query answering in description logics: The DL-Lite family. *J. Autom. Reasoning* 39(3):385–429.

Giacomo, G. D.; Lenzerini, M.; Poggi, A.; and Rosati, R. 2007. On the approximation of instance level update and erasure in description logics. In *Proc. AAAI*, 403–408.

Kontchakov, R.; Wolter, F.; and Zakharyaschev, M. 2008. Can you tell the difference between DL-Lite ontologies? In *Proc. KR*, 285–295. AAAI Press.

Liu, H.; Lutz, C.; Milicic, M.; and Wolter, F. 2006. Updating description logic ABoxes. In *Proc. KR*, 46–56.

Meyer, T.; Lee, K.; and Booth, R. 2005. Knowledge integration for description logics. In *Proc. AAAI*, 645–650.

Qi, G., and Du, J. 2009. Model-based revision operators for terminologies in description logics. In *Proc. IJCAI*, 891–897.

Wang, Z., and Wang, K., and Topor, R., and Pan, J. Z. 2010. Forgetting for knowledge bases in DL-Lite. *Annals of Mathematics and Artificial Intelligence* (accepted for publication). http://hobbit.ict.griffith.edu.au/~kewen/forgetting.pdf.